CONTROL SYSTEM

Introduction	2
Transfer Function	6
Block Diagram	12
Signal Flow Graph	15
Physical Systems	24
Components of a Control System	32
Stability Analysis and Routh Stability Criterion	60
Time Domain Analysis	69
Frequency Response	92
Root Locus	96
Bode Plot	111
Nyquist Plot	127
Control Action	147
Nichols Chart	157
State Variable Analysis	159
Compensators	225
Miscellaneous	234

INTRODUCTION

Multiple Choice Type Questions

1. Feedback control system is basically [WBUT 2009, 2012]
   a) high pass filter b) band pass filter
   c) low pass filter d) band stop filter

Answer: (c)

2. The input-output relationship of a linear system is given by [WBUT 2009]
   a) $y = a_0x^2 + a_1x + a_0$ b) $y = a_1x + a_0$ c) $y = a_1x$ d) $y = a_0$

Answer: (c)

3. Regenerative feedback means the output is feedback with [WBUT 2009]
   a) positive sign b) negative sign c) step input d) oscillation

Answer: (a)

4. When a human being tries to approach an object, his brain acts as [WBUT 2010]
   a) an error measuring device b) a controller
   c) an actuator d) an amplifier

Answer: (b)

5. A servomechanism is [WBUT 2015]
   a) an automatic regulating system b) a position control system
   c) a process control system d) a closed-loop system

Answer: (d)

Short Answer Type Questions

1. a) Draw a general block diagram of a closed loop control system, describe the function of each block. [WBUT 2014]

Answer:

In closed loop control system, the output is fedback to the input through a feedback element to maintain a prescribed relationship between them even in the presence of disturbances. Closed control system uses the difference or error e(t) between controlled variable (output) and set point (input) and sends this generated e(t) to the controller for necessary actions to reduce the error e(t). Thus, the controlled variable is continuously fedback and compared with the input signal. If disturbances are present, naturally the output will change and an e(t) will generate. The controller once again will try to reduce the error. So, the closed loop control system provides a self-corrective process against the variation of output.

Fig. 1. depicts the block diagram of a closed loop control system, which is also known as a fedback control system.

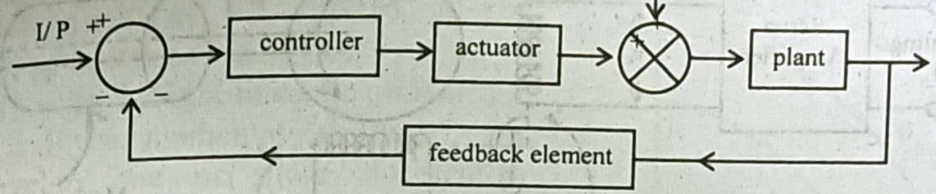


Fig: 1 A closed loop system

b) Compare between the open loop and closed loop control system. [WBUT 2014]

Answer:

Open-loop System	Closed-loop System
(a) No feedback used.	(a) Feedback is there for comparison between desired output and reference input.
(b) Open-loop system are generally stable.	(b) Closed-loop system can become unstable under certain conditions.
(c) Their accuracy is determined by the calibration of their elements. Simple to develop and cheap.	(c) They are more complex. Complicated to construct and costly.
(d) Affected by non-linearities in the system.	(d) Adjust to the effects of non-linearities present in the system.
Examples: Washing machine, fixed time traffic control system, room heater, etc.	Examples: Servomotor control, generator output voltage control system and so on.

Long Answer Type Questions

1. Explain the theory and operation of a two-phase servomotor and explain how a position control scheme can be made up by using the motor. [WBUT 2017]

Answer:

AC Servo

- The AC servomotors are basically two-phase, reversible, induction motors modified for servo operation.
- Ac servomotors are used in applications requiring rapid and accurate response characteristics.
- To achieve these characteristics, these ac servomotors have small diameter, high resistance rotors.
- The ac servomotor's small diameter provides low inertia for fast starts, stops, and reversals.
- High resistance provides nearly linear speed-torque characteristics for accurate servo motor control.

Construction

It has 2 stator coils. The axes of the windings of the two coils are in space quadrature as shown in the following Fig. 1.

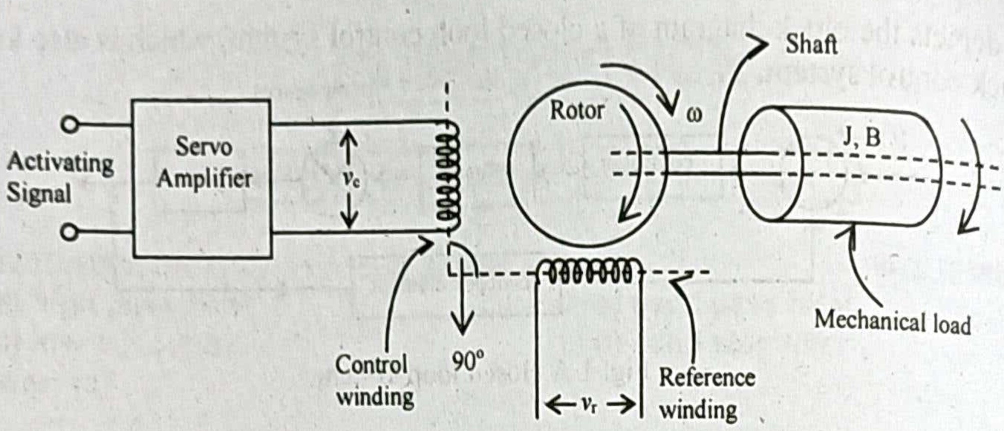


Fig: 1 Schematic diagram of AC servo

One stator coil is called control winding and the other reference winding.
Voltages in the two stator windings are equal in magnitude with a phase difference of 90° Fig. 2.

i.e., v_c = control voltage = $V_m \sin \omega t$

v_r = reference voltage = $V_m \sin(\omega t + 90°)$

This phase difference of 90° in two stator windings produces a rotating magnetic field.

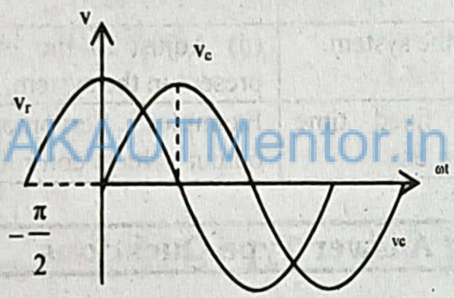


Fig: 2 Voltage waveforms in two stator windings

- ***Power supply***

The two stator windings are normally excited by a $2-\varphi$ power supply.
If a 2φ voltage supply is not available, then single phase supply along with an additional circuit is used to generate a phase difference of 90° between the voltage of two stator windings.

- ***Rotor***

It is a squirrel-cage type having high Electrical Resistance.
Its diameter to length $\left(\frac{d}{l}\right)$ ratio is kept small to reduce the moment of inertia of the rotor. (Fig: 3)

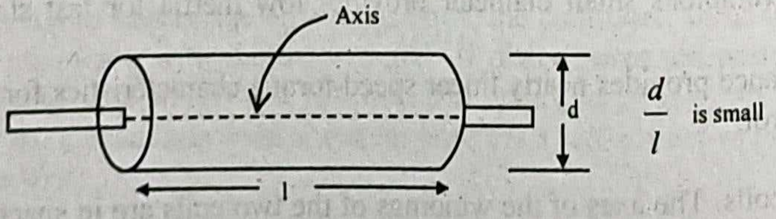


Fig: 3

- ***Characteristic Curves***

A group/set of torque-speed curves are plotted as shown in figure 6.16d when

(i) fixed phase winding is fed with a rated voltage.
(ii) different voltages are applied to the control phase winding.

These curves are not straight lines, i.e., linear. To develop a linear mathematical model, we have to develop one or more differential equations.
So, using linearization techniques these non-linear curves are approximated to linear curves with negative slopes.

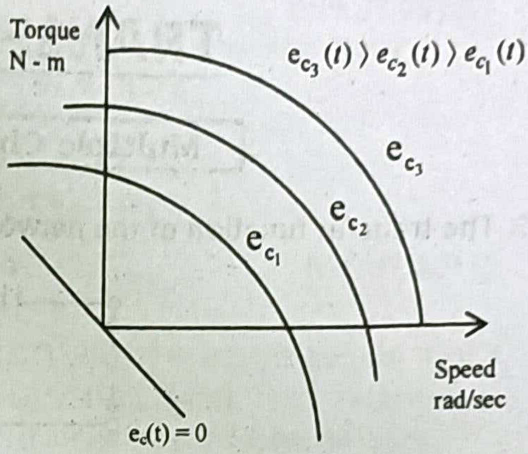


Fig: 4 Characteristic curves

- ***Evaluation of Transfer Function***

Step 1: In ac servo motor, torque T is a function of
(i) Motor's angular speed $\dot{\theta}$
(ii) Control voltage $e_c(t)$

i.e., $T = f\left(\dot{\theta}, e_c(t)\right)$

Considering linearized torque-speed characteristics, the equation for a torque speed line is

$$T = -K\dot{\theta}(t) + K_c e_c(t) \quad \text{.... (1)}$$

where K and K_c are constants.

From free body diagram (Fig. 6) we have the Torque Balance equation as

$$T = J\frac{d^2\theta(t)}{dt^2} + B\frac{d\theta(t)}{dt} \quad \text{.... (2)}$$

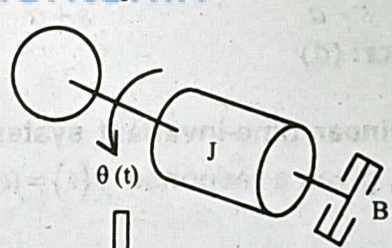


Fig: 5

Step 2: Taking Laplace transform of (1) and (2)

$$T(s) = -Ks\,\Theta(s) + K_c E_c(s) \quad \text{.... (3)}$$

$$T(s) = Js^2\Theta(s) + Bs\Theta(s) \quad \text{.... (4)}$$

From Eqn. (3) and (4), we get

$$Js^2\Theta(s) + Bs\Theta(s) = -Ks\Theta(s) + K_c E_c(s)$$

$$\left(Js^2 + Bs + Ks\right)\Theta(s) = K_c E_c(s) \quad \text{.... (5)}$$

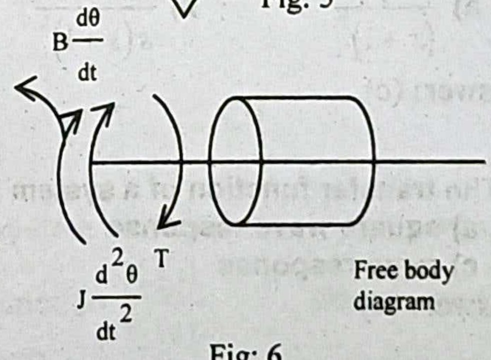


Fig: 6

Step 3: To get the Transfer function.

$$\frac{\Theta(s)}{E_c(s)} = \frac{K_c}{\left(Js^2 + Bs + Ks\right)} \quad \text{(from Eqn. 5)}$$

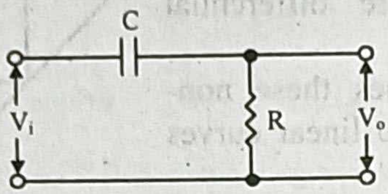
Step 4: To draw the Block diagram.

$E_c(s) \longrightarrow \boxed{K_c/\left(Js^2 + Bs + Ks\right)} \longrightarrow \Theta(s)$

TRANSFER FUNCTION

Multiple Choice Type Questions

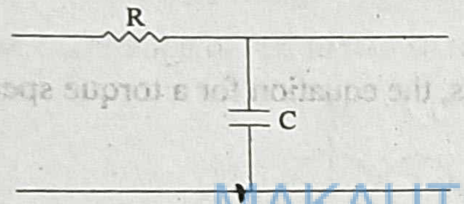
1. The transfer function of the network given below is **[WBUT 2009, 2013, 2016]**



a) $\frac{1}{1+sRC}$ b) $\frac{sRC}{1+sRC}$ c) $\frac{RC}{1+sRC}$ d) $\frac{1+sRC}{1-sRC}$

Answer: (b)

2. Transfer function of a simple R-C integrator circuit shown in figure is given by **[WBUT 2011]**



$a=\frac{1}{RC}$

a) $\frac{1}{s-a}$ b) $\frac{1}{s+a}$ c) $\frac{a}{s-a}$ d) $\frac{a}{s+a}$

Answer: (d)

3. A linear time-invariant system initially at rest, when subjected to an unit step input, gives a response $y(t)=te^{-t}$, the transfer function of the system is **[WBUT 2017]**

a) $\frac{1}{(s+1)^2}$ b) $\frac{1}{s(s+1)^2}$ c) $\frac{s}{(s+1)^2}$ d) $\frac{1}{s(s+1)}$

Answer: (c)

4. The transfer function of a system is its **[WBUT 2017]**

a) square wave response
b) step response
c) ramp response
d) impulse response

Answer: (d)

5. The phase margin of the system for which the loop gain $GH(j\omega)=\frac{1}{(1+j\omega)^3}$ is **[WBUT 2017]**

a) $-\pi$ b) π c) 0 d) $\pi/2$

Answer: (d)

6. A linear time invariant system obeys **[WBUT 2019]**
a) the principle of superposition **b) the principle of homogeneity**
c) both of the principles in (a) and (b) **d) none of these**

Answer: (c)

Short Answer Type Questions

1. Define the transfer function of a system. **[WBUT 2014]**

Answer:
The Transfer Function of a Linear Time Invariant (LTI) control system is defined as the ratio of the Laplace Transform of the response function to the Laplace Transformation of driving or excitation function under the assumption that all initial conditions are zero.

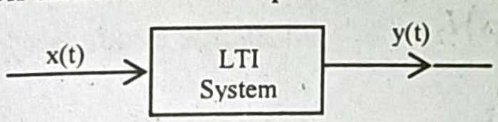


Fig: 1 LTI system

Let
x(t) is the driving Function and y(t) is the response Function.
A linear time-invariant system may be defined by the following differential equation.

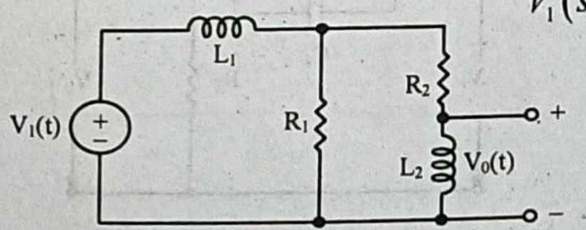
$$a_0\frac{d^n y}{dt^n}+a_1\frac{d^{n-1}y}{dt^{n-1}}+.....a_{n-1}\frac{dy}{dt}+a_n y$$
$$=b_0\frac{d^m x}{dt^m}+b_1\frac{d^{m-1}x}{dt^{m-1}}+......+b_m x \quad ......(1)$$
$$(n \ge m)$$

Initial conditions are zero.
Taking Laplace transformation of equation (1):
$a_0 s^n Y(s) + a_1 s^{n-1} Y(s) + ...+ a_{n-1} sY(s) + a_n Y(s)$
$= b_0 s^m X(s) + b_1 s^{m-1} X(s)+....+b_{n-1} sY(s) + b_n X(s)$
or, $Y(s)\,[a_0 s^n + a_1 s^{n-1} +.....+ a_{n-1}s + a_n] = X(s)\,[\,b_0 s^m + b_1 s^{m-1} + ...+ b_{n-1}s + b_n]$

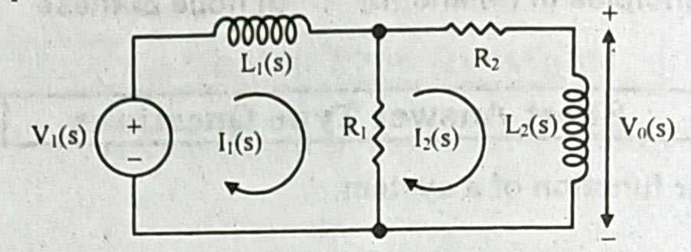
or, $\dfrac{Y(s)}{X(s)}=\dfrac{\text{Laplace Transformation of response function}}{\text{Laplace Transformation of driving function}}$ | Initial conditions = 0

$$= T.F. = G(s) = \frac{\left[b_0 s^m + b_1 s^{m-1} + ... + b_{n-1}s + b_n\right]}{\left[a_0 s^n + a_1 s^n + ... + a_{n-1}s + a_n\right]} \quad ..... (2)$$

2. Draw a block diagram representation for the system shown below, representing every element by a block. Also find the transfer function $\dfrac{V_0(s)}{V_1(s)}$. **[WBUT 2015]**



Answer:
The block diagram representation for the system is shown below:



$$V_1(s) = I_1(L_1 s + R_1) - I_2 R_1 \quad \text{.... (1)}$$

$$0 = (R_1 + R_2 + L_2 s) I_2 - I_1 R_1$$

$$\therefore \quad I_1 R_1 = (R_1 + R_2 + L_2 s) I_2$$

$$\therefore \quad I_1 = \frac{R_1 + R_2 + L_2 s}{R_1} I_2$$

$\therefore$ From Eqn. (1) we get

$$V_1(s) = \left(\frac{R_1 + R_2 + L_2 s}{R_1}\right)(L_1 s + R_1) I_2 - R_1 I_2$$

$$= \frac{R_1 L_1 s + R_1^2 + R_2 L_1 s + R_1 R_2 + L_1 L_2 s^2 + R_1 L_2 s - R_1^2}{R_1} \cdot I_2$$

$$= \frac{R_1 R_2 + R_1 L_1 s + R_1 L_2 s + R_2 L_1 s + L_1 L_2 s^2}{R_1} \cdot I_2$$

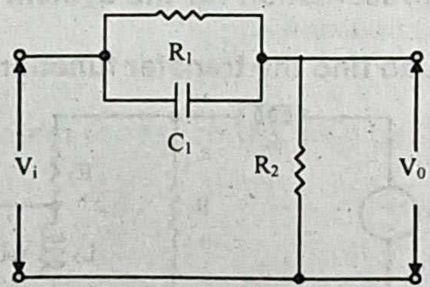
$$\therefore \quad V_0(s) = I_2 \cdot L_2 s = \frac{R_1 L_2 s V_1(s)}{R_1 R_2 + R_1 L_1 s + R_1 L_2 s + R_2 L_1 s + L_1 L_2 s^2}$$

The transfer function

$$\frac{V_0(s)}{V_1(s)} = \frac{R_1 L_2 s}{R_1 R_2 + R_1 L_1 s + R_1 L_2 s + R_2 L_1 s + L_1 L_2 2s^2} \quad \textbf{(Ans.)}$$

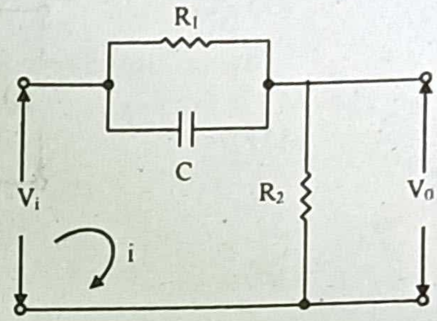
3. Derive the transfer function $\frac{V_0(s)}{V_i(s)}$, for the electrical network shown below.

[WBUT 2016]



Answer:

It Z_1 is the equivalent impedance of the parallel combination of R_1 and C then



$$\frac{1}{z_1}=\frac{1}{R_1}+j\omega C$$

$$\therefore Z_1=\frac{R_1}{1+j\omega CR_1}=\frac{R_1}{1+sCR_1}(j\omega=s)$$

Writing differential equations by Kirchoff's laws

We get

$$V_i(t)=Z_1 i(t)+R_2 i(t)$$

$$V_0(t)=R_2 i(t)$$

Taking Laplace transform of the above equations.

We get

$$v_i(s)=Z_1 I(s)+R_2 I(s)$$

and $v_0(s)=R_2 I(s)$ $\quad\quad \therefore I(s)=\dfrac{v_0(s)}{R_2}$

or $\quad v_i(s)=z_1\dfrac{v_0(s)}{R_2}+v_0(s)$

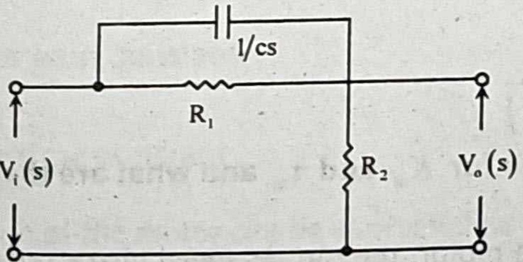
or $\quad v_i(s)=v_0(s)\left[\dfrac{z_1}{R_2}+1\right]$

Substituting the value of z_1 we get

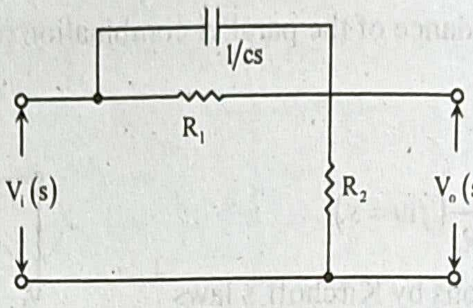
$$v_i(s)=v_0(s)\left(\frac{R_1}{1+SCR_1}\frac{1}{R_2}+1\right)$$

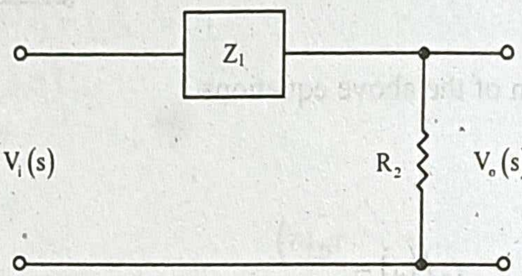
$$\frac{v_i(s)}{v_0(s)}=\frac{R_2(1+SCR_1)}{R_1+R_2+SCR_1R_2}=\frac{R_2}{R_1+R_2}\left[\frac{1+SCR_1}{1+\dfrac{SCR_1R_2}{R_1+R_2}}\right]$$

4. Find the transfer function of the system shown in the figure: **[WBUT 2018]**



Answer:





where impedance $Z_1 = R_1 \parallel \frac{1}{cs} = \frac{\frac{R_1}{cs}}{R_1 + \frac{1}{cs}} = \frac{R_1}{1 + R_1 cs}$

$$\therefore \quad V_o(s) = \frac{R_2 V_i(s)}{R_2 + Z_1}$$

$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + \frac{R_1}{1 + R_1 cs}}$$

$$\therefore \text{ Transfer function, } \quad \frac{V_o(s)}{V_i(s)} = \frac{R_2(1 + R_1 cs)}{R_1 + R_2(1 + R_1 cs)}$$

Long Answer Type Questions

1. Show that the transfer function of a two-phase induction motor can be written in form

$$\frac{\theta_m(s)}{V_z(s)} = \frac{K_m}{s(1 + s\tau_m)}$$

What are the expressions for K_m and τ_m and what are they called? [WBUT 2018]

Answer:

For a two-phase induction motor, the torque-speed characteristic shows that at a specific winding voltage, torque varies linearly with the speed, however, the slope is negative as shown in the figure below. The torque characteristics can be linearised at the operating point and transfer function of the motor can be established about that point. In general, the torque is a function of speed and the control voltage

$$T_M = f(\dot{\theta}, E) \qquad \text{.... (1)}$$

Expanding the above Eqn. (1), in Taylor's series about the operating point, we

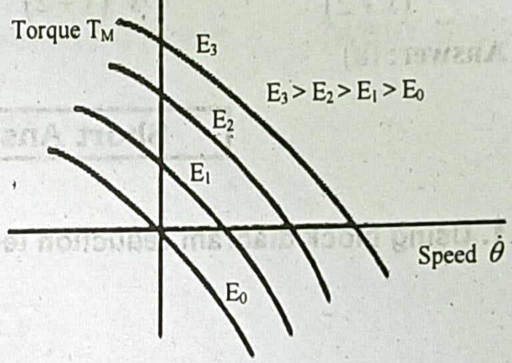
$$T_M = T_{MO} + \left.\frac{\partial T_M}{\partial E}\right|_{\substack{E=E_0\\ \dot{\theta}=\theta_0}} (E - E_0) + \left.\frac{\partial T_M}{\partial \dot{\theta}}\right|_{\substack{E=E_0\\ \dot{\theta}=\theta_0}} (\dot{\theta} - \dot{\theta}_0) \qquad \text{.... (2)}$$

The Eqn. (2) can be written as

$$T_M = T_{MO} = K(E - E_0) - B(\dot{\theta} - \dot{\theta}_0) \qquad \text{.... (3)}$$

where, T_{MO} = torque, E_0 = voltage and $\dot{\theta}$ = speed at the operating point and

$$K = \left.\frac{\partial T_M}{\partial E}\right|_{\substack{E=E_0\\ \dot{\theta}=\theta_0}} \text{ and } B = \left.\frac{\partial T_M}{\partial \dot{\theta}}\right|_{\substack{E=E_0\\ \dot{\theta}=\theta_0}}$$



Since, $\frac{\partial T_M}{\partial \dot{\theta}}$ is negative, B will be positive

The Eqn. (3) can be presented as

$$\Delta T_M^{'} = K\Delta E - B\Delta\dot{\theta}$$

From the mechanical section of the motor,

The torque experienced by the motor has to overcome the inertial force (due to inertia J) of the moving parts and damping force (dissipative in nature) with the damping coefficient B_0. Therefore, the mechanical equation can be written as

$$J\Delta\ddot{\theta} + B_0\Delta\dot{\theta} = \Delta T_M$$

or, $$J\Delta\ddot{\theta} + B_0\Delta\dot{\theta} = K\Delta E - B\Delta\dot{\theta}$$

Taking Laplace transform at both sides of the mechanical equation

$$Js^2\theta(s) + (B_0 + B)s\theta(s) = KE(s)$$

$$\Rightarrow \frac{\theta(s)}{E(s)} = \frac{K}{s[Js + (B + B_0)]} \quad \Rightarrow \quad \frac{\theta(s)}{E(s)} = \frac{\frac{K}{(B+B_0)}}{s\left[\frac{J}{B+B_0}s + 1\right]} = \frac{K_m}{s(\tau_m s + 1)}$$

where $K_m = \frac{K}{B_0 + B}$ = Motor gain constant

and $\tau_m = \frac{J}{B_0 + B}$ = Motor time constant

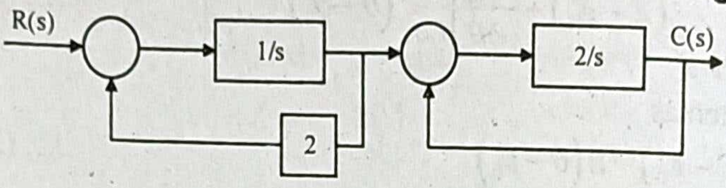
Therefore, the transfer function of the motor can be expressed as

$$\frac{\theta(s)}{E(s)} = \frac{K_m}{s(\tau_m s + 1)}$$

BLOCK DIAGRAM

Multiple Choice Type Questions

1. The transfer function of a system shown in the block-diagram is **[WBUT 2015]**

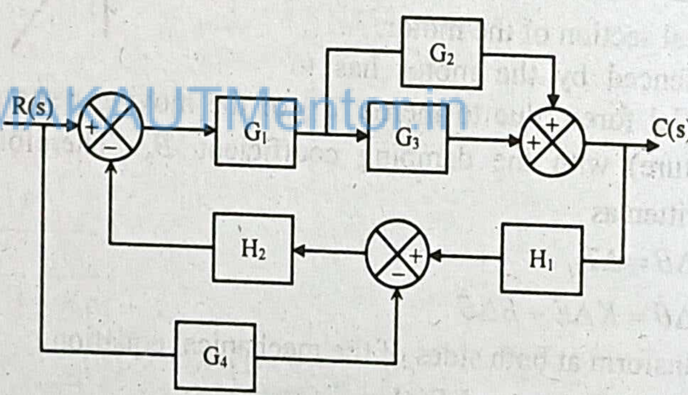


a) $\frac{2}{(s+2)^2}$ b) $\frac{1}{s^2(s+2)}$ c) $\frac{2}{s^2+2}$ d) $\frac{s}{s^2+2}$

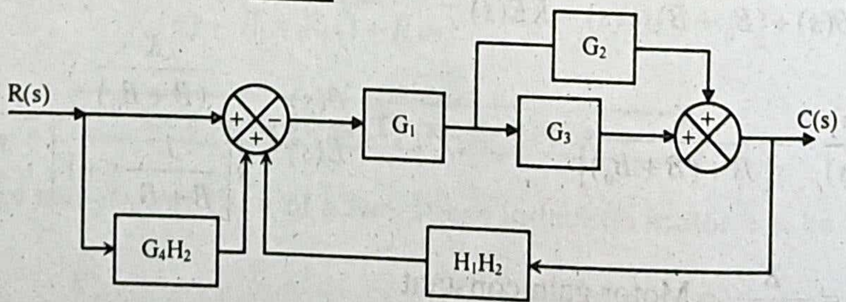
Answer: (a)

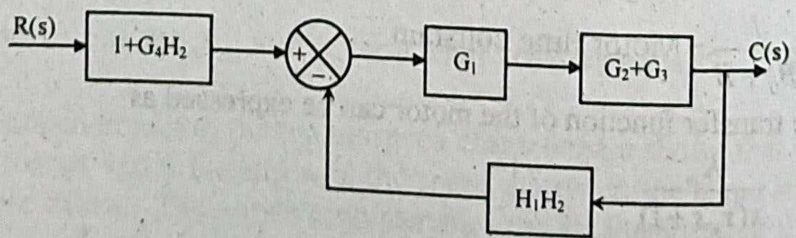
Short Answer Type Questions

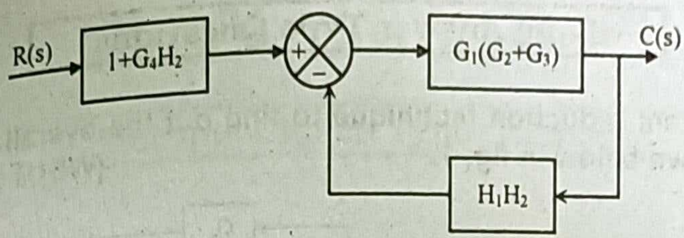
1. Using block diagram reduction technique find $\frac{C}{R}$. **[WBUT 2010]**

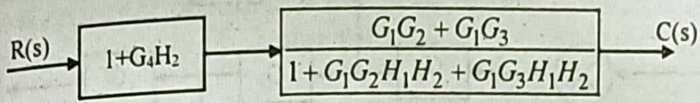


Answer:









$$\therefore \frac{C(s)}{R(s)} = \frac{G_1(1+G_4H_2)(G_2+G_3)}{1+G_1G_2H_1H_2+G_1G_3H_1H_2}$$

2. Derive the transfer function of armature controlled DC Motor. **[WBUT 2019]**

Answer:

Transfer function of an armature controlled DC motor

$$\text{CLTF} = \frac{H(s)}{E_a(s)} = \frac{G(s)}{1+G(s)H(s)}$$

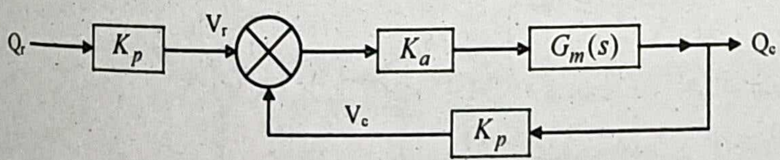
where $G(s) = \text{Forward path T.F.} = \dfrac{K_a}{s(Js+B)(L_as+R_a)}$

$H(s) = \text{Feedback path T.F.} = K_b s$

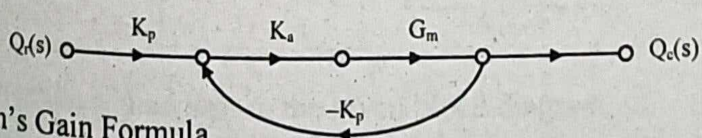
$$\frac{H(s)}{E_a(s)} = \frac{\dfrac{K_a}{s(Js+B)(L_as+R_a)}}{1+\dfrac{K_a}{s(Js+B)(L_as+R_a)} \times K_b s}$$

$$= \frac{K_a}{s\left[JL_as^2+(L_aB+R_aJ)s\right]+K_aK_bs} = \frac{K_a}{s\left[JL_as^2+(R_aJ+L_aB)s+K_aK_b\right]}$$

The block diagram of the complete system is shown below:



The SFG can be drawn as



Using Mason's Gain Formula

$$\frac{Q_c(s)}{Q_r(s)} = \frac{K_pK_aG_m}{1+K_aK_pG_m}$$

Long Answer Type Questions

1. Use block diagram reduction technique to find out the overall transfer function of the system shown below in fig. 1. [WBUT 2006, 2007, 2017]

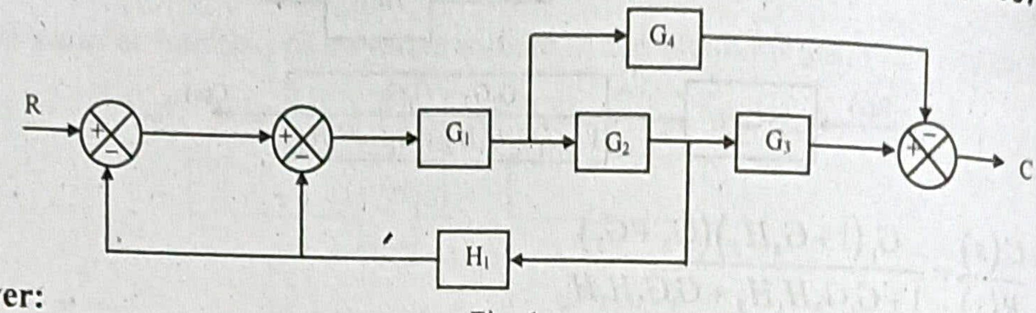
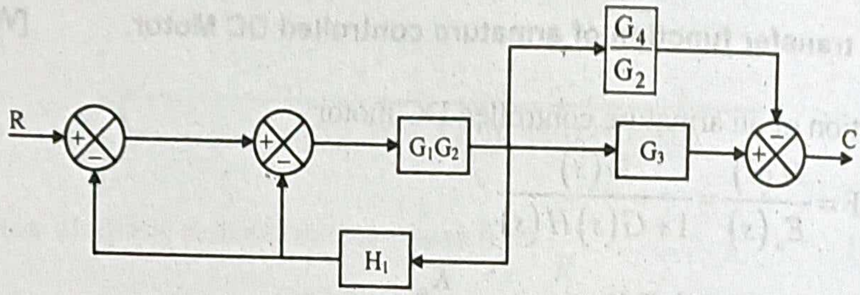
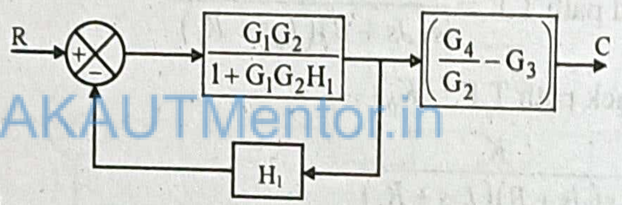
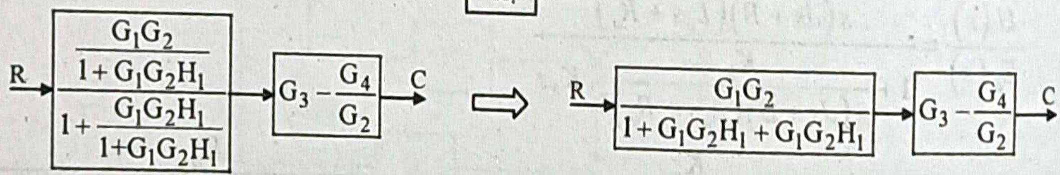


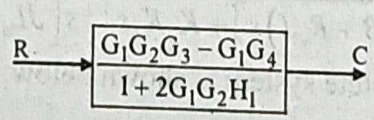
Fig: 1

Answer:









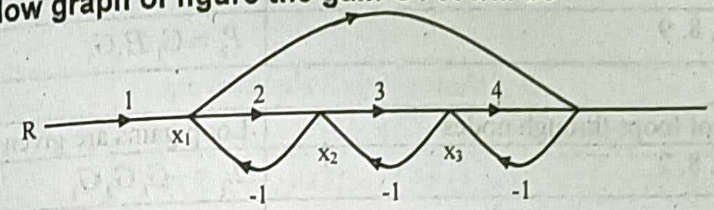
SIGNAL FLOW GRAPH

Multiple Choice Type Questions

1. Signal flow graph is [WBUT 2009, 2013]
   a) topological representation of a set of differential equations
   b) Bode plot c) polar plot d) locus of roots

Answer: (a)

2. In the signal flow graph of figure the gain C / R will be [WBUT 2012]



a) 11/9 b) 22/15 c) 24/23 d) 44/23

Answer: (d)

3. Signal flow graph is used to obtain the [WBUT 2013]
   a) Stability of a system b) Transfer function of a system
   c) Controllability of a system d) Observability of a system

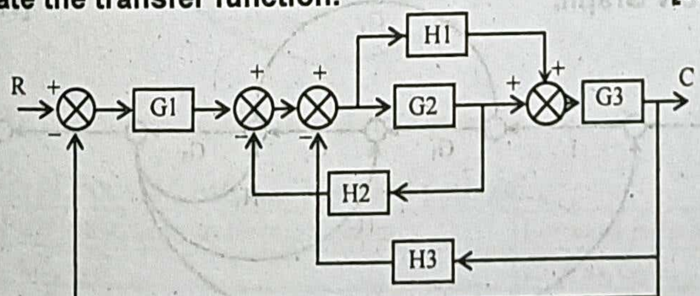
Answer: (b)

4. A signal flow graph is used to determine the [WBUT 2015]
   a) steady state error in the system b) stability of the system
   c) transfer function of the system d) dynamic error co-efficient

Answer: (c)

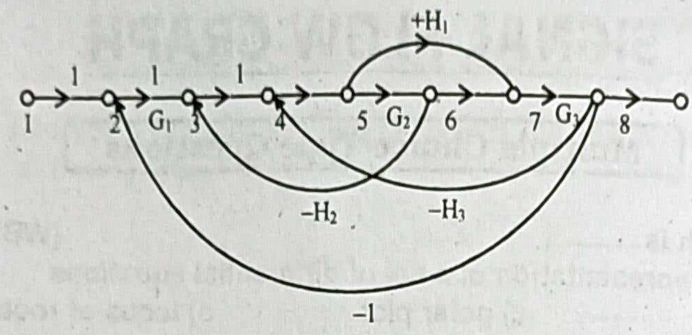
Short Answer Type Questions

1. Construct an equivalent signal flow graph for the block diagram shown in figure below and evaluate the transfer function. [WBUT 2008, 2012]



Answer:

Signal flow graph is drawn from the given block diagram



Identification of forward paths through nodes	Forward path gains are given by
1, 2, 3, 4, 5, 6, 7, 8, 9	$P_1 = G_1 G_2 G_3$
1, 2, 3, 4, 5, 7, 8, 9	$P_2 = G_1 H_1 G_3$

Identification of loops through nodes	Loop gains are given by
2, 3, 4, 5, 6, 7, 8, 2	$L_1 = -G_1 G_2 G_3$
3, 4, 5, 6, 3	$L_2 = -G_2 H_2$
4, 5, 6, 7, 8, 4	$L_3 = -G_2 G_3 H_3$
2, 3, 4, 5, 7, 8, 2	$L_4 = -G_1 H_1 G_3$
4, 5, 7, 8, 4	$L_5 = -H_1 G_3 H_3$

No non-touching loop is there in the drawn signal flow graph.

$$\Delta = 1-(L_1+L_2+L_3+L_4+L_5) = 1+G_1G_2G_3+G_1G_3H_1+G_2H_2+G_2G_3H_3+G_3H_1H_3$$

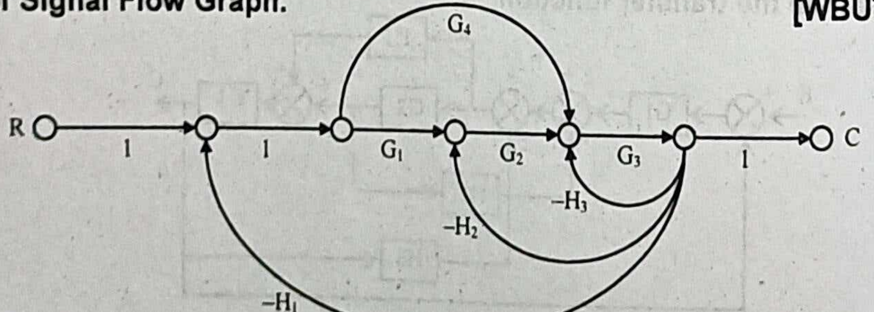
$\Delta_1 = 1;$

$\Delta_2 = 1$

As per Mason's gain formula

$$T = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 + G_1H_1G_3}{1+G_1G_2G_3+G_1H_1G_3+G_2H_2+G_2G_3H_3+G_3H_1H_2}.$$

2. Find out the overall transfer function C/R of the following system using the rules of Signal Flow Graph. **[WBUT 2009, 2013]**

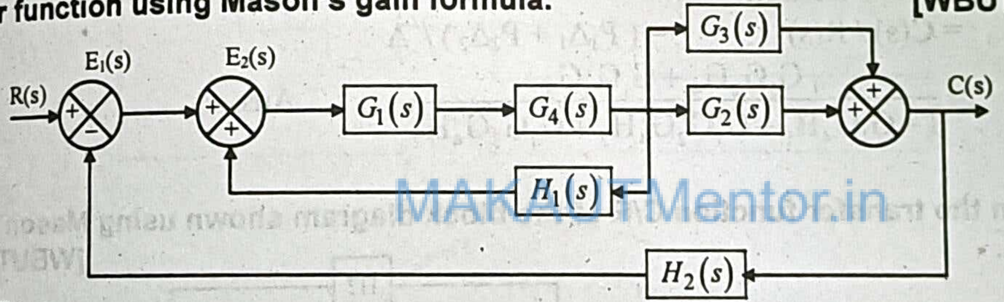


Answer:

Forward path gains	$P_1 = G_1 \times G_2 \times G_3$ $P_2 = G_4 \times G_3$

Loop gains	$L_1 = -(G_1 \times G_2 \times G_3 \times H_1)$ $L_2 = -(G2 \times G_3 \times H_2)$ $L_3 = -(G_3 \times H_3)$ $L_4 = -(G_4 \times G_3 \times H_1)$
Non touching loops	NIL
Finding Δ	$\Delta = \begin{bmatrix} 1 + G_1 \times G_2 \times G_3 \times H_1 + G_1 \times G_2 \times G_3 \times H_1 + G_3 \times H_3 \\ + G_4 \times G_3 \times H_1 \end{bmatrix}$
Co-factors	$\Delta_1 = 1$ $\Delta_2 = 1$
T.F.	Using Mason's Gain Formula $\dfrac{(G_1 \times G_2 \times G_3 + G_4 \times G_3)}{1 + G_1 \times G_2 \times G_3 \times H_1 + G_1 \times G_2 \times G_3 \times H_1 + G_3 \times H_3 + G_4 \times G_3 \times H_1}$

3. Convert the following block diagram into its signal flow graph and find its transfer function using Mason's gain formula. **[WBUT 2014]**



Answer:

Step I: Represent the system variables or signals by nodes (Fig: below)

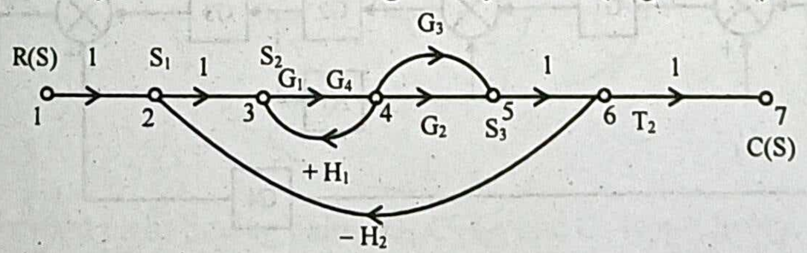


Fig: Signal flow graph

Step II: Evaluation of Forward paths & Forward path gains:

Forward Path Identification through nodes	Forward path gains
a) 1, 2, 3, 4, 5, 6, 7	$P1 = G_1G_2G_4$
b) 1, 2, 3, 4, 5, 6, 7	$P2 = G_1G_4G_3$

Step III: Evaluation of loops & Loop gains:

Loop Identification through nodes	Loop gains
a) 3, 4, 3	$L_1 = G_1G_4H_1$
b) 2, 3, 4, 5, 6, 2	$L_2 = -G_1G_2G_4H_2$
c) 2, 3, 4, 5, 6, 2	$L_3 = -G_1G_3G_4H_2$

Step IV: Evaluation of Non-Touching Loops:
The given signal flow graph has no non-touching loop as all the three loops passes a least one common node.

Step V: Evaluation of Δ:

$$\Delta = 1 - (L_1 + L_2 + L_3) = 1 - (G_1G_4H_1 - G_1G_2G_4H_2 - G_1G_3G_4H_2)$$
$$= 1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2$$

Step VI: Evaluation of Co-factors:
As the no of forward path is 2, number of Co-factors will also be 2.
Cofactor for forward path 1:
All the three loops touch forward path P_1 (As P_1 and all the loops are having common nodes)

$\therefore \quad \Delta_1 = 1$

Cofactor for forward path 2:
All the three loops touch forward path P_2 (As P_2 and all the loops are having common nodes)

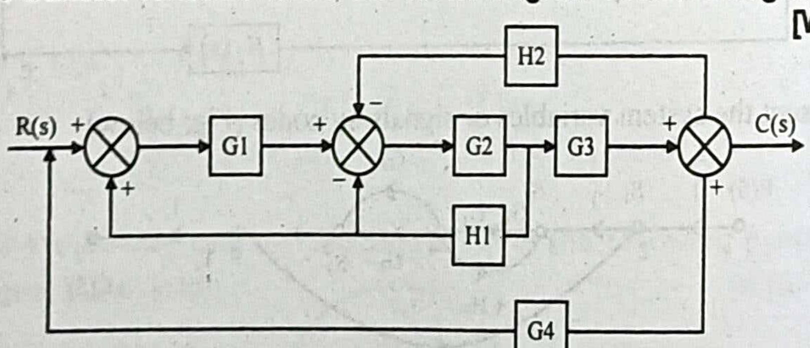
$\therefore \quad \Delta_2 = 1$

Step VII: From Mason's Gain Formula:
Over all Transfer Function

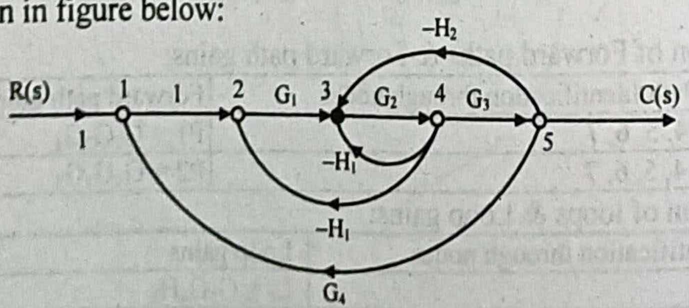
$$= C(s)/R(s) \quad = (P_1\Delta_1 + P_2\Delta_2)/\Delta$$

$$= \frac{G_1G_2G_4 + G_1G_4G_3}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2} \quad \text{...Ans.}$$

4. Obtain the transfer function C/R of the block diagram shown using Mason's Gain Formula. **[WBUT 2015]**



Answer:
The SFG is shown in figure below:



Forward path gains $\quad (P_1) = G_1G_2G_3$

Loop gains are:

$$L_1 = G_1G_2G_3G_4$$
$$L_2 = -G_1G_2H_1$$
$$L_3 = -G_2H_1$$
$$L_4 = -G_2G_3H_2$$

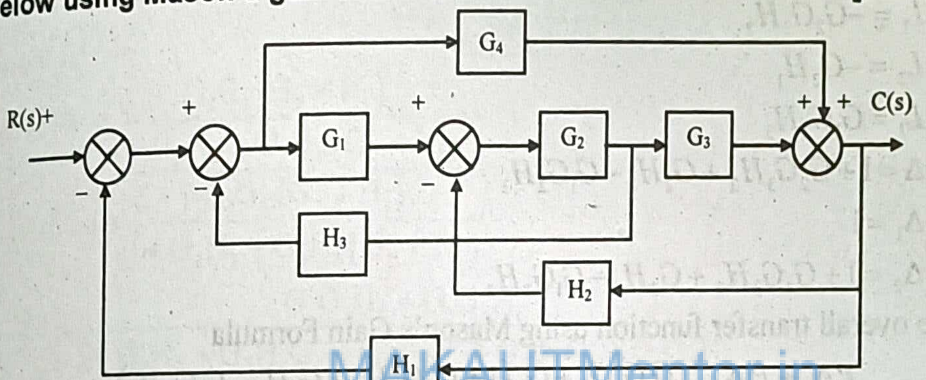
$\therefore$

$$\Delta = 1-(L_1+L_2+L_3+L_4) = 1-G_1G_2G_3G_4+G_1G_2H_1+G_2H_1+G_2G_3H_2$$

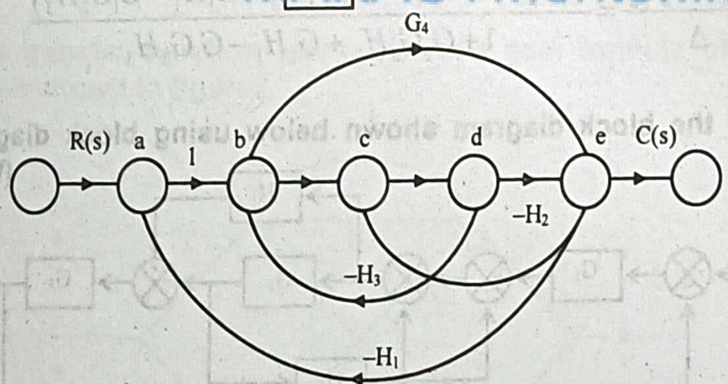
$\therefore$ Over all transfer function

$$=\frac{C(s)}{R(s)}=\frac{P_1\Delta_1}{\Delta}=\frac{G_1G_2G_3}{1-G_1G_2G_3G_4+G_1G_2H_1+G_2H_1+G_2G_3H_2}$$

5. Find the overall transfer function of the system whose block diagram is given in figure below using Mason's gain formula. [WBUT 2016]



Answer:



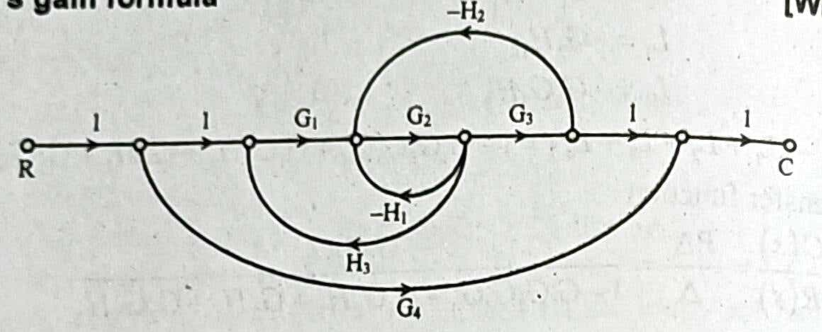
There are two forward paths having gains. $G_1G_2G_3$ and G_4 respectively.

Loops	Gain
1. b-c-d-e-a-b	$G_1G_2G_3(-H_1) = -G_1G_2G_3H_1$
2. b-c-d-b	$G_1G_2(-H_3) = -G_1G_2H_3$
3. c-d-e-c	$G_2G_3(-H_2) = -G_2G_3H_2$
4. b-e-a-b	$G_4(-H_1) = -G_4H_1$
5. c-d-b-e-c	$G_2(-H_3)G_4(-H_2) = G_2G_4H_2H_3$

Non touching loops Nil

$$\therefore \frac{C(s)}{R(s)}=\frac{G_1G_2G_3+G_4}{1+G_1G_2G_3H_1+G_1G_2H_3+G_2G_3H_2+G_4H_1-G_2G_4H_2H_3}$$

6. Determine the overall transfer function of the signal flow graph given below using Mason's gain formula **[WBUT 2018]**



Answer:

$P_1 = G_1G_2G_3$

$P_2 = G_4$

$L_1 = -G_2G_3H_2$

$L_2 = -G_2H_1$

$L_3 = G_1G_2H_3$

$\Delta = 1 + G_2G_3H_2 + G_2H_1 - G_1G_2H_3$

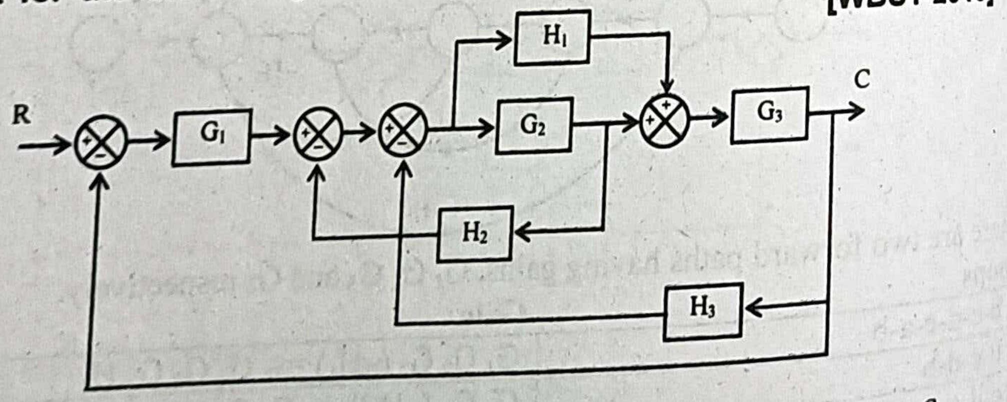
$\Delta_1 = 1$

$\Delta_2 = 1 + G_2G_3H_2 + G_2H_1 - G_1G_2H_3$

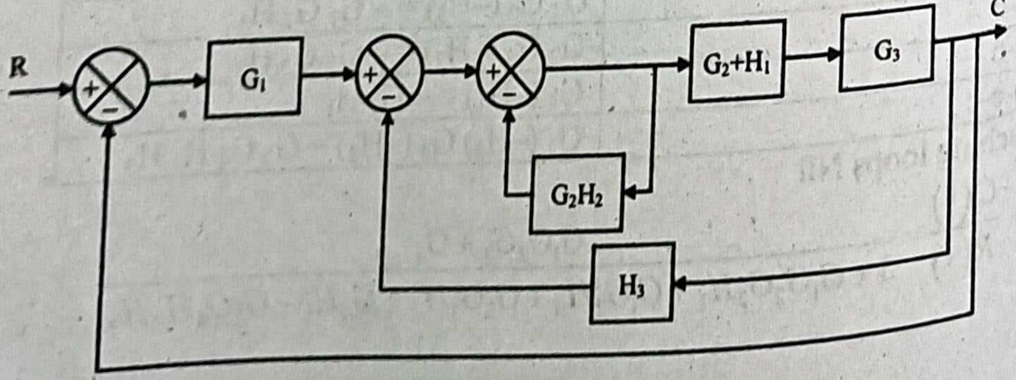
$\therefore$ So the overall transfer function using Mason's Gain Formula

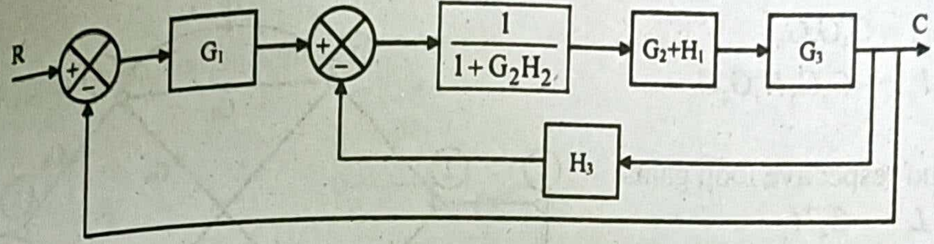
$$TF = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 + G_4\left(1 + G_2G_3H_2 + G_1H_1 - G_1G_2H_3\right)}{1 + G_2G_3H_2 + G_1H_1 - G_1G_2H_3}$$

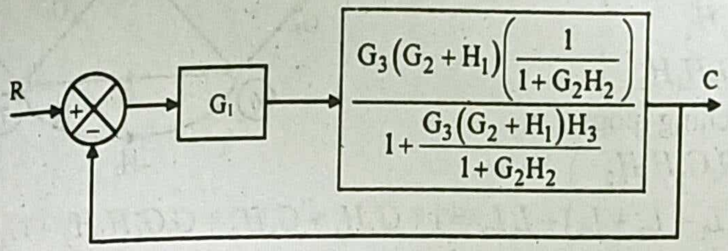
7. Find C/R for the block diagram shown below using block diagram reduction techniques. **[WBUT 2019]**

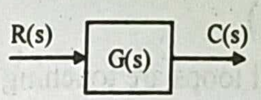


Answer:





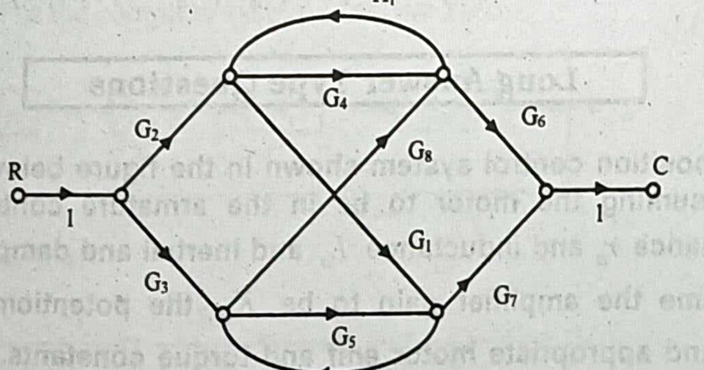




$$G(s)=\frac{\dfrac{G_1G_3(G_2+H_1)}{1+G_2H_2+G_3H_3(G_2+H_1)}}{1+\dfrac{G_1G_3(G_2+H_1)}{1+G_2H_2+G_3H_3(G_2+H_1)}}$$

$$=\frac{G_1G_3G_2+G_1G_3H_1}{1+G_2H_2+G_3H_3G_2+G_3H_3H_1+G_1G_2G_3+G_1G_3H_1}$$

8. Evaluate the transfer function using Mason's gain formula of the equivalent signal-flow graph shown in figure. **[WBUT 2019]**



Answer:

Step 1: Forward paths and respective gains

$P_1=G_2G_4G_6$

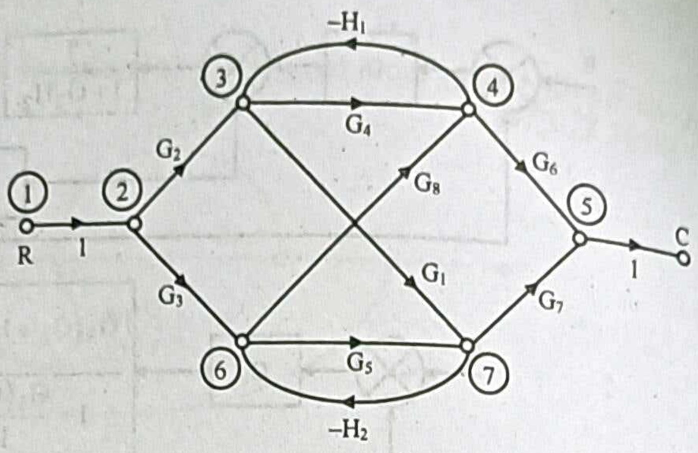
$P_2=G_2G_1G_7$

$P_3=-G_2G_4H_2G_8G_6$

$P_4=G_3G_5G_7$

$P_5 = G_3G_8G_6$

$P_6 = -G_3G_5H_1G_4G_6$



Step 2:
Loops and respective loop gains

$L_1 = -G_4H_1$

$L_2 = -G_5H_2$

$L_3 = G_1G_8H_1H_2$

Step 3: Non-touching loops

$L_1L_2 = G_4G_5H_1H_2$

Step 4: $\Delta = 1-(L_1+L_2+L_3)+L_1L_2 = 1+G_4H_1+G_5H_2+G_1G_5H_1H_2+G_4G_5H_1H_2$

Step 5: $\Delta_1 = (1-L_2)(1+G_5H_2)$

$\Delta_2 = (1)$ [As all loops are touching them forward path]

$\Delta_3 = (1-0) = 1$ [As all loops are touching them forward path]

$\Delta_4 = (1-L_1) = (1+G_4H_1)$

$\Delta_5 = (1-0) = 1$ [As all loops are touching them forward path]

$\Delta_6 = (1-0) = 1$ [As all loops are touching them forward path]

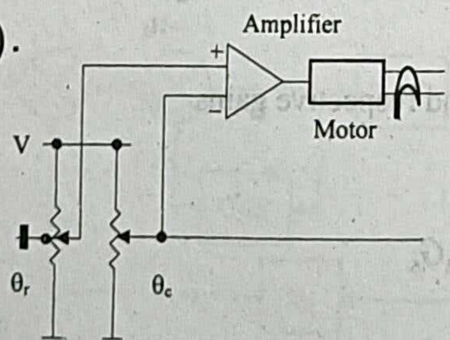
Step 6: Overall transfer function using Mason's gain formula

$$= \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

$$= \frac{G_2G_4G_6(1+G_5H_2)+G_2G_1G_7-G_2G_4G_8G_6H_2+G_3G_5G_7+G_3G_6G_8-G_3G_8G_4G_6H_1}{1+G_4H_1+G_5H_2+G_1G_8H_1H_2+G_4G_5H_1H_2}$$

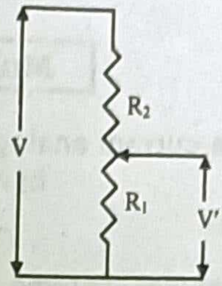
Long Answer Type Questions

1. For the DC position control system shown in the figure below draw the signal flow graph assuming the motor to be in the armature controlled mode with armature resistance r_a and inductance L_a and inertial and damping constants J_m and F_m. Assume the amplifier gain to be K_a, the potentiometer gains to be K_p V/radians and appropriate motor emf and torque constants. Hence derive the transfer function $Q_c(s)/Q_r(s)$. **[WBUT 2011]**

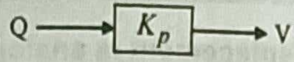


Answer:

Transfer function of a potentiometer



$$Q = KV$$



$$V' = \frac{R_1}{R_1 + R_2} \cdot V$$

Again $V' = K'Q$

$$\therefore \quad Q = \frac{1}{K'}\left(\frac{R_1}{R_1 + R_2}\right) \cdot V = K_p V$$

Transfer function of an armature controlled DC motor

$$\text{CLTF} = \frac{H(s)}{E_a(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

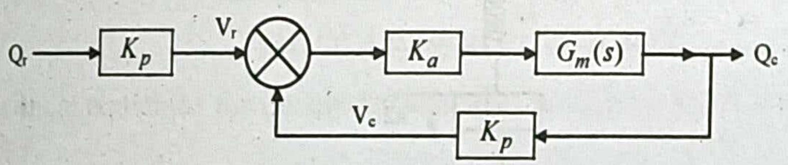
where $G(s) = \text{Forward path T.F.} = \dfrac{K_a}{s(Js + B)(L_a s + R_a)}$

$H(s) = \text{Feedback path T.F.} = K_b s$

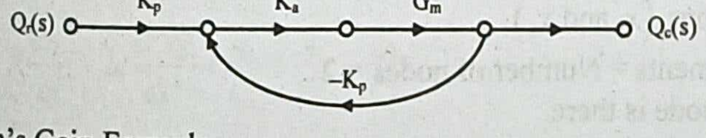
$$\frac{H(s)}{E_a(s)} = \frac{\dfrac{K_a}{s(Js + B)(L_a s + R_a)}}{1 + \dfrac{K_a}{s(Js + B)(L_a s + R_a)} \times K_b s}$$

$$= \frac{K_a}{s\left[JL_a s^2 + (L_a B + R_a J)s\right] + K_a K_b s} = \frac{K_a}{s\left[JL_a s^2 + (R_a J + L_a B)s + K_a K_b\right]}$$

The block diagram of the complete system is shown below:



The SFG can be drawn as



Using Mason's Gain Formula

$$\frac{Q_c(s)}{Q_r(s)} = \frac{K_p K_a G_m}{1 + K_a K_p G_m}$$

PHYSICAL SYSTEMS

Multiple Choice Type Questions

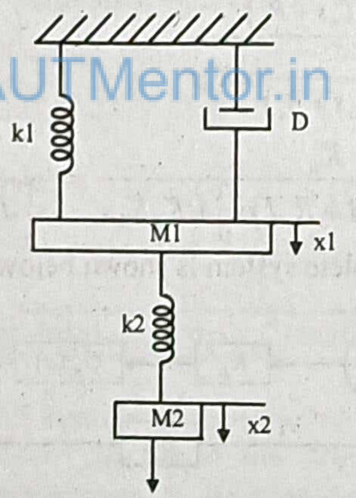
1. In torque-current analogy, displacement is analogous to [WBUT 2010, 2013]
a) flux b) moment of inertia c) voltage d) current
Answer: (a)

2. In mechanical systems, the spring force is proportional to [WBUT 2010]
a) motion b) displacement c) acceleration d) mass
Answer: (b)

3. The capacitance, in force-current analogy is analogous to [WBUT 2016]
a) velocity b) momentum c) displacement d) mass
Answer: (d)

Short Answer Type Questions

1. Draw the electrical analogous circuit using force-voltage analogy for the mechanical system shown in the figure. [WBUT 2012]



Answer:
1st Part:
To obtain nodal equations $(x_1 \text{ and } x_2)$
1: Number of displacements = Number of nodes = 2
In addition, reference node is there.
2: M_1 is connected to x_1, M_2 is connected to x_2.
3: K_1 is connected to x_1 and reference, K_2 is connected to x_1 and x_2.
4: B is connected to x_1 and reference.
5: Force $f(t)$ is applied on M_1. So, with this information we draw the equivalent mechanical network [figure (a)].
6: From the mechanical network of figure (a),

For Node 'x_1':

$$f(t) = M_1\ddot{x}_1 + B\dot{x}_1 + K_1x_1 + K_2(x_1 - x_2) \quad \text{.... (i)}$$

Taking Laplace transform, we get

$$F(s) = (s^2M_1 + Bs + K_1 + K_2)X_1(s) - K_2X_2(s) \quad \text{.... (ii)}$$

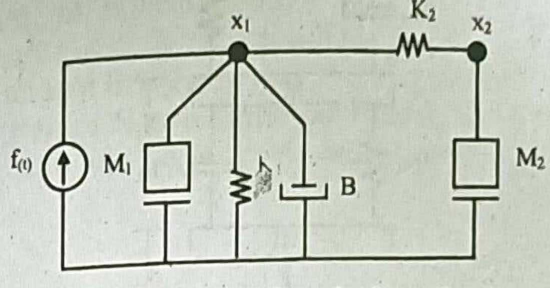


Fig: (a) Mechanical equivalent circuit

For Node 'x_2':

$$K_2(x_1 - x_2) = M_2\ddot{x}_2 \quad \text{.... (iii)}$$

Taking Laplace transform at both sides of the above equation

$$(s^2M_2 + K_2)X_2(s) - K_2X_1(s) = 0 \quad \text{.... (iv)}$$

2nd Part:

To draw analogous electrical circuit.

Using force-voltage analogy, we have the figure (b).

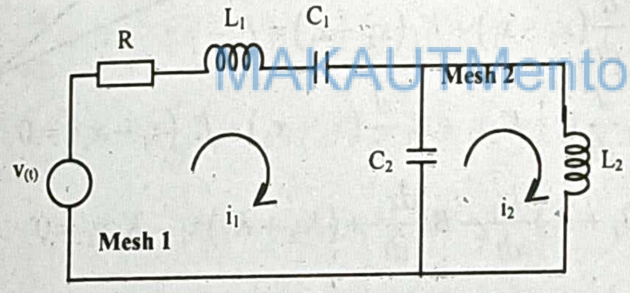


Fig: (b) Analogous electrical circuit

The differential equations for the electrical analog are (using mesh analysis approach)

For mesh 1

$$v(t) = Ri_1 + L_1\frac{di_1}{dt} + \frac{1}{C_1}\int i_1 dt + \frac{1}{C_2}\int i_1 dt - \frac{1}{C_2}\int i_2 dt$$

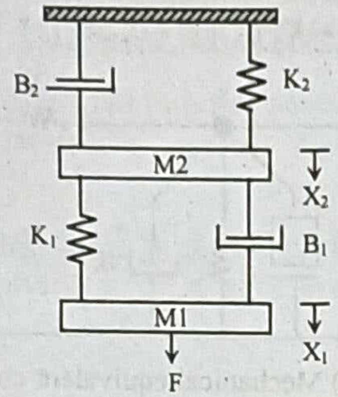
or, $$v(t) = R\frac{dq_1}{dt} + L_1\frac{d^2q_1}{dt^2} + \frac{q_1}{C_1} + \frac{q_1}{C_2} - \frac{q_2}{C_2} \quad \text{.... (v)}$$

For mesh 2

$$L_2\frac{di_2}{dt} + \frac{1}{C_2}\int i_2 dt - \frac{1}{C_2}\int i_1 dt = 0$$

or, $$L_2\frac{d^2q_2}{dt} + \frac{q_2}{C_2} - \frac{q_1}{C_2} = 0 \quad \text{.... (vi)}$$

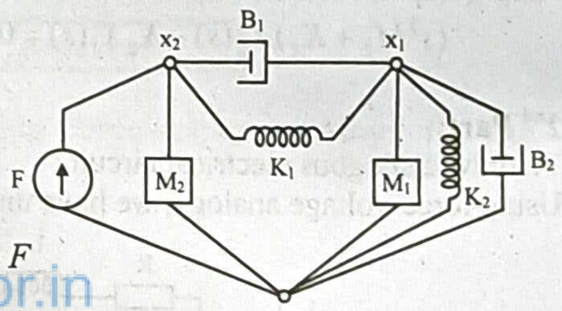
2. **Consider the following mechanical translational system. F denotes force, x denotes displacement, M denotes mass, B denotes friction coefficient and K denotes spring constant.** **[WBUT 2018]**



a) Write down the differential equations governing the above system.
b) Draw the corresponding electrical equivalent circuit using force-voltage analogy scheme.

Answer:

a)

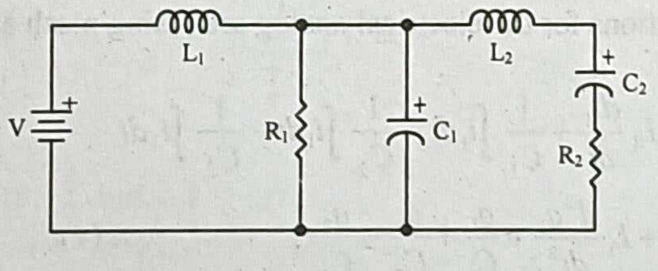


$$M_1\frac{d^2x_2}{dt^2}+B_1\frac{d}{dt}(x_2-x_1)+K_1(x_2-x_1)=F$$

$$M_2\frac{d^2x_1}{dt^2}+B_2\frac{d}{dt}x_1+K_2x_1+B_1\frac{d}{dt}(x_1-x_2)+K_1(x_1-x_2)=0$$

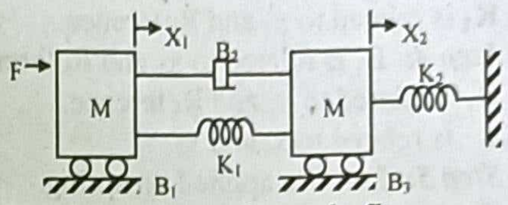
$$\Rightarrow \quad M_2\frac{d^2x_1}{dt^2}+(B_2+B_1)\frac{dx_1}{dt}-B_1\frac{dx_2}{dt}+(K_2+K_1)x_1-K_1x_2=0$$

b) Force-voltage analogy is used to draw the electrical circuit



Long Answer Type Questions

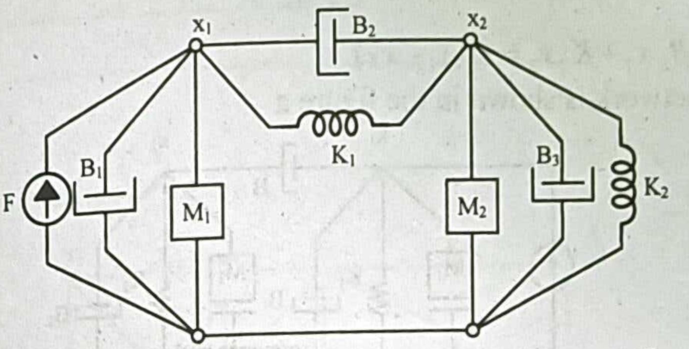
1. Consider the following mechanical translation system. F denotes force, X denotes displacement, M denotes mass, B denotes friction coefficient & K denotes spring constant. As shown in fig.



a) Write down the differential equations governing the system shown in fig.
b) Draw the corresponding electrical equivalent circuit using force-voltage analogy. **[WBUT 2006, 2007, 2017]**

Answer:

a)

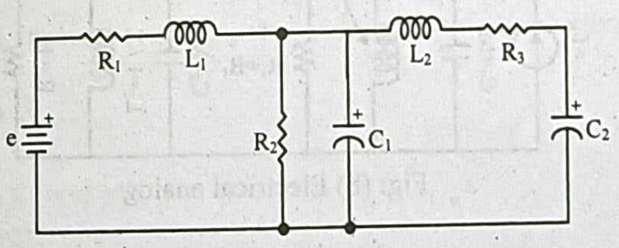


The differential equation governing the system are written as (from the mechanical network as shown above)

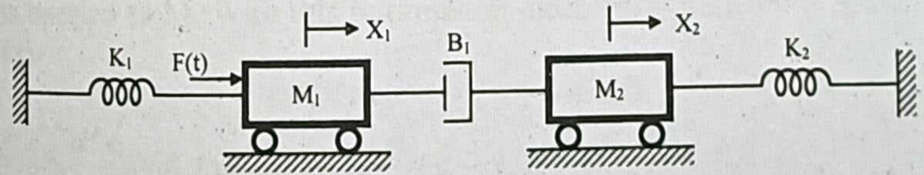
$$M_1\frac{d^2x_1}{dt^2}+B_1\frac{dx_1}{dt}+B_2\frac{d}{dt}(x_1-x_2)+K(x_1-x_2)=F$$

and $$M_2\frac{d^2x_2}{dt^2}+B_3\frac{d^2x_2}{dt^2}+B_2\frac{d}{dt}(x_2-x_1)+K_2x=0$$

b)



2. Obtain the differential equation of mechanical system show in following figure. Draw the electrical analogous circuit based on force-current analogy. [WBUT 2013]



Answer:

First part: To obtain the nodal equation.

Step 1: Number of displacements = Number of nodes = 2

In addition reference Node is there.

Step 2: M_1 is related to x_1 and Reference

M_2 is related to x_2 and Reference.
Step 3: K_1 is related to x_1 and Reference
K_2 is related to x_2 and Reference.
Step 4: B_1 is related to x_1 and Reference
B_2 is related to x_2 and Reference.
B is related to x_1 and x_2
Step 5: Force is applied on spring.
Step 6: *Node 'x_1'*

$$M_1\ddot{x}_1 + B_1\dot{x}_1 + B(\dot{x}_1 - \dot{x}_2) + K_1x_1 = f(t)$$

Node 'x_2'

$$M_2\,x_2 + B_2\,x_2 + K_2x_2 = B(x_1 - x_2)$$

The mechanical network is shown in the figure a

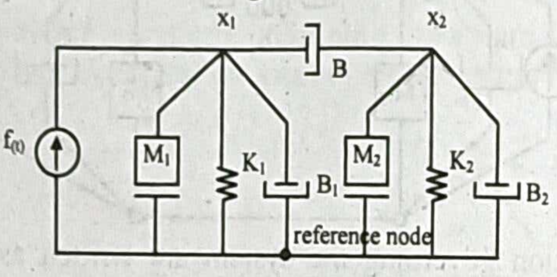


Fig: (a) Mechanical network

Second part: To draw electrical analog.

Electrical analog circuit based on force-current analogy is shown in the figure b.

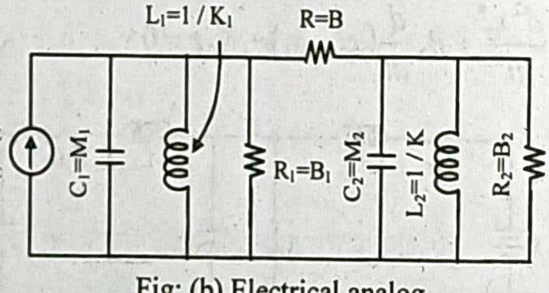
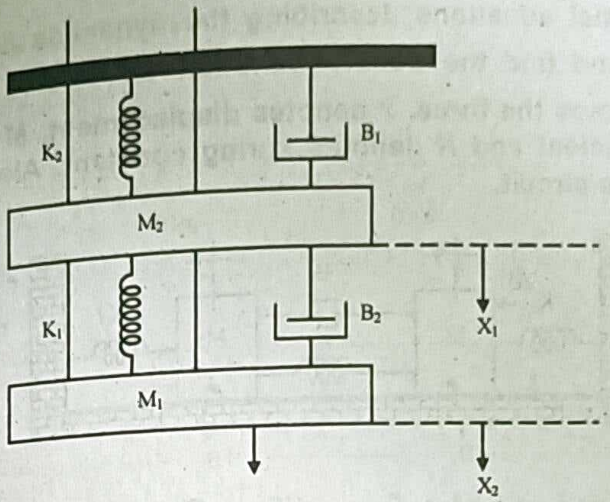


Fig: (b) Electrical analog

3. Obtain the mathematical model of the mechanical system given below. Draw the electrical analogous circuit based on force-voltage analogy. **[WBUT 2014, 2016]**



Answer:
To get the mechanical network [fig.1(a)],

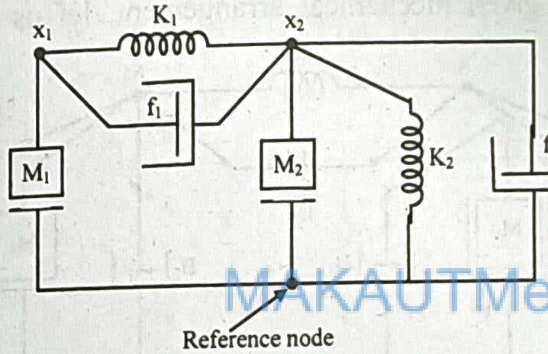


Fig: 1(a)

Step 1: There are two nodes x_1 and x_2 and reference of the system corresponds to the Reference Node.

Step 2: M_1 is related to x_1,
M_2 is related to x_2

Step 3: K_1 is related to x_1 and x_2,
K_2 is related to x_2 and reference.

Step 4: B_1 is connected to x_1 and x_2,
f_2 is connected to x_2 and reference.

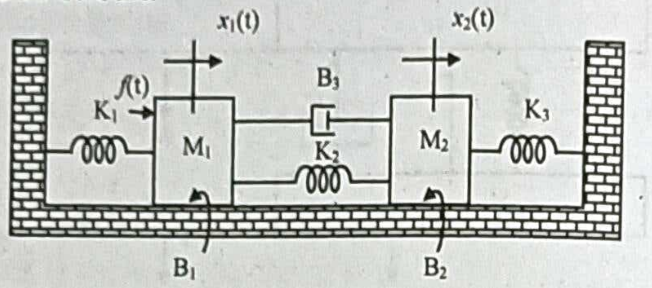
Step 5: F is applied to M. With this information mechanical network is drawn as shown in figure 1(a).

Step 6:

At x_1, $F = M_1\frac{d^2x_1}{dt^2} + K_1(x_1 - x_2) + B_1\frac{d}{dt}(x_1 - x_2)$ ...(i)

At x_2, $B_1\frac{d}{dt}(x_1 - x_2) + K_1(x_1 - x_2) = M_2\frac{d^2x_2}{dt^2} + K_2x_2 + B_2\frac{dx_2}{dt}$ ...(ii)

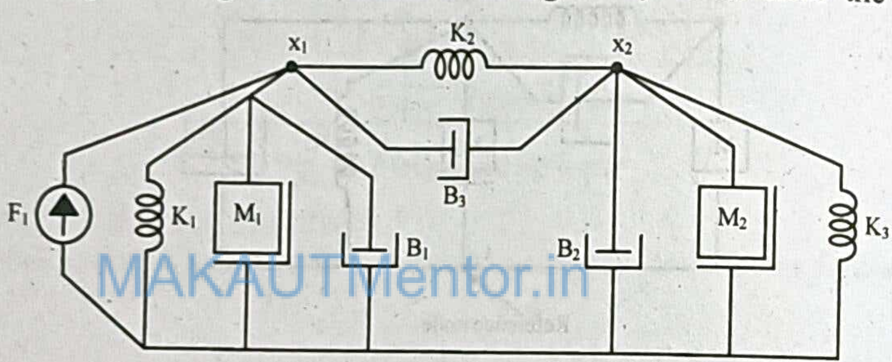
4. a) Define transfer function of a system.
b) Obtain the differential equations describing the dynamics of the mechanical translational system and find the transfer function $X_2(s)/F(s)$ for the system shown in figure; *f* denotes the force, *x* denotes displacement, *M* denotes mass, *B* denotes friction co-efficient and *K* denotes spring constant. Also draw its force-voltage (F-V) analogous circuit.
[WBUT 2019]



Answer:

a) ***Refer to Question No. 1 of Short Answer Type Questions.***

b) Corresponding to the given mechanical arrangement, let us have the mechanical network.



From the above mechanical network, the differential equations may be written as

1st Part:

$$M_1\frac{d^2x_1}{dt^2}+B_1\frac{dx_1}{dt}+K_1x_1+K_2\left(x_1-x_2\right)+B_3\frac{d}{dt}\left(x_1-x_2\right)=F_1 \qquad \text{.... (1)}$$

$$M_2\frac{d^2x_2}{dt^2}+B_2\frac{dx_2}{dt}+K_3x_2+K_2\left(x_2-x_1\right)+B_3\frac{d}{dt}\left(x_2-x_1\right)=0 \qquad \text{.... (2)}$$

2nd Part:

Taking Laplace transform of Eqn. (1)

$$M_1s^2X_1(s)+B_1sX_1(s)+K_1X_1(s)+K_2X_1(s)-K_2X_2(s)-B_3sX_2(s)+B_3sX_1(s)=F_1(s)$$

$$\Rightarrow \quad X_1(s)\left[M_1s^2+\left(B_1+B_3\right)s+\left(K_1+K_2\right)\right]-\left(B_3s+K_2\right)X_2(s)=F_1(s) \qquad \text{.... (3)}$$

Taking Laplace transform of Eqn. (2)

$$M_2s^2X_2(s)+B_2sX_2(s)+K_3X_2(s)+K_2X_2(s)-K_2X_1(s)+B_3sX_2(s)-B_3sX_1(s)=0$$

$$\Rightarrow \quad \left[M_2s^2+\left(B_2+B_3\right)s+\left(K_2+K_3\right)\right]X_2(s)=\left(B_3s+K_2\right)X_1(s)$$

$$\Rightarrow \quad X_2(s) = \frac{(B_3 s + K_2) X_1(s)}{\left[M_2 s^2 + (B_2 + B_3)s + (K_2 + K_3)\right]} \quad \ldots (4)$$

Combining Eqns. (3) & (4)

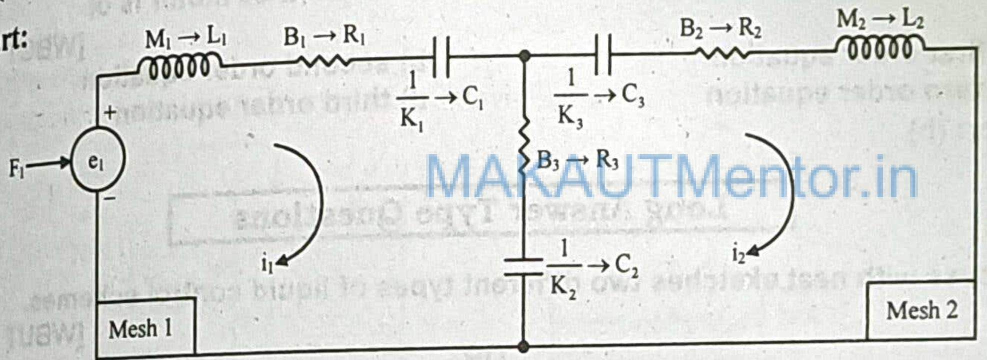
$$\left[M_1 s^2 + (B_1 + B_3)s + (K_1 + K_2)\right] X_1(s) - \frac{(B_3 s + K_2) X_1(s)}{M_2 s^2 + (B_2 + B_3)s + (K_2 + K_3)} = F_1(s)$$

$$\Rightarrow \quad \frac{\left[M_1 s^2 + (B_1 + B_3)s + (K_1 + K_2)\right] \times M_2 s^2 + (B_2 + B_3)s + (K_2 + K_3) - (B_3 s + K_2) X_1(s)}{M_2 s^2 + (B_2 + B_3)s + (K_2 + K_3)} = F_1(s)$$

$$\Rightarrow \quad \frac{X_1(s)}{F_1(s)} = \frac{M_2 s^2 + (B_2 + B_3)s + (K_2 + K_3) - (B_3 s + K_2)}{\left[M_1 s^2 + (B_1 + B_3)s + (K_1 + K_2)\right]\left[M_2 s^2 + (B_2 + B_3)s + (K_2 + K_3)\right]}$$

$$\Rightarrow \quad \frac{X_1(s)}{F_1(s)} = \frac{M_2 s^2 + B_2 s + K_3}{\left[M_1 s^2 + (B_1 + B_3)s + (K_1 + K_2)\right]\left[M_2 s^2 + (B_2 + B_3)s + (K_2 + K_3)\right]}$$

3rd Part:



COMPONENTS OF A CONTROL SYSTEM

Multiple Choice Type Questions

1. AC servomotor is basically a [WBUT 2007, 2010, 2017]
   a) universal motor
   b) single-phase induction motor
   c) two-phase induction motor
   d) three-phase induction motor

Answer: (c)

2. "Synchros" are popularly used as transmitter of [WBUT 2007]
   a) digital data
   b) mathematical data
   c) angular data
   d) all of these

Answer: (c)

3. A potentiometer converts linear/rotational displacement into [WBUT 2008, 2010, 2019]
   a) current
   b) power
   c) voltage
   d) torque

Answer: (c)

4. The characteristic equation of an armature controlled *dc* motor is of [WBUT 2009]
   a) first order equation
   b) second order equation
   c) zero order equation
   d) third order equation

Answer: (b)

Long Answer Type Questions

1. Discuss with neat sketches two different types of liquid control schemes. [WBUT 2007]

OR,

Write short note on Liquid level control. [WBUT 2012, 2016]

Answer:

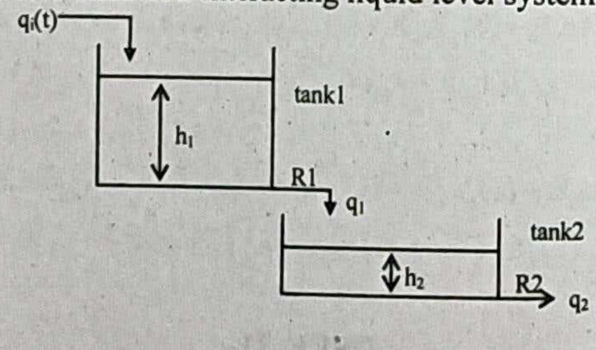
Two-tank Level System may be referred to as two first-order tank Level System connected in series.

Depending upon piping arrangements, two-tank level system may be classified as

1. Non Interacting System
2. Interacting System.

1. Non-Interacting System

Figure below shows a two-tank non-interacting liquid level system.



Here, outflow from tank1 discharges directly into the atmosphere before falling into tank 2. Liquid flow through R1 and R2 depend on respective liquid levels in tanks1 and 2. But the variation in h2 in tank 2 has no effect on the variation occurring in tank1. So, the name non-interacting.

Analysis

Assumptions:

1. Liquid density should remain the same
2. Tanks to have 'uniform' cross-sectional area.

Writing Mass-balance around each tank

(Mass flow in) – (Mass flow out) = Rate of accumulation of mass in the tank.

For tank1 $q_i - q_1 = A_1 \dfrac{dh_1}{dt}$

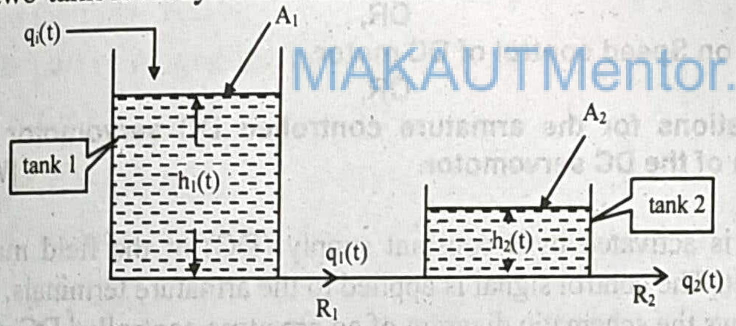
For tank2 $q_1 - q_2 = A_2 \dfrac{dh_2}{dt}$

Again from flow-head relationships

$q_1 = \dfrac{h_1}{R_1}$ $\qquad q_2 = \dfrac{h_2}{R_2}$

2. Interacting System

An interacting two-tank level system is shown below:



The above system is said to be interacting because the flow through R_1 depends on the differences between $h_1(t)$ and $h_2(t)$.

Analysis

Assumptions

- Density (ρ) of the liquid should remain constant.
- Cross-sectional areas of the tanks (1) and (2) should be uniform.

Writing mass balance equations

For tank 1

(Mass flow in) – (Mass flow out) = Rate of accumulation of mass in the tank.

$$\rho \cdot q_i(t) - \rho q_1(t) = \frac{d}{dt}\rho A h_1(t)$$

$$\Rightarrow \quad q_i(t) - q_1(t) = A_1 \frac{dh_1(t)}{dt}$$

For tank 2

$$q_1(t) - q_2(t) = A_2 \frac{dh_2(t)}{dt}$$

Writing the flow-head relationships

For tank 1

$$q_1(t) = \frac{h_1(t) - h_2(t)}{R_1}$$

For tank 2

$$q_2(t) = \frac{h_2(t)}{R_2}.$$

2. Find out the transfer function of an armature controlled DC motor. **[WBUT 2008]**

OR,

Obtain mathematical model of armature controlled DC motor and then determine the transfer function of the system. **[WBUT 2009]**

OR,

Derive the transfer function of armature controlled DC motor. **[WBUT 2010]**

OR,

Find out the overall transfer function of an armature controlled *d.c.* servo potion control system. **[WBUT 2013]**

OR,

Write short note on Speed control of DC motor. **[WBUT 2016]**

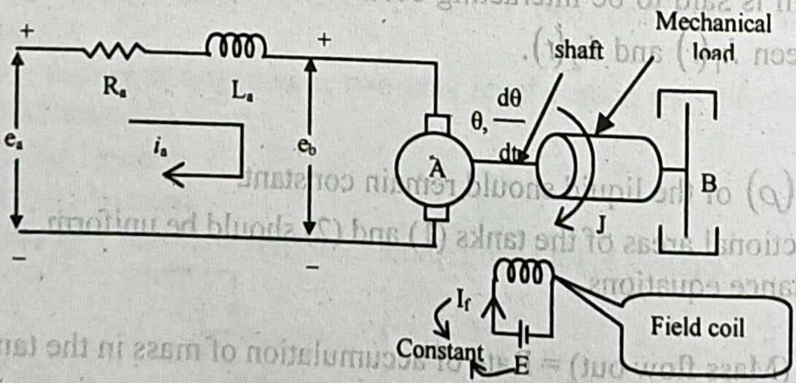
OR,

Obtain the equations for the armature controlled DC servomotor and find the transfer function of the DC servomotor. **[WBUT 2018]**

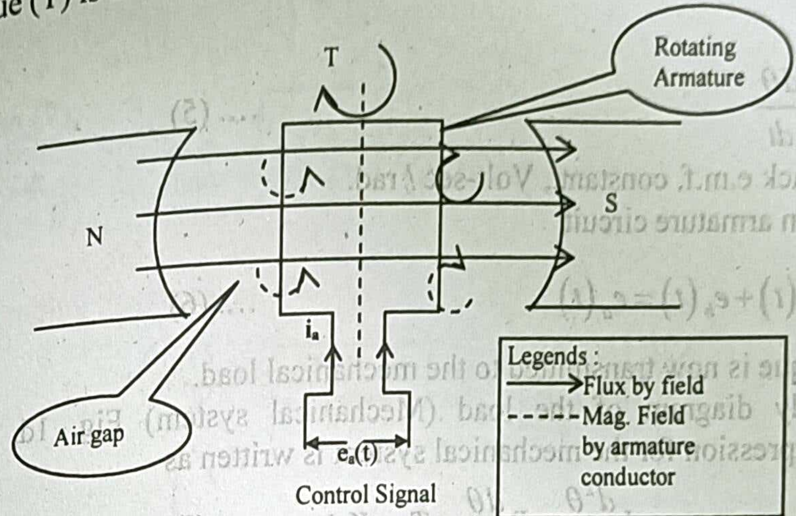
Answer:

The field circuit is activated by a constant supply (DC) or the field may be due to a permanent magnet. The control signal is applied to the armature terminals.

Figure below shows the schematic diagram of an armature controlled DC motor.



In the above figure, the armature circuit is modeled as a linear network of passive elements, where armature resistance R_a is connected in series with an inductance L_a (armature). The control signal $e_a(t)$ is applied to the armature terminals as the applied voltage. Due to $e_a(t)$, an armature current $i_a(t)$ flows through the armature circuit. The field current is constant as the supply voltage to field is constant. The magnetic field generated by the armature circuit interacts with magnetic field due to the field circuit. As a result a torque (T) is developed as shows in the figure below.



Torque delivered by the motor is proportional to the product of armature current (i_a) and air gap flux (ϕ) i.e., $T \propto \phi . i_a$ .... (1)

But, in a d.c. servomotor, field winding is operated in linear portion of magnetization curve as shown in the figure below.

$\therefore \phi \propto i_f$ [Linear part of magnetization curve] .... (2)

$$\therefore T \propto i_f i_a$$

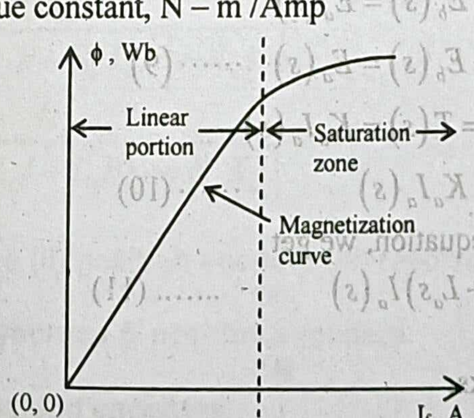
$$or, T = K_1 i_f . i_a \quad \cdots (3)$$

But $i_f = \text{Constant} = K_2$

$$\therefore T = K_1 K_2 . i_a$$

$$T = K i_a \quad .... (4)$$

where K ⇒ Motor's torque constant, N – m /Amp



As the motor rotates, a voltage proportional to the product of flux and angular velocity is induced in the armature circuit as per Faraday's laws of induction.
This voltage is known as back e.m.f. / counter e.m.f., e_b

$\therefore e_b \propto \phi.\dfrac{d\theta}{dt}$, in armature controlled d.c. servomotor ϕ is constant.

$$\therefore \qquad e_b \propto \frac{d\theta}{dt}$$

or, $$e_b = K_b \frac{d\theta}{dt} \qquad \text{.... (5)}$$

where, $K_b \Rightarrow$ back e.m.f. constant., Volt-sec / rad.

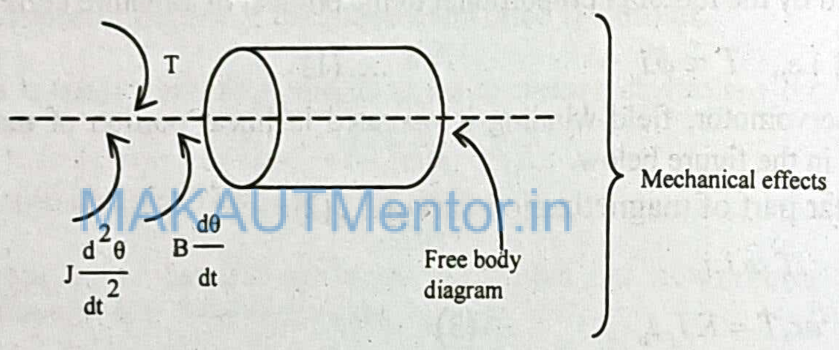
Applying KVL in armature circuit

$$L_a \frac{di_a(t)}{dt} + R_a i_a(t) + e_b(t) = e_a(t) \qquad \text{.... (6)}$$

The motor's torque is now transmitted to the mechanical load.

From Free body diagram of the load (Mechanical system) Fig. 1d, we get the mathematical expression for the mechanical system is written as

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T = K_a i_a \qquad \text{.... (7)}$$



Steps for constructing Block Diagram

Step 1: We take Laplace transform of equation (5), (6) and (7) to have

$$E_b(s) = K_b s\Theta(s) \qquad \text{.... (8)}$$

$$L_a s I_a(s) + R_a I_a(s) + E_b(s) = E_a(s)$$

$$\text{or,} (R_a + L_a s) I_a(s) + E_b(s) = E_a(s) \qquad \text{...... (9)}$$

$$Js^2 H(s) + Bs\Theta(s) = T(s) = K_a I_a(s)$$

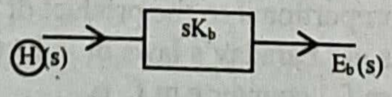
$$\text{or,} (Js + B)s\Theta(s) = K_a I_a(s) \qquad \text{...... (10)}$$

Rearranging equation above equation, we get

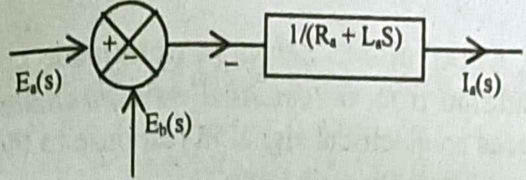
$$E_a(s) - E_b(s) = (R_a + L_a s) I_a(s) \qquad \text{....... (11)}$$

Step 2: Development of blocks:

From equation (8),

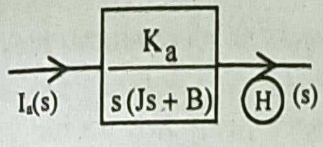


From equation (11),

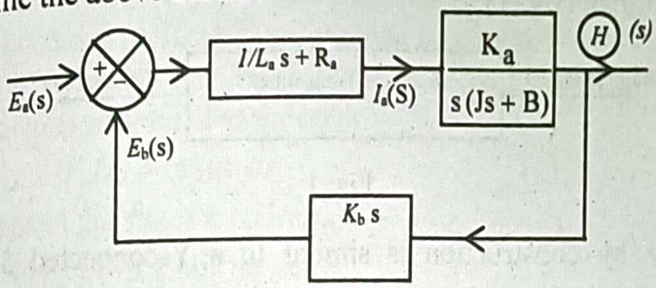


From equation (9),

$$\frac{H(s)}{I_a(s)} = \frac{K_a}{s(Js+B)}$$



Step 3: Recombine the above blocks.



We know,

$$\text{CLTF} = \frac{H(s)}{E_a(s)} = \frac{G(s)}{1+G(s)H(s)}$$

where $G(s) =$ Forward path T.F. $= \dfrac{K_a}{s(Js+B)(L_a s+R_a)}$ .... (12)

$H(s) =$ Feedback path T.F. $= K_b s$ .... (13)

$$\frac{H(s)}{E_a(s)} = \frac{\dfrac{K_a}{s(Js+B)(L_a s+R_a)}}{1+\dfrac{K_a}{s(Js+B)(L_a s+R_a)} \times K_b s}$$

$$= \frac{K_a}{s\left[JL_a s^2 + (L_a B + R_a J)s\right] + K_a K_b s}$$

$$= \frac{K_a}{s\left[JL_a s^2 + (R_a J + L_a B)s + K_a K_b\right]} \quad \text{.... (14)}$$

3. What are (i) synchros (ii) position encoder (iii) resolvers? [WBUT 2009]

OR,

Write short note on Synchros & position encoders. [WBUT 2012]

OR,

Write short note on Position encoders. [WBUT 2015]

Answer:

i) Synchro: A ***Synchro***, named also as Selsyn (a word made up from *self-synchronizing*) or Autosyn (a word made up from *automatically-synchronizing*), is an electromagnetic transducer, which produces an electrical signal in response to the angular displacement. A Synchro is basically consists of two sections.

1. Synchro transmitter
2. Synchro receiver

Synchro Transmitter

The synchro transmitter converts the angular position of its rotor (mechanical input) into an electrical output signal. (fig: 1).

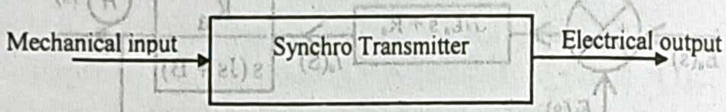


Fig: 1

- ***Construction***

Synchro Transmitter by construction is similar to a Y-connected 3-phase alternator, having a stator part and a rotor part.

- **Stator part**

The Stator, which is stationary, is made up of laminated silicon steel and is slotted to wind a ***balanced 3-phase*** winding which is of concentric coil type.

The axes of the coils are displaced 120° apart with each other and Y connected. (figure 1.a)

The stator windings provide electrical output.

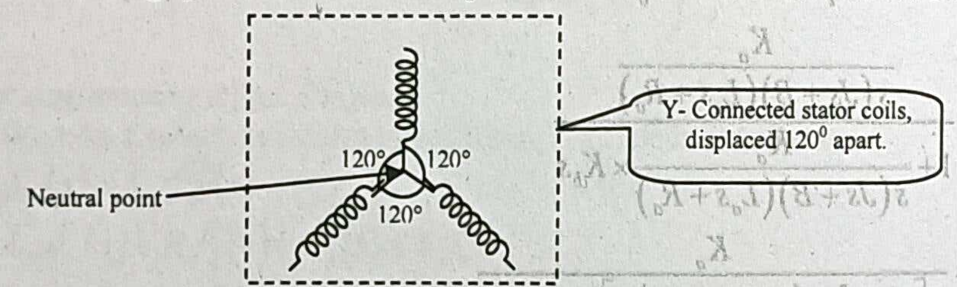


Fig: 1(a)

- ***Rotor part:***

The rotor is of dumb bell shaped. It is a salient pole type wound with concentric coils. Through the ship rings, an a.c. voltage is fed to the rotor winding.

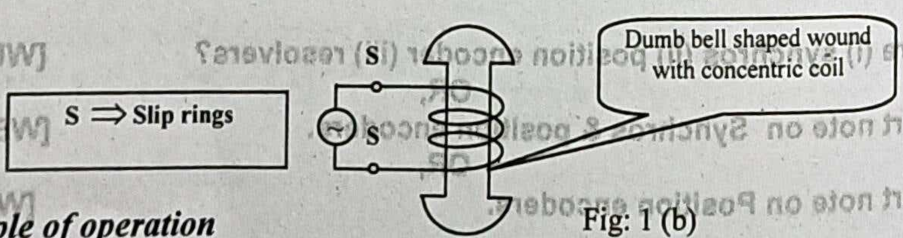


Fig: 1 (b)

- ***Principle of operation***

The synchro transmitter is based on Faraday's laws of inductions and acts as a transformer with

- rotor as primary side.
- stator as secondary side where the windings are displaced 120° apart from each other.

When an ac voltage is applied to the rotor of a synchro transmitter, the following events occur

- an alternating current produces an ac magnetic field around the rotor windings.
- the lines of force cut through the windings of the three stator coils and, by transformer action, induce voltage into the stator coils.
- the effective voltage induced in any stator coil depends upon the cosine of the angular position (figure 1.c) of that coil's axis with respect to the rotor axis.
- when the maximum effective coil voltage is known, the effective voltage induced into a stator coil at any angular displacement can be determined.

Let us consider, ac voltage applied to the rotor is

$$V_r(t) = A\sin \omega t \quad \text{...... (1)}$$

and induced voltages in the stator windings are

$$\left.\begin{aligned} V_{S1}(t) &= KA \sin \omega t \cos\theta \\ V_{S2}(t) &= KA \sin \omega t \cos(120+\theta) \\ V_{S3}(t) &= KA \sin \omega t \cos(240+\theta) \end{aligned}\right\} \text{...... (2)}$$

and the corresponding line voltages are

$$\begin{aligned} V_{L1} &= V_{S1} - V_{S2} \\ &= KA \sin \omega t[\cos\theta - \cos(120+\theta)] \\ &= KA \sin \omega t \cdot 2 \cdot \sin(60+\theta) \cdot \sin 60 \\ &= KA \sin \omega t 2 \cdot \sin(60+\theta)\frac{\sqrt{3}}{2} \\ &= \sqrt{3}\ KA \sin \omega t \cdot \sin(60+\theta) \quad \text{...... (3)} \end{aligned}$$

Similarly
$$\begin{aligned} V_{L2} &= V_{S2} - V_{S3} \\ &= KA \sin \omega t\ [\cos(120+\theta) - \cos(240+\theta)] \\ &= KA \sin \omega t\ 2 \sin(180+\theta)\sin 60 \\ &= \sqrt{3} KA \sin \omega t \cdot \sin(180+\theta) \quad \text{...... (4)} \end{aligned}$$

and
$$\begin{aligned} V_{L3} &= V_{S3} - V_{S1} \\ &= KA \sin \omega t[\cos(240+\theta) - \cos\theta] \\ &= -2KA\sin \omega t \sin\left(\frac{240+\theta+\theta}{2}\right)\sin\frac{240}{2} \\ &= -2KA \sin \omega t \cdot \sin(120+\theta) \cdot \sin 120 \\ &= -\sqrt{3}\ KA\sin \omega t \sin(120+\theta) \quad \text{...... (5)} \\ &= \sqrt{3}\ KA \sin \omega t \sin(240+\theta) \quad \text{...... (6)} \end{aligned}$$

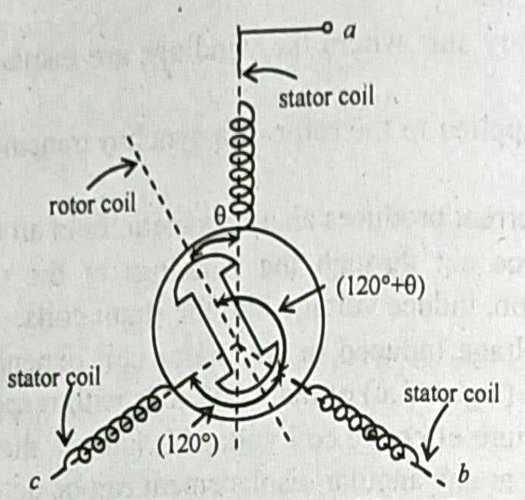


Fig: 1 (c) Relative angular positions of rotor

So, we find that for an input of angular position of the rotor shaft relative to the stator windings, the synchro transmitter gives a set of three lines voltages. Thus, synchro stator windings give electrical outputs.

- ***Electrical Zero:*** In figure 1d when θ = 0, V_{S1} has maximum value of voltage

  = KA sin ωt (from equation 2)

and $V_{L2} = \sqrt{3}\, KA \sin \omega t \cdot \sin(180 + \theta) = 0$

The position at which V_{S1} is maximum and V_{L2} = 0 is known as Electrical zero or reference position of the rotor in synchro transmitter.

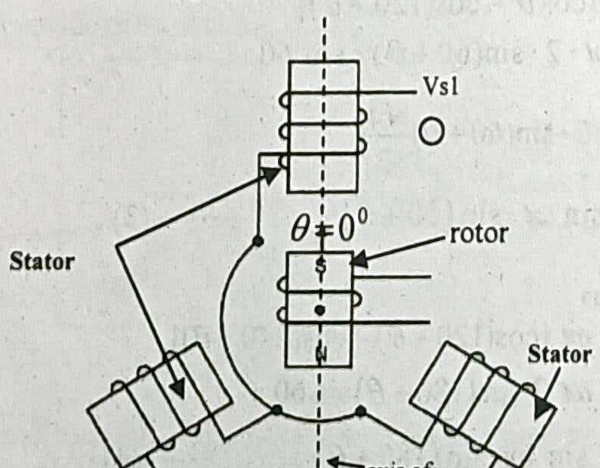


Fig: 1 (d) Electrical zero in synchro transmitter

Synchro Receiver / Control Transformer

When the stator windings (output) of a synchro transmitter is coupled to a synchro control transformer as shown in the figure 1.e, the complete system is called Synchro Error Detector or simply Synchro.

Unlike the synchro transmitter, the receiver has an electrical input to its stator and a mechanical output from its rotor.

The synchro receiver's function is to convert the electrical signal at its stator from the transmitter, back to a mechanical angular position through the movement of its rotor. Synchro Error Detector compares the angular positions of the two rotors.

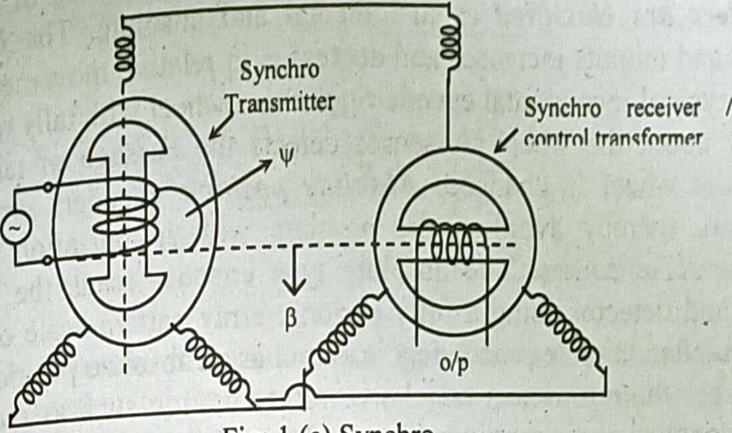


Fig: 1 (e) Synchro

- ***Working***

Since stators of both Synchro transmitter and Synchro control transformer are identical and output signal from synchro transmitter is fed as the input to the stator of the control transformer, the flux patterns are identical in both the systems.

A voltage will be induced in the rotor of the control transformer. The induced voltage will be proportional to the cosine of the angle between two rotors

$\therefore \quad e(t) = K_1 A \sin \omega t \cos \Psi$ ...... (7)

where, $\psi \Rightarrow$ angular displacement between the rotors.

and $K_1 \Rightarrow$ proportionality constant

When $\Psi = 90^\circ$, $e(t) = 0$. This position is known as *electrical zero of the error detector.*

Let us consider, $\theta_1 \Rightarrow$ Angular displacement of the rotor of transmitter

& $\beta \Rightarrow$ Angular displacement of the control transformer

$\therefore$ net angular $e(t) = K_1 A \sin \omega t \cos(90 + \theta - \beta)$

$= K_1 A \sin \omega t \sin(\beta - \theta)$ ...... (8)

For $(\beta - \theta)$ to be small, $\sin(\beta - \theta) \rightarrow (\beta - \theta)$ ......(9)

So, from equations 8 and 9 we have $e(t) = K_1 A \sin \omega t \cdot (\beta - \theta)$

$\Rightarrow e(t) \propto (\beta - \theta)$

$\Rightarrow$ Synchro-transmitter and control transformer pair acts as an Error detector.

ii) Position encoder:

Encoder is a device that detects rotational and linear positions of machines such as servomotors, linear actuators, tachometers, and the like. They allow accurate positioning of such machines, and determination of such quantities as derivatives of position i.e. velocity and acceleration.

Position encoders are used to generate an electronic signal that indicates an absolute mechanical position, or an incremental mechanical movement relative to a reference position. There are many known ways of generating a position signal, including magnetic sensors, capacitive sensors, and optical sensors. Linear and rotary type encoders are used extensively as detectors for control over feed rates and stop positions of various actuators. ***Position encoders are classified*** as incremental and absolute. The ***incremental*** type encoder detects and outputs increases and decreases in relative movement between coder and detector. A typical incremental encoder includes a wheel with tally marks arranged in a circular array about the wheel. A sensor detects the passage of tally marks as the orientation of the wheel is changed. ***Absolute position*** encoders provide readout of absolute position, thereby avoiding the problem with initialization movements that plagues incremental encoders. The absolute type encoder reads the relative position between coder and detector using a fully periodic array pattern scale of the coder. The relative position, after suitable processing, is output as an absolute position.

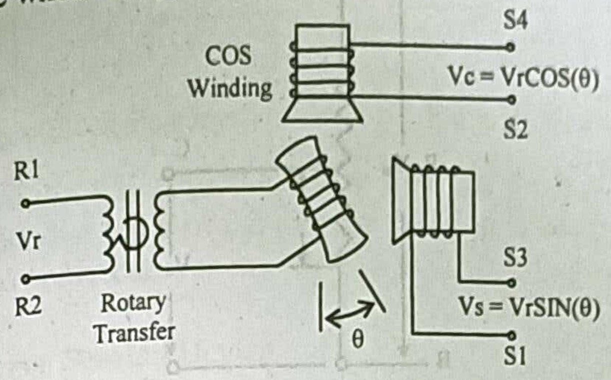
Different ways are there to detect angular or rotary motion such as ***mechanical*** means utilizing brush contacts or magnet/inductive methods. But the most common, reliable and widely used devices are ***non-contact optical*** receptors employed by optical encoders. An optical encoder comprises a light emitting unit, which emits a light beam, and ***photoelectric*** converting elements, which are disposed behind two diffraction gratings. A photoelectric rotary encoder is a kind of a sensor that is used to detect rotation angle, rotation number, rotational speed and the like of a rotating unit as digital signals. ***Optical shaft encoders*** generally consist of an optical shutter, such as a disk or drum, which is rigidly attached to the shaft whose position is to be determined. Such encoders are used in various types of machinery and machine tools where information concerning the precise angular relationship or speed between a shaft and another component is needed.

iii) Resolver

A resolver, an analog device, is a type of rotary electrical transformer that measures degrees of rotation. Pulse encoder is its digital counterpart.

A resolver is a rotary transformer where the magnitude of the energy through the resolver windings varies sinusoidally as the shaft rotates. A resolver control transmitter has one primary winding, the Reference Winding, and two secondary windings, the SIN and COS windings. The Reference Winding is located in the rotor of the resolver, the SIN and COS Windings in the stator. The SIN and COS Windings are mechanically displaced 90 degrees from each other. In a brushless resolver, energy is supplied to the Reference Winding (rotor) through a rotary transformer. This eliminates brushes and slip rings in the resolver and the reliability problems associated with them.

In general, in a control transmitter, the Reference Winding is excited by an AC voltage called the Reference Voltage (Vr). The induced voltages in the SIN and COS Windings are equal to the value of the Reference Voltage multiplied by the SIN or COS of the angle of the input shaft from a fixed zero point. Thus, the resolver provides two voltages whose ratio represents the absolute position of the input shaft. (SIN θ / COS θ = TAN θ, where θ = shaft angle.) Because the ratio of the SIN and COS voltages is considered, any changes in the resolvers' characteristics, such as those caused by aging or a change in temperature, are ignored. An additional advantage of this SIN / COS ratio is that the shaft angle is absolute. Even if the shaft is rotated with power removed, the resolver will take its new position value when energy is restored.



4. a) Draw the schematic diagram of an armature controlled *dc* servo portion control system showing all its components. Use potentiometers as the position error sensor. Draw the block diagram. **[WBUT 2012]**

Answer:

1st Part:

Armature Controlled DC Motor:

Refer to Question No. 2 of Long Answer Type Questions.

2nd Part:

Potentiometer

A potentiometer is a device, which has a resistance with three terminals. The Resistance is fixed in between two end terminals. The resistance of the third terminal (i.e., wiper) can be varied with respect to the end terminals as shown in the figure 1.

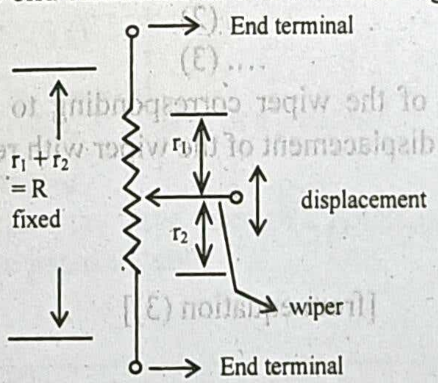


Fig: 1 A potentiometer

A potentiometer is a passive transducer which converts the mechanical displacement of the wiper into equivalent electrical signal. Figure 1(a) shows the block diagram of a potentiometer.

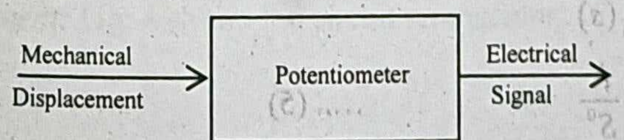


Fig: 1(a) Block diagram of a potentiometer

Working Principle

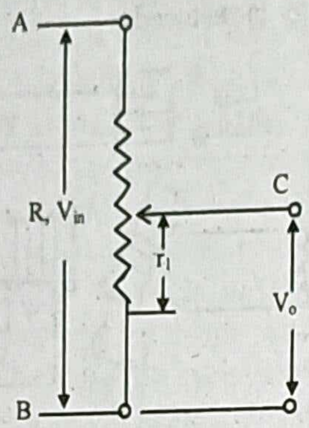


Fig: 2 Working a potentiometer

A and B are two fixed terminals having resistance R. If a constant d.c. voltage (V_{in}) is applied across the fixed end terminals (A and B), then an output voltage V_0 is obtained from the variable terminal (C) with respect to an end terminal. (say, B) as shown in the figure 2.

If the wiper is displaced then we get a variable output voltage V_0 i.e. the voltage output of the movable terminal / wiper with respect to one of the fixed terminals (say, B) is proportional to the displacement of the wiper with respect to terminal B.

Referring to the figure 2, due to voltage division,

$$v_0 = \frac{r_1}{R}.V_{in} = \frac{V_{in}}{R}r_1 \quad \text{.... (1)}$$

assuming uniform spreading out of the resistance,

$r_1 \propto x_1 \Rightarrow r_1 = K_1 x_1$ .... (2)

and $\quad R = K_1 x$ .... (3)

where, $x_1 \Rightarrow$ displacement of the wiper corresponding to r_1 and $K_1 \Rightarrow$ proportionality constant, $x \Rightarrow$ total sensible displacement of the wiper with respect to the terminal B.

$$\therefore \quad v_0 = \frac{V_{in}K_1}{R}.x_1$$

$$\text{or,} \quad v_0 = \left(\frac{V_{in}}{x}\right)\cdot x_1 \quad \text{[from equation (3)]}$$

$$\Rightarrow \quad v_0 \propto x_1$$

$$\Rightarrow \quad v_0 = K.x_1 \quad \text{.... (4)}$$

To Generate T.F.

Taking Laplace transform of equation (4), we have

$$V_0(s) = KX_1(s)$$

$$\Rightarrow \frac{V_0(s)}{X_1(s)} = K = \frac{K}{1} = \frac{K}{S^0} \quad \text{..... (5)}$$

Block Diagram Presentation

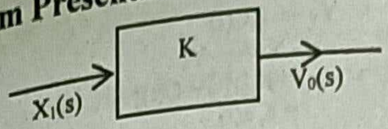


Fig: b Block diagram

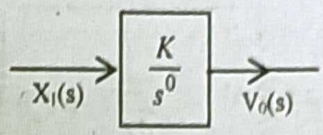


Fig: c Block diagram

From the expression of the T.F. (equation 5) we find that the highest power of s in the denominator is zero. So, the order of the transfer function is zero. Hence, we say that a potentiometer is a **zero order component.**

Error Detector or Error Sensing Device

Error detector is a component of control system; which is used to generate a difference signal.

In control system, the error detector is a device, which generates a difference signal, called error signal, depending upon the magnitude and polarity of the set point variable and controlled variable.

Symbol

Figure 3 depicts the symbol of a comparator, where, $r(t)$ and $c(t)$ are the inputs to it and $e(t)$ is the output from it providing the difference of the inputs.

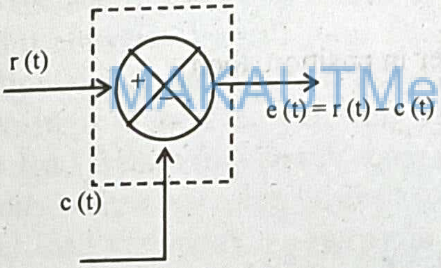


Fig. 3: Symbol of a comparator

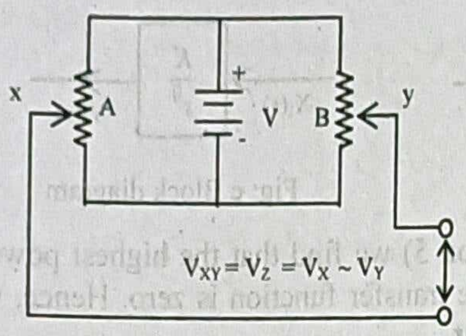
Realization of an Error Detector

Realization of an error detector may have through a potentiometric error detector, whose components and functions are listed in table.

- ***Components***

Components	Functions
2 potentiometers	Potentiometers, connected in parallel, are used as variable voltage sources.
Fixed dc voltage source	Applied across the potentiometers and the polarity of error voltage will determine the relative positions of the shafts / wipers.

- ***Circuit Arrangement:*** Fig. 4 shows the circuit arrangement of a potentiometric error detector.



$V_X = Kr(t)$
$V_Y = KC(t)$
$\therefore V_Z = V_X \sim V_Y$
$= K[r(t) \sim C(t)]$
$= K\, e(t)$
where K⇒ Proportionality constant
r(t) ⇒ Set Value
C(t) ⇒ Controlled Variable

Fig: 4 Potentiometric error detector

- ***Operations***

Step 1: One of the potentiometers, say A, in figure 4, is used to generate reference signal through its wiper. The position of the wiper (x) corresponds to an electrical signal (V_x).
Step 2: The other potentiometer's wiper position is governed by the controlled signal. Corresponding to a value of controlled signal, wiper takes a position, say y. This wiper position corresponds to an electrical signal, say V_y.
Step 3: The voltage V_{xy} developed between these two wiper will indicate the difference between the positions of two potentiometer wipers.
If x and y both at same physical level, then, voltages at these two points V_y and V_x are equal.

$\therefore \quad V_x - V_y = V_{xy} = 0$

However, if points x and y differ in position, then

$V_x \neq V_y$

$\therefore \quad V_{xy} \neq 0$

If $V_x > V_y$, then $V_{xy} > 0$
If $V_x < V_y$, then $V_{xy} < 0$
and $V_{xy} \propto$ (difference in levels of x and y)
and will correspond to an error signal.

$\therefore \quad V_{xy} = K.\, e(t) \qquad .....(6)$

where K⇒ sensitivity of the error detector

- ***Transfer Characteristics:*** Transfer characteristic of a potentiometric error detector is shown in figure 5.

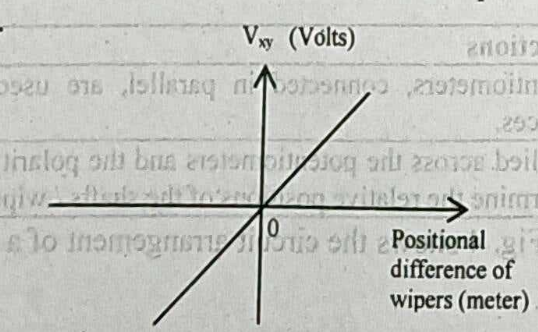


Fig: 5 Transfer characteristic of a potentiometric error detector

b) Find the overall transfer function of the system. Assume all relevant parameters and variables of the system. **[WBUT 2012]**

Answer:

Refer to Question No. 2 of Long Answer Type Questions.

5. Write short notes on the following:

a) Servo mechanisms **[WBUT 2008, 2013]**
b) AC tachometers **[WBUT 2009, 2012, 2019]**
OR,
AC tacho generator **[WBUT 2009, 2013]**
c) Servomotors **[WBUT 2010, 2015]**
d) Synchros **[WBUT 2013, 2015, 2019]**
e) DC tachogenerator **[WBUT 2013, 2016, 2019]**
f) Potentiometer. **[WBUT 2014]**
g) Block diagram of speed control of dc motor **[WBUT 2015]**

Answer:

a) Servo mechanisms:

A closed-circuit feedback system used in the automatic control of machi nes, involving an error-sensor using a small amount of energy, an amplifier, and a servomotor dispensing large amounts of power. In other words servomechanism deals with a feedback system that comprises of a sensing element, amplifier, and servomotor, used in the automatic control of a mechanical device by means of negative feedback. A control system with servomechanism converts a small force into a larger force. It is used to control the mechanical position and its derivative i.e. velocity and acceleration.

The purpose of servomechanism is to meet the following objectives: Power amplification i.e. control of a high-power load from a low-power command signal. Automatic control i.e. accurate and precise control of motion without the presence of human attendants. To take care of any mechanical load variations, power supply fluctuations, changes in the environment, and derated performance of components due to aging etc.

A control system may use single-loop servomechanism or, simply, a servo loop. More complex control system one may use two or more loops (multiloop servo).

Figure below shows a block diagram of a single loop servomechanism.

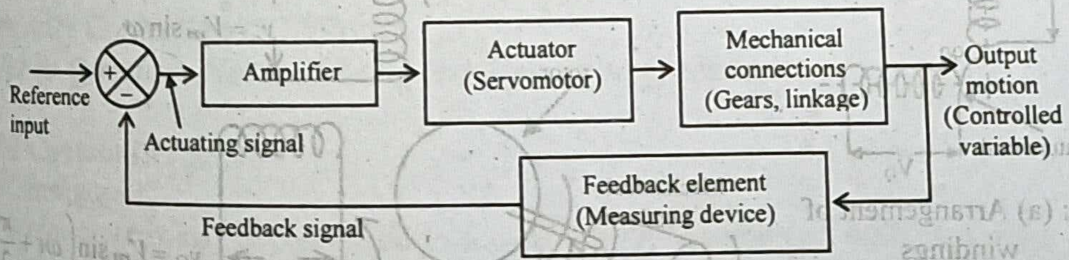


Fig: Block diagram of a single loop servomechanism

Figure below shows an example of Servomechanism where the azimuth of the antenna is controlled through the servomechanism.

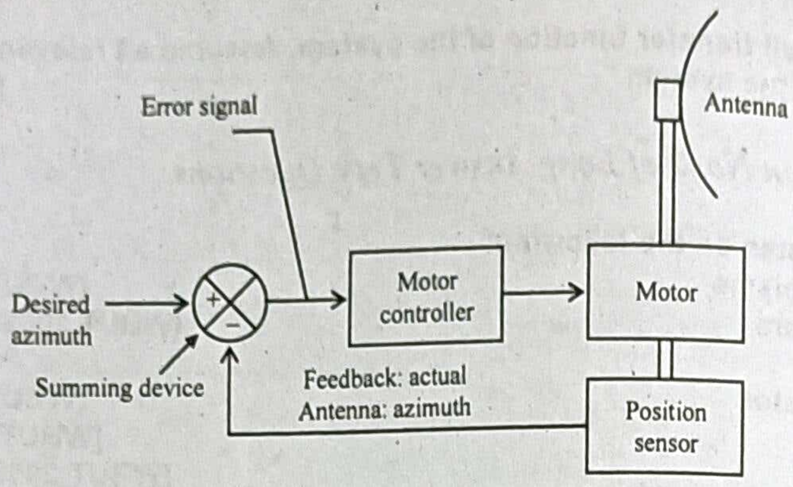


Fig. An example of Servomechanism

The position sensor senses any change in azimuth. The summing device generates an error signal depending upon the desired value of the azimuth. Motor controller after getting the feedback from the summing device drives the motor in such a way that the antenna regains its desired value.

b) AC tachometers or AC tacho generator:

Construction

The construction of an a.c. Tachometer is similar to a two-phase a.c. servomotor. Its construction is classified as

i) Stator part
ii) Rotor or Armature part

Stator part: It comprises of two coils oriented at right angles to each other, i.e., in space quadrature. One coil is called reference coil and other quadrature coil. The two windings are excited by voltages having 90° phase difference. [see figure (a)]

Rotor part: It comprises of a thin aluminium or copper cup which rotates in the air gap between a fixed magnetic structure. Inertia of the rotor is very low and the rotor is wound with highly conducting materials to provide a uniform short-circuited path.

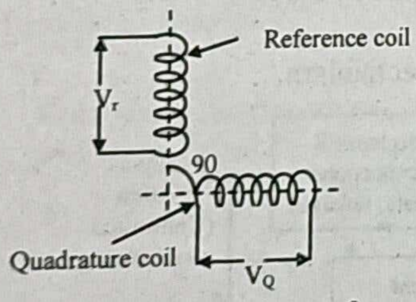


Fig: (a) Arrangement of windings

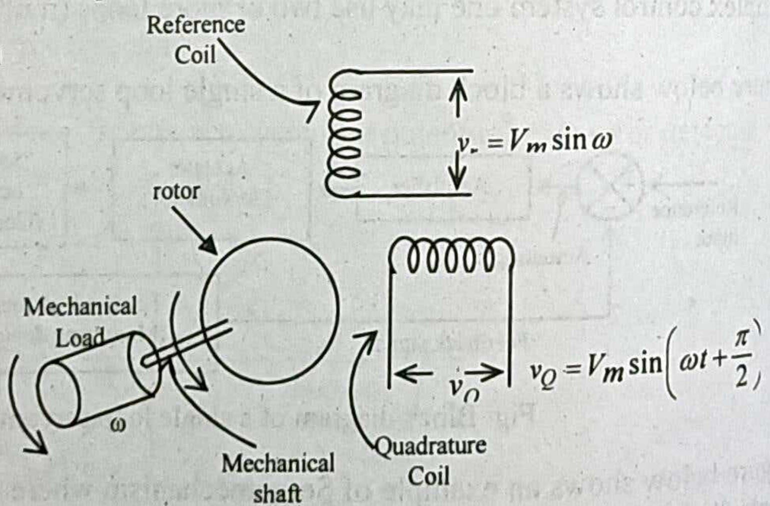


Fig: (b) Schematic diagram of an ac tachometer

Operation

Step 1: A sinusoidal rated voltage is applied to the reference winding.

Step 2: Since the rotor is coupled mechanically to an object whose speed is to be measured, the rotor rotates with the same speed as the object.

Step 3: The voltage at the quadrature coil is produced which is proportional to the speed of the rotor. Mathematically it can be represented as follows:

$$e_0(t) = K_t \cdot \frac{d}{dt}\theta(t)$$

where $e_0(t) \rightarrow$ output voltage, Volt

$\theta(t) \rightarrow$ angular displacement of rotor, Radian

$K_t \rightarrow$ tachometer constant, V/rad/sec.

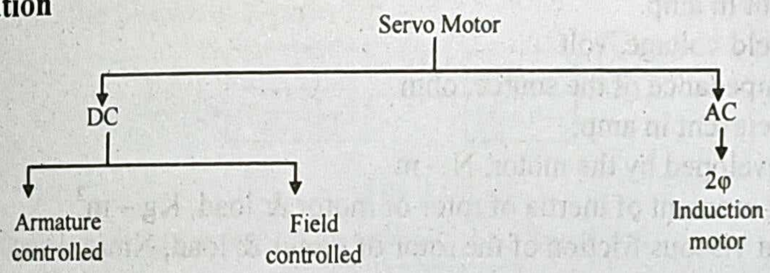
c) Servomotors

The word 'Servo' has basically been derived from the word 'Servant', who follows the instructions given by the master. Hence, a DC servomotor is used as an actuator in the control loop to drive a load as instructed by the controller.

It is usually a DC motor of low power rating, and having high ratio of Torque to Inertia (T/J) so bearing a faster dynamic response.

A Servomotor is a component of a control system in which the controlled variable is a mechanical angular position or rate of change of angular position. It converts electrical signal into equivalent mechanical system.

- **Classification**



Armature Controlled DC Motor:

Refer to Question No. 2 of Long Answer Type Questions.

Field Control DC Servo

Here, in figure 1,

(i) armature terminal is connected to a constant DC source (e_a) with very high internal impedance(R). As a result, armature current (i_a) remains constant.

(ii) control signal (from controller) is fed to the field windings with resistance R_f and inductance L_f.

(iii) armature is mechanically coupled to the load.

- **Operation**

Torque T developed by the motor is proportional to the product of air gap flux ϕ_f and armature current i_a

$\therefore \quad T \propto \phi_f i_a$

As i_a is constant,

$\therefore \quad T \propto \phi_f$

In linear part of magnetization curve $\quad \phi_f \propto i_f$

$\therefore \quad T \propto i_f$

or, $\quad T = K_f . i_f \quad \cdots\cdots (1)$

where, K_f = Motor's field torque constant, N – m/Amp.

Schematic diagram of a field controlled dc motor is as shown in figure 1.

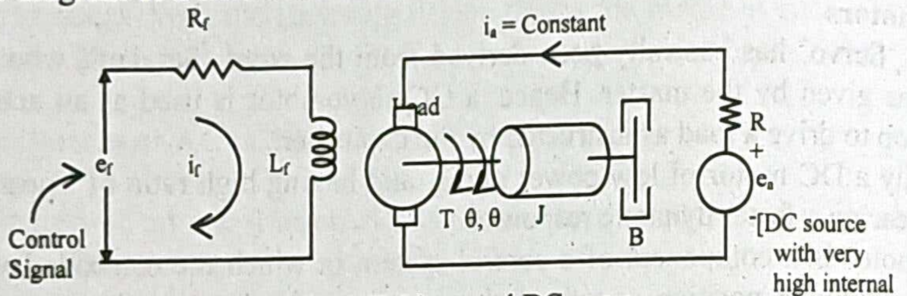


Fig: 1 Field control DC servo

Here, R_f = Field winding resistance, ohm
L_f = Field winding inductance, henry
i_f = Field current in amp.
e_f = Applied field voltage, volt
R = Internal Impedance of the source, ohm
i_a = Armature current in amp.
T = Torque developed by the motor, N – m
J = Equivalent moment of inertia of rotor of motor & load, Kg – m^2
B = Equivalent viscous friction of the rotor of motor & load, Nm/rad/sec.

From field circuit we have, $e_f = R_f i_f + L_f \dfrac{d.i_f}{dt} \quad \cdots\cdots (2)$

From the mechanical system, we draw the free body diagram (fig: 1.a).

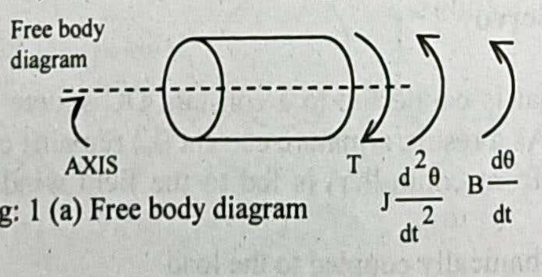


Fig: 1 (a) Free body diagram

From the free body diagram, balancing the different forces

$$T - \left(J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} \right) = 0 \quad \cdots\cdots (3)$$

or, $$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = K_f i_f \quad \cdots\cdots \quad (4)$$

[From equation (1)]

- ***Generation of Block diagram***

Step 1: Take Laplace transformation of time-domain differential equations (2) and (4). We thus get,

$$E_f(s) = R_f I_f(s) + sL_f I_f(s)$$

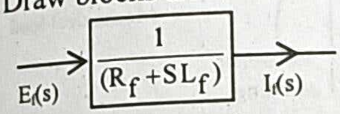
$$E_f(s) = (R_f + sL_f) I_f(s) \quad \cdots\cdots (5)$$

or,

and $$Js^2\Theta(s) + Bs\Theta(s) = K_f I_f(s)$$

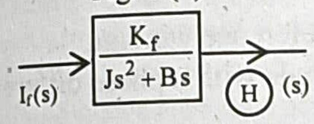
or, $$\Theta(s)\left[Js^2 + Bs\right] = K_f I_f(s) \quad \cdots\cdots (6)$$

Step 2: Draw blocks from individual equation in Laplace form



[From equation (5)]

Fig: 1 (b)



[From equation (6)]

Fig: 1 (c)

Step 3: Combine the blocks in figures 1.b and 1.c

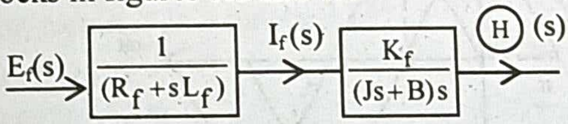


Fig: 1 (d)

Step 4: Apply block diagram reduction technique in figure 1.d

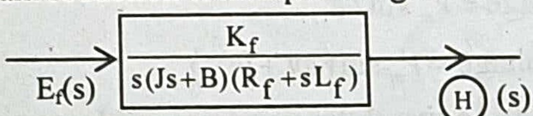


Fig: 1 (e) Represents the block diagram of a field controlled DC serve-motor

(a) It is a type-1, 3rd order system.

(b) Like armature controlled DC motor, it does not provide any inherent feedback path. So, from stability point of view it does not provide a good option.

AC Servo

1. The AC servomotors are basically two-phase, reversible, induction motors modified for servo operation.
2. Ac servomotors are used in applications requiring rapid and accurate response characteristics.

3. To achieve these characteristics, these ac servomotors have small diameter, high resistance rotors.
4. The ac servomotor's small diameter provides low inertia for fast starts, stops, and reversals.
5. High resistance provides nearly linear speed-torque characteristics for accurate servo motor control.

- **Construction**

It has 2 stator coils. The axes of the windings of the two coils are in space quadrature as shown in the following figure:

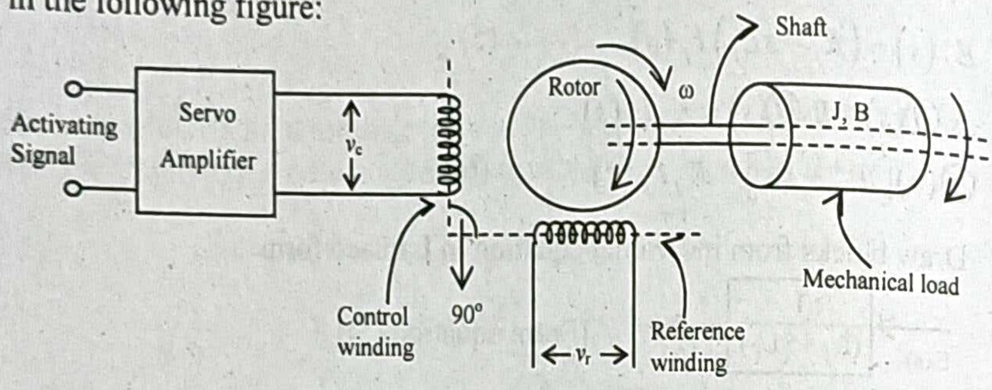


Fig: 2 (a) Schematic diagram of AC servo

One stator coil is called control winding and the other reference winding.
Voltages in the two stator windings are equal in magnitude with a phase difference of 90° figure 2 (b).

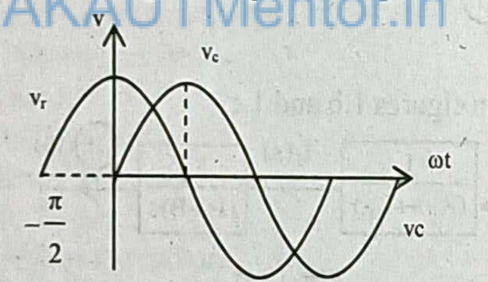


Fig: 2 (b) Voltage waveforms in two stator windings

i.e., v_c = control voltage $= V_m \sin \omega t$

v_r = reference voltage $= V_m \sin(\omega t + 90^\circ)$

This phase difference of 90° in two stator windings produces a rotating magnetic field.

- **Power supply**

The two stator windings are normally excited by a $2-\varphi$ power supply.
If a 2φ voltage supply is not available, then single phase supply along with an additional circuit is used to generate a phase difference of 90° between the voltage of two stator windings.

- **Rotor**

It is a squirrel-cage type having high Electrical Resistance.
Its diameter to length (d/l) ratio is kept small to reduce the moment of inertia of the rotor. (Fig: 2.c)

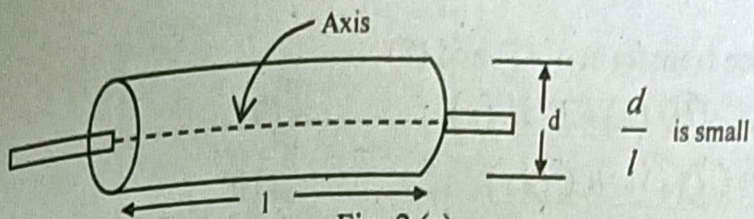


Fig: 2 (c)

- **Characteristic Curves**

A group/set of torque-speed curves are plotted as shown in fig: 2 (d) when

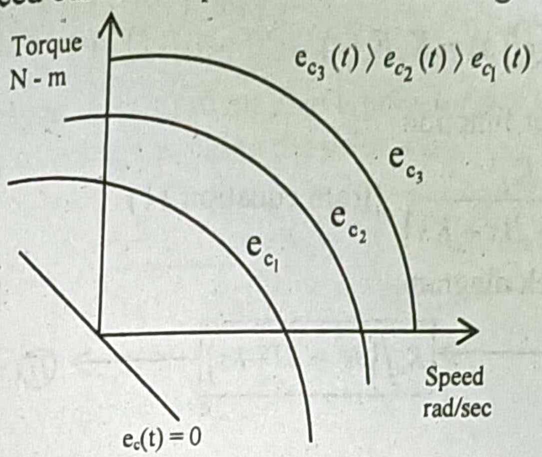


Fig: 2 (d) Characteristic curves

(i) fixed phase winding is fed with a rated voltage.
(ii) different voltages are applied to the control phase winding.

These curves are not straight lines, i.e., linear. To develop a linear mathematical model, we have to develop one or more differential equations. So, using linearization techniques these non-linear curves are approximated to linear curves with negative slopes.

- **Evaluation of Transfer Function**

Step 1: In ac servo motor, torque T is a function of

(iii) Motor's angular speed $\dot{\theta}$
(iv) Control voltage $e_c(t)$

i.e., $T = f\left(\dot{\theta}, e_c(t)\right)$

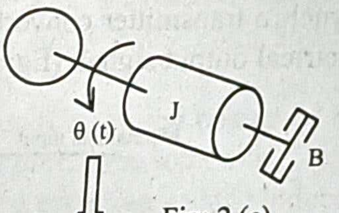


Fig: 2 (e)

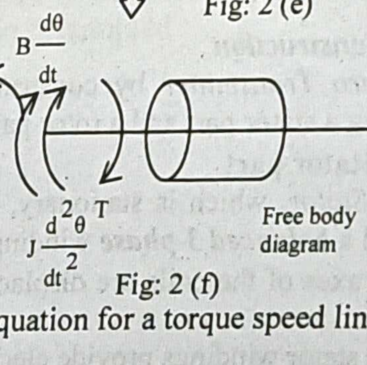


Fig: 2 (f)

Considering linearized torque-speed characteristics, the equation for a torque speed line is

$$T = -K\dot{\theta}(t) + K_c e_c(t) \quad ......(7)$$

where K and K_c are constants.

From free body diagram (fig: 2.f) we have the Torque Balance equation as

$$T = J\frac{d^2\theta(t)}{dt^2} + B\frac{d\theta(t)}{dt} \quad ....(8)$$

Step 2: Taking Laplace transform of (7) and (8)

$$T(s) = -Ks\Theta(s) + K_c E_c(s) \quad \text{.....(9)}$$

$$T(s) = Js^2\Theta(s) + Bs\Theta(s) \quad \text{.....(10)}$$

From equation (9) and (10), we get

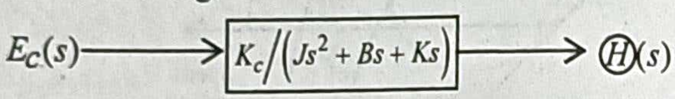
$$Js^2\Theta(s) + Bs\Theta(s) = -Ks\Theta(s) + K_c E_c(s)$$

$$\left(Js^2 + Bs + Ks\right)\Theta(s) = K_c E_c(s) \quad \text{......(11)}$$

Step 3: To get the Transfer function.

$$\frac{\Theta(s)}{E_c(s)} = \frac{K_c}{\left(Js^2 + Bs + Ks\right)} \quad \text{(from equation 11)}$$

Step 4: To draw the Block diagram.



d) Synchros:

Synchro: A ***Synchro***, named also as Selsyn (a word made up from *self-synchronizing*) or Autosyn (a word made up from *automatically-synchronizing*), is an electromagnetic transducer, which produces an electrical signal in response to the angular displacement. A Synchro is basically consists of two sections

1. Synchro transmitter
2. Synchro receiver

Synchro Transmitter

The synchro transmitter converts the angular position of its rotor (mechanical input) into an electrical output signal. (fig: 1)

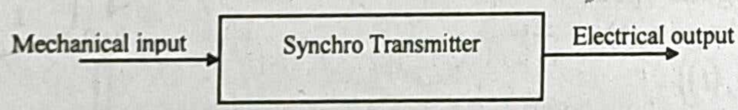


Fig: 1

- ***Construction***

Synchro Transmitter by construction is similar to a Y-connected 3-phase alternator, having a stator part and a rotor part.

- **Stator part**

The Stator, which is stationary, is made up of laminated silicon steel and is slotted to wind a ***balanced 3-phase*** winding which is of concentric coil type.

The axes of the coils are displaced 120° apart with each other and Y connected. (figure 1.a).

The stator windings provide electrical output.

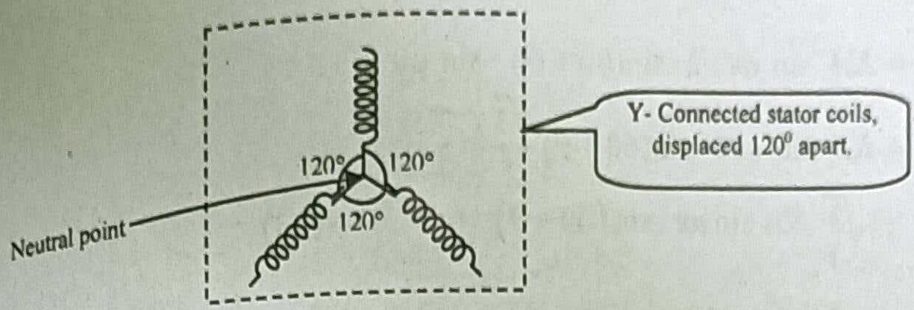


Fig: 1(a)

- ***Rotor part:***

The rotor is of dumb bell shaped. It is a salient pole type wound with concentric coils. Through the ship rings, an a.c. voltage is fed to the rotor winding.

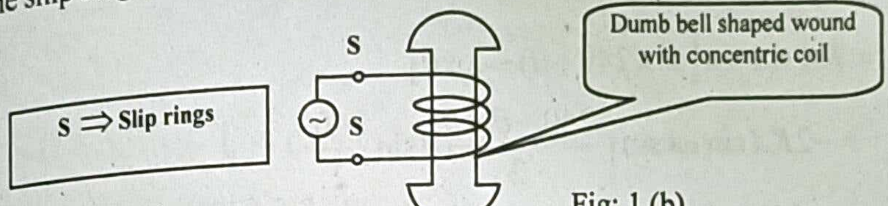


Fig: 1 (b)

- ***Principle of operation***

The synchro transmitter is based on Faraday's laws of inductions and acts as a transformer with

- rotor as primary side.
- stator as secondary side where the windings are displaced 120° apart from each other.

When an ac voltage is applied to the rotor of a synchro transmitter, the following events occur

- an alternating current produces an ac magnetic field around the rotor windings.
- the lines of force cut through the windings of the three stator coils and, by transformer action, induce voltage into the stator coils.
- the effective voltage induced in any stator coil depends upon the cosine of the angular position (figure 1.c) of that coil's axis with respect to the rotor axis.
- when the maximum effective coil voltage is known, the effective voltage induced into a stator coil at any angular displacement can be determined.

Let us consider, ac voltage applied to the rotor is

$$V_r(t) = A\sin\omega t \qquad \cdots\cdots(1)$$

and induced voltages in the stator windings are

$$\left.\begin{aligned} V_{S1}(t) &= KA\sin\omega t\cos\theta \\ V_{S2}(t) &= KA\sin\omega t\cos(120+\theta) \\ V_{S3}(t) &= KA\sin\omega t\cos(240+\theta) \end{aligned}\right\} \cdots\cdots(2)$$

and the corresponding line voltages are

$$V_{L1} = V_{S1} - V_{S2}$$

$$= KA\sin\omega t[\cos\theta - \cos(120+\theta)]$$

$$= KA\ \sin\omega t \cdot 2 \cdot \sin(60+\theta)\cdot \sin 60$$
$$= KA\ \sin\omega t 2 \cdot \sin(60+\theta)\frac{\sqrt{3}}{2}$$
$$= \sqrt{3}\ KA \sin\omega t \cdot \sin(60+\theta) \quad \cdots\cdots(3)$$

Similarly $V_{L2} = V_{S2} - V_{S3}$

$$= KA \sin\omega t\ [\cos(120+\theta) - \cos(240+\theta)]$$
$$= KA \sin\omega t\ 2\sin(180+\theta)\sin 60$$
$$= \sqrt{3}\ KA\ \sin\omega t \cdot \sin(180+\theta) \quad \cdots\cdots(4)$$

and $V_{L3} = V_{S3} - V_{S1}$

$$= KA \sin\ \omega t[\cos(240+\theta) - \cos\theta]$$
$$= -2KA\sin\omega t\sin\left(\frac{240+\theta+\theta}{2}\right)\sin\frac{240}{2}$$
$$= -2KA\ \sin\omega t\cdot \sin(120+\theta)\cdot \sin 120$$
$$= -\sqrt{3}\ KA\sin\ \omega t\sin(120+\theta) \quad \cdots\cdots(5)$$
$$= \sqrt{3}\ KA\sin\ \omega t\ \sin(240+\theta) \quad \cdots\cdots(6)$$

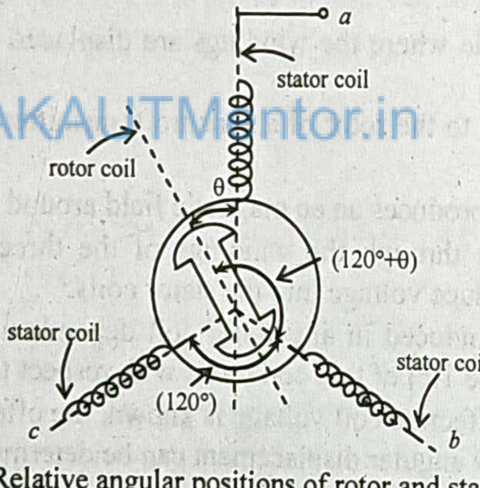


Fig: 1 (c) Relative angular positions of rotor and stator coils

So, we find that for an input of angular position of the rotor shaft relative to the stator windings, the synchro transmitter gives a set of three lines voltages. Thus, synchro stator windings give electrical outputs.

- ***Electrical Zero:*** In figure 1d when $\theta = 0$, V_{S1} has maximum value of voltage $= KA \sin\omega t$ (from equation 2)

and $V_{L2} = \sqrt{3}\ KA\ \sin\ \omega t \cdot \sin(180+\theta) = 0$

The position at which V_{S1} is maximum and $V_{L2} = 0$ is known as Electrical zero or reference position of the rotor in synchro transmitter.

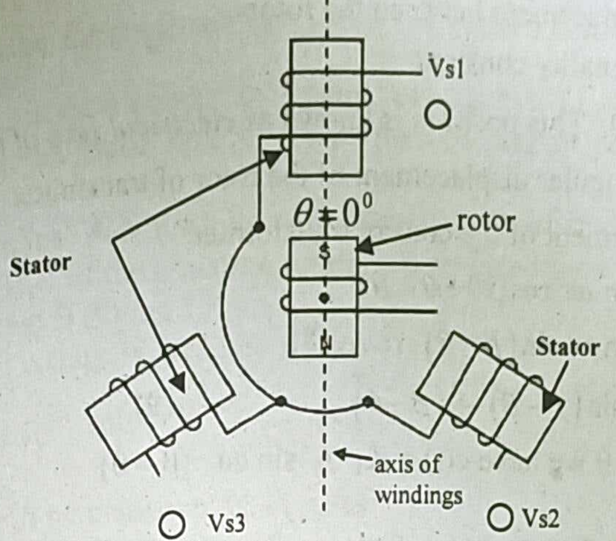


Fig: 1 (d) Electrical zero in synchro transmitter

Synchro Receiver / Control Transformer

When the stator windings (output) of a synchro transmitter is coupled to a synchro control transformer as shown in the figure 1.e, the complete system is called Synchro Error Detector or simply Synchro.

Unlike the synchro transmitter, the receiver has an electrical input to its stator and a mechanical output from its rotor.

The synchro receiver's function is to convert the electrical signal at its stator from the transmitter, back to a mechanical angular position through the movement of its rotor.

Synchro Error Detector compares the angular positions of the two rotors.

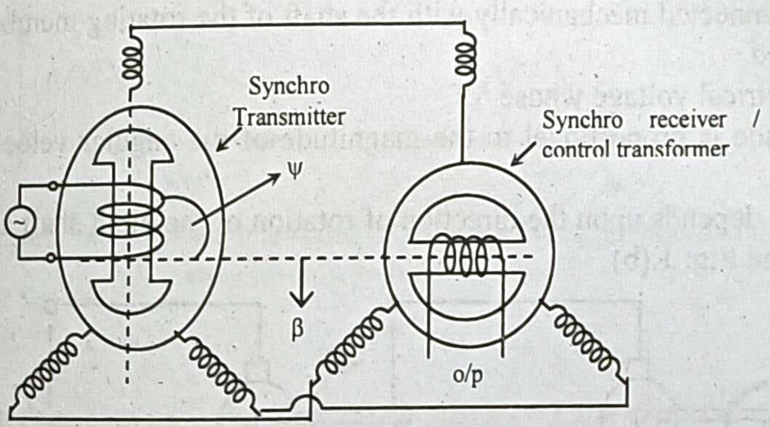


Fig: 1 (e) Synchro

- ***Working***

Since stators of both Synchro transmitter and Synchro control transformer are identical and output signal from synchro transmitter is fed as the input to the stator of the control transformer, the flux patterns are identical in both the systems.

A voltage will be induced in the rotor of the control transformer. The induced voltage will be proportional to the cosine of the angle between two rotors

$\therefore \quad e(t) = K_1 A \sin \omega t \cos \Psi \qquad \ldots\ldots (7)$

where, $\psi \Rightarrow$ angular displacement between the rotors.

and $K_1 \Rightarrow$ proportionality constant

When $\Psi = 90^\circ$, $e(t) = 0$. This position is known as *electrical zero of the error detector.*

Let us consider, $\theta_1 \Rightarrow$ Angular displacement of the rotor of transmitter

& $\beta \Rightarrow$ Angular displacement of the control transformer

$\therefore$ net angular $e(t) = K_1 A \sin \omega t \cos(90 + \theta - \beta)$

$= K_1 A \sin \omega t \sin(\beta - \theta)$ ...... (8)

For $(\beta - \theta)$ to be small, $\sin(\beta - \theta) \to (\beta - \theta)$ ......(9)

So, from equations 8 and 9 we have $e(t) = K_1 A \sin \omega t \cdot (\beta - \theta)$

$\Rightarrow e(t) \propto (\beta - \theta)$

$\Rightarrow$ Synchro-transmitter and control transformer pair acts as an Error detector.

e) DC tachogenerator:

Construction: The construction of a d.c. tachometer is similar to a small d.c. generator. Its construction is classified as

i) stator part
ii) rotor or armature part

Stator part: As the name implies this part remains stationary. It comprises of several permanent magnet poles which produce magnetic field.

Rotor or Armature part: A wound rotor is provided with a commutator and brushes. The output voltage is picked up from the brushes.

Rotor shaft is connected mechanically with the shaft of the rotating member whose speed is to be measured.

It generates electrical voltage whose

a) magnitude is proportional to the magnitude of the angular velocity of the input shaft.
b) polarity depends upon the direction of rotation of the input shaft as shown in Fig. 1. (a) and Fig: 1.(b)

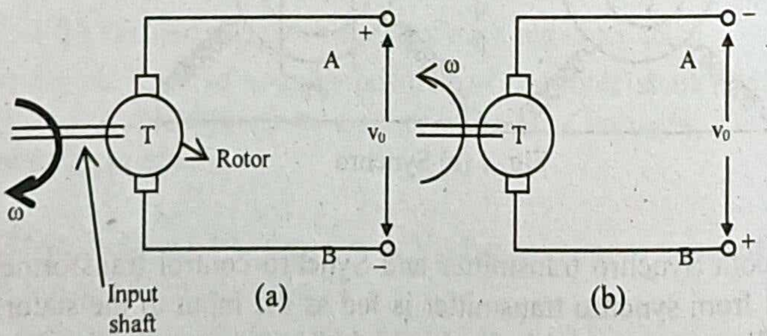


Fig: 1 Dependence of polarity on rotation

Principle of operation: The operation of a DC tachometer is based on Faraday's law of induction i.e. when a conductor rotates in a magnetic field, it generates an electrical voltage across the conductor, which is proportional to the rotation of the conductor. i.e.

$v_0 \propto \frac{d\theta}{dt}$ where, v_0 is the electrical voltage (volts) and θ(rad.) is the angular displacement of the conductor. So, when a conductor wound rotor, provided with a commutator and brushes, rotates in the magnetic flux due to permanent magnet poles, a D.C. voltage is produced.

In figure 1.a, the terminal A is positive with respect to B when the armature rotates in clockwise direction (say). If the armature rotation is reversed then A becomes negatively polarised with respect to B (figure 1.b).

The magnitude of DC voltage is given by

$$v_0 = \frac{n_p n_c N\phi}{60 n_{pp}} \times 10^{-8} \text{ volts.} \qquad \text{........ (1)}$$

where, n_p = number of permanent magnet poles

n_c = number of conductors in armature

n_{pp} = number of parallel paths between the positive and negative brushes

N = angular speed is r.p.m.

ϕ = flux produced by poles; Wb.

Since, n_p, n_c, n_{pp} and ϕ are constants.

So, $V_0 \propto N$.

The tachometer's voltage waveform (V_0) follows the pattern of the angular speed (N) of the rotating member as shown by the figures 2.a and 2.b.

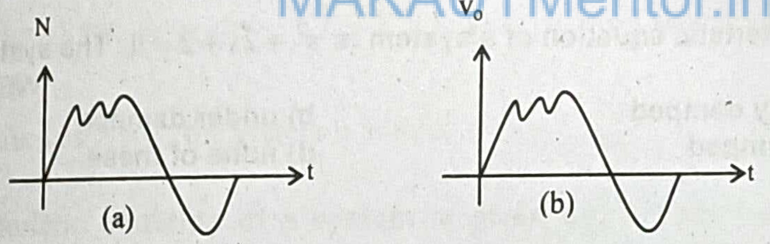


Fig: 2 waveforms of the angular speed (N) of the rotating member and the tachometer's voltage (V_o)

f) **Potentiometer:**

Refer to Question No. 4(a) (2nd Part) of Long Answer Type Questions.

g) **Block diagram of speed control of dc motor:**

Refer to Question No. 2 of Long Answer Type Questions.

STABILITY ANALYSIS AND ROUTH STABILITY CRITERION

Multiple Choice Type Questions

1. If the system has multiple poles on the $j\omega$-axis, the system is [WBUT 2012]
   a) stable b) unstable c) marginally stable d) conditionally stable

Answer: (c)

2. The condition for stability of a closed loop system with characteristic equation $s^3 + Bs^2 + Cs + 1 = 0$, the positive coefficient is [WBUT 2013, 2017]
   a) $B+C>1$ b) $BC>1$ c) $B=C$ d) $B>C$

Answer: (b)

3. The open loop transfer function of a unity feedback control system is $G(s)H(s) = \frac{30}{s(s+1)(s+T)}$, where T is a variable parameter. The closed-loop system will be stable for all values of [WBUT 2013]
   a) $T>0$ b) $0<T<3$ c) $T>5$ d) $3<T<5$

Answer: (c)

4. The characteristic equation of a system is $s^2 + 2s + 2 = 0$. The system is [WBUT 2014, 2019]
   a) critically damped b) under damped
   c) over damped d) none of these

Answer: (b)

5. The characteristic equation $1 + G(s)H(s) = 0$ of a system is given by $s^4 + 6s^3 + 11s^2 + 6s + K = 0$
   For the system to remain stable, the value of gain K should be [WBUT 2018]
   a) zero
   b) greater than zero but less than 10
   c) greater than 10 but less than 20
   d) greater than 20 but less than 30

Answer: (b)

6. If the gain *k* of the system increases, the steady state error of the system [WBUT 2019]
   a) decreases b) increases
   c) may increase or decrease d) remains unchanged

Answer: (a)

7. The open loop transfer function of a unity feedback control system is $G(s)H(s) = 100(1+2s)/s^2(1+5s)$. If the system is subjected to an input $r(t) = 1 + t + t^2/2 (t \geq 0)$, the steady state error of the system will be [WBUT 2019]

a) 1 b) 0.1 c) 0.01 d) 100

Answer: (c)

Short Answer Type Questions

1. A system is defined by the transfer function $G(s) = \dfrac{k}{(T_1 s+1)(T_2 s+1)}$, $T_1, T_2 > 0$, Prove mathematically that the system will be stable. [WBUT 2011]

Answer:

Characteristic equation

$$T_1T_2s^2 + (T_1+T_2)s + (1+k) = 0$$

Routh's array is formed as shown below:

$$\begin{array}{c|cc} s^2 & T_1T_2 & 1+k \\ s & (T_1+T_2) & 0 \\ s^0 & 1+k & 0 \end{array}$$

$\because T_1$ and $T_2 > 0$ $\therefore T_1T_2$ is positive, (T_1+T_2) is also positive, which says for stability $1+k>0, k>-1$.

Since, for all values of $k \geq 0$, the system is stable.

2. The characteristic equation of a system is given by $s^3 + 3ks^2 + (k+2)s + 4 = 0$. Find the range of k for which the system is stable. [WBUT 2012, 2013]

Answer:

To write the characteristic equation $s^3 + 3Ks^2 + (K+2)s + 4 = 0$

To check the necessary conditions:

Necessary conditions hold good.

To check the sufficient condition:

We form Routh array:

	Col 1	Col 2
Row 1 s^3	1	$K+2$
Row 2 s^2	3K	4
Row 3 s^1	$\dfrac{3K(K+2)-4}{3K}$	0
Row 4 s^0	4	

To check for stability:
To have a stable system there should not be any change in sign of Routh elements in the first column of Routh array, i.e.,

$$3K > 0 \quad \text{(From row 2)}$$
$$\Rightarrow \quad K > 0$$
$$\& \quad \frac{3K(K+2)-4}{3K} > 0 \quad \text{(From row 3)}$$
$$\Rightarrow \quad 3K(K+2)-4>0$$
$$\Rightarrow \quad 3K^2+6K-4>0$$

i.e., $K_1, K_2 = \dfrac{-6 \pm \sqrt{36+48}}{6} = \dfrac{-6 \pm \sqrt{84}}{6} = 0.53 - 2.52 = -1.99$

But $K = -2.52$ is not acceptable. Thus the range of K for stability is $0 < K < 0.53$.

3. How many roots of the given polynomial area on the RHP, LHP and on the $j\omega$**-axis?**

$$s^7+3s^6+7s^5+10s^4+11s^3+11s^2+2s+6=0$$

Hence, comment on the stability of the system. **[WBUT 2012]**

Answer:

	Col 1	Col 2	Col 3	Col 4
Row 1 s^7	1	7	11	2
Row 2 s^6	3	10	11	6
Row 3 s^5	$\frac{21-10}{3}=\frac{11}{3}$	$\frac{33-11}{3}=\frac{22}{3}$	$\frac{6-6}{3}=0$	0
Row 4 s^4	$\frac{\frac{11}{3}\cdot 10-22}{\frac{11}{3}} = \frac{\frac{110}{3}-\frac{66}{3}}{\frac{11}{3}}=4$	$\frac{\frac{11}{3}\cdot 11-0}{\frac{11}{3}}=11$	$\frac{\frac{11}{3}\cdot 6-0}{\frac{11}{3}}=6$	0
Row 5 s^3 Sign change	$\frac{4\times\frac{22}{3}-11\times\frac{11}{3}}{4} = \frac{\frac{88}{3}-\frac{121}{3}}{4}=-\frac{11}{4}$	$\frac{4\cdot 0-6\cdot\frac{11}{3}}{4}=-\frac{11}{2}$	0	0
Row 6 s^2	$\frac{\frac{-121}{4}+22}{\frac{-11}{4}}=3$	6	0	0
Row 7 s^1	0	0	0	0
Row 8 s^0	0	0	0	0

Auxiliary equation $A(s) = 3s^2+6=0$

$\frac{dA(s)}{ds} = 6s$

Row 7 s^1	6	0	0	0
Row 8 s^0	6	0	0	0

There are two sign changes in the first column i.e., 4 to –11/4 and –11/4 to 3.
Therefore, two of the seven roots are in the right half of s-plane.
From the auxiliary equation

$3s^2 + 6 = 0$

or, $s^2 + 2 = 0$

$\Rightarrow$ $s^2 = -2$

$\Rightarrow$ $s = \pm j1.41$

The solution of the auxiliary equation say that the system has two roots lying on the imaginary axis of s-plane.
So number of roots lying left side of s-plane is three and two roots line on the $j\omega$-axis.
So the system is unstable.

4. Using Routh's criterion determine the stability, indicating the number of roots in the right half s-plane, of a closed-loop system that has the characteristic equation $s^5 + 2s^4 + 4s^3 + 8s^2 + 16s + 32 = 0$. **[WBUT 2014]**

Answer:

The characteristic equation:

$s^5 + 2s^4 + 4s^3 + 8s^2 + 16s + 32 = 0$

$\therefore$ From Routh Array:

s5	1	4	16
s4	2	8	32
s3	0	0	

All the Routh elements in Row 3 are zero.

$\therefore$ The Auxiliary equation.

$A(s) = 2s^4 + 8s^2 + 32 = 0$

$\Rightarrow$ $s^4 + 4s^2 + 16 = 0$

Now, different the Auxiliary equation with respect to S.

$4s^3 + 8s = 0$

$\therefore$ $s^3 + 2s = 0.$

5. The characteristic equation of a feedback system is **[WBUT 2015]**

$s^4 + 4s^3 + 16s^2 + 16s + 48 = 0$

Check whether the response is oscillatory or not. If so, determine the frequency of oscillation.

Answer:

From the Routh Array:

s^4	1	16	48
s^3	4	16	
s^2	12	48	
s^1	0		

∴ Auxiliary equation

$$A(s) = 12s^2 + 48$$

$$\therefore \quad \frac{dA(s)}{ds} = 24s = 0$$

s^4	1	16	48
s^3	4	16	
s^2	24		
s^1	16		

$$12(j\omega)^2 + 48 = 0$$

$$\Rightarrow \quad 12j^2\omega^2 = -48$$

$$\Rightarrow \quad 12\omega^2 = 48$$

$$\Rightarrow \quad \omega^2 = 4$$

$$\Rightarrow \quad \omega = 2 \text{ rad/sec}$$

6. The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{K}{s(1+sT_1)(1+sT_2)}.$$

Using the Routh-Hurwitz method, determine the necessary conditions for the system to be stable. **[WBUT 2017]**

Answer:

The characteristic equation of the system is

$T_1 T_2 s^3 + (T_1 + T_2)s + s + K = 0$

The Routh's array is formed as:

s^3	T_1T_2	1
s^2	(T_1+T_2)	K
s	$[(T_1+T_2) - K\,T_1T_2]/(T_1+T_2)$	0
s^0	K	

For stability, $K > 0$ and $[(T_1+T_2) - K\,T_1T_2] > 0$ or $K < (T_1+T_2)/\,T_1T_2$

The range of K for stability: $0 < K < (T_1+T_2)/\,T_1T_2$

7. A feedback system has an open loop transfer function $G(s)H(s) = ke^{-s}/s(s^2+2s+1)$. Determine by use of Routh-Hurwitz criterion the maximum value of k for the closed loop system to be stable. Also find the frequency of sustained oscillations. **[WBUT 2019]**

Answer:

Given $G(s)H(s) = \dfrac{ke^{-s}}{s(s^2+2s+1)}$

For low frequency

$$G(s)H(s) = \frac{k(1-s)}{s(s^2+2s+1)} \quad \left[e^{-s} = 1 - s + \frac{s^2}{2}\cdots\cdots\right]$$

$\therefore$ Characteristic equation

$$= 1 + G(s)H(s)$$

$$= 1 + \frac{k(1-s)}{s^3+2s^2+s} = 0$$

$$\Rightarrow \quad s^3 + 2s^2 + s + k - ks = 0$$

$$\Rightarrow \quad s^3 + 2s^2 + (1-k)s + k = 0$$

s^3	1	$(1-k)$
s^2	2	k
s^1	$\dfrac{2(1-k)-k}{2}$	0
s^0	k	0

For maximum value of k

$$2(1-k) - k = 0 \;\Rightarrow\; 2 - 3k = 0$$

$\therefore \quad k = \dfrac{2}{3}$ is the maximum value

$\therefore$ Auxiliary equation

$$2s^2 + k = 0$$

$$s^2 = \frac{-k}{2} = \frac{-\frac{2}{3}}{3} = \frac{-2}{9}$$

$\therefore \quad s_{1,2} = \pm j\dfrac{\sqrt{2}}{3} = \pm j0.47$

$\therefore$ Frequency of sustained oscillation $= \omega = 0.47$ rad/sec.

Long Answer Type Questions

1. a) State the Routh stability criterion. **[WBUT 2006, 2007]**

b) The open loop transfer function of a unity feedback control system is given by

$$G(S)=\frac{K}{(S+2)(S+4)(S^2+6S+25)}$$

By applying the Routh criterion, discuss the stability of the closed loop system as a function of K. Determine the values of which will cause sustained oscillations in the closed loop system. What are the corresponding frequencies?

[WBUT 2006, 2007, 2016]

Answer:

a) A Routh stability criterion is an approach or a tool to find out the stability of a linear time-invariant differential equation system by considering the characteristic equation of the system. We know that for a system to be stable, the poles must lie in the left half of s-plane. For systems with lower order (say up to third order), one can find the pole locations; but for higher order systems, it becomes difficult to find the pole locations and to know about the stability of the system. Routh stability method provides an answer to this problem.

Routh's ***necessary conditions*** for a stable system

For the ***characteristics equation*** of a system

a) There should not be any missing power of s.
b) All the coefficients of the polynomial should be real.
c) All the coefficients of the polynomial should have same sign.

Sufficient condition for a stable system

There should be no change in sign in the elements of the first column of Routh array.

b) $G(s)=\frac{K}{(s^2+6s+25)(s+2)(s+4)}$ $\qquad H(s)=1$

The characteristic equation is $\quad 1+G(s)H(s)=0$

or, $\quad 1+\frac{K}{(s^2+6s+25)(s+2)(s+4)}=0$

or, $\quad s^4+12s^3+69s^2+198s+200+K=0$

The Routh's array is

s^4	1	69	$200+K$
s^3	12	1.98	0
s^2	52.5	$200+K$	
s^1	$198-\frac{(200+K)}{52.5}$	0	
s^0	$200+K$		

For stability $200+K>0$ or $K>-200$

Also, $198-\frac{(200+K)12}{52.5}>0$ or $K<666.25$

The range for K for stability is $\quad -200<K<666.25$

If $K = 666.25$ then the first column is $(1, 12, 52.5, 0, 866.25)$.

2. Using Routh–Hurwitz Stability Criterion determine the maximum feedback gain K for which the closed loop system will be stable if the open-loop transfer function is $G(s) = \frac{5(1-0.2s)}{(s^2+3.2s+4)}$. Calculate the frequency of oscillation at this gain.

[WBUT 2011]

Answer:

$G(s) = \frac{4(1-0.25)}{s^2+3.2s+4}$ $\qquad H(s) = K$

The characteristic equation is

$$1 + G(s)1 + (s) = 0$$

$$\Rightarrow 1 + \frac{K5(1-0.2s)}{s^2+3.2s+4} = 0$$

$$\Rightarrow s^2 + 3.2s + 4 + 5K(1-0.2s) = 0$$

$$\Rightarrow s^2 + 3.2s + 4 + 5K - Ks = 0$$

$$\Rightarrow s^2 + (3.2-k)s + (4-5k) = 0$$

Routh's Array is

s^2	1	$4+5k$
s^1	$3.2-k$	–
s^0	$4+5k$	–

$3.2 - k > 1$

$k < 3.2$

For stability

a) $K > 1$

b) $K < 3.2$

c) $K < -\frac{4}{5}$

Range of stability

$0 < K < 3.2$

The system will become stable of $K = 3.2$

Oscillations will occur when $K = 3.2$

$$s^2 + (4+5K) = 0$$

$$\Rightarrow s^2 + (4+5\times3.2) = 0$$

$$\Rightarrow s^2 + (4+16) = 0$$

$$\Rightarrow s^2 = -20$$

$$\Rightarrow s = \pm j4.4721$$

Frequency of oscillation is 4.4721 rad./sec.

3. Write short notes on the following:
a) Routh's stability criterion [WBUT 2014]
b) Relative stability and steady state stability [WBUT 2014]

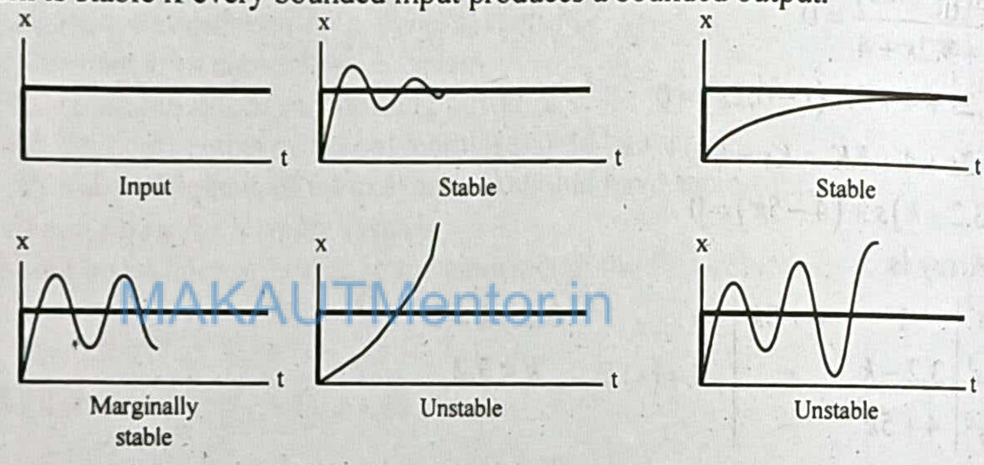
Answer:

a) Routh's stability criterion:

Refer to Question No. 1(a) of Long Answer Type Questions.

b) Relative stability and steady state stability:

The stability of a system relates to its response to inputs or disturbances. A system which remains in a constant state unless affected by an external action and which returns to a constant state when the external action is removed can be considered to be stable. A systems stability can be defined in terms of its response to external impulse inputs.. A system is stable if its impulse response approaches zero as time approaches infinity. The system stability is defined in terms of bounded inputs and outputs.
A system is stable if every bounded input produces a bounded output.



In above examples at t = infinity i.e. at steady state, the systems are showing stability, instability and marginal stability. So the steady state stability of a system implies its ability to return to its initial state or to to some other state, which is not changing with time after a small disturbance from the state.

It's not enough to know that a system is stable or unstable. If a system is just barely stable, then a small gain in a system parameter could push the system over the edge, and one will often want to design systems with some margin of error. Relative stability refers towards the degree of goodness of a system in terms of stability. To assess the relative stability of a system we use two frequency domain specifications, such as: gain margin and phase margins. Higher the values of margins, both in positive side, better is the stability. Phase margin is the most widely used measure of relative stability when working in the frequency domain. We define gain margin as the amount that the frequency response would have to increase to move to the -1 point. Phase margin is defined as as the angle that the frequency response would have to change to move to the -1 point.

TIME DOMAIN ANALYSIS

Multiple Choice Type Questions

1. If maximum overshoot is 100%, the damping ratio is [WBUT 2006, 2012, 2019]
a) 1 b) 0 c) 0.5 d) infinity
Answer: (b)

2. The type of a transfer function denotes the number of [WBUT 2007, 2010, 2014, 2016]
a) zeros at origin b) poles at infinity c) poles at origin d) finite poles
Answer: (c)

3. The characteristics equation of a system is $s^2 + 2s + 2 = 0$. The system is [WBUT 2007, 2010, 2018]
a) critically damped
b) underdamped
c) overdamped
d) none of these
Answer: (b)

4. Addition of a pole to the closed loop transfer function [WBUT 2007, 2010, 2014]
a) increases rise time
b) decreases rise time
c) increases overshoot
d) has no effect
Answer: (c)

5. A 2nd order system exhibits 100% overshoot. Its damping coefficient is [WBUT 2007, 2011]
a) equal to 0 b) equal to 1 c) < 1 d) > 1
Answer: (a)

6. A system has a single pole at origin. Its impulse response will be [WBUT 2008, 2010, 2011]
a) constant b) ramp c) decaying exponential d) oscillatory
Answer: (a)

7. The setting time for a second order system responding to a step input with 5% overshoot is [WBUT 2008, 2012]
a) $4 / \xi\omega_n$ b) $2 / \xi\omega_n$ c) $3 / \xi\omega_n$ d) $5 / \xi\omega_n$
Answer: (c)

8. The setting time for an under damped second order system [WBUT 2011]
a) increases with damping ratio
b) reduces with damping ratio
c) does not depend on damping ratio
d) may increase or decrease with damping ratio
Answer: (d)

9. Without affecting steady–state error, maximum overshoot can be decreased by [WBUT 2011, 2016]

a) derivative error control
b) integral error control
c) gain adjustment
d) proportional error control

Answer: (a)

10. Area under a unit impulse function is [WBUT 2011]

a) infinity
b) zero
c) unity
d) none of these

Answer: (c)

11. A feedback system with the transfer function is a [WBUT 2011]

$$G(s)=\frac{6\,(s+1)\,(s+6)}{s^3\,(s+2)\,(s+4)}$$

a) type 5 system
b) type 3 system
c) type 3 system
d) type 0 system

Answer: (c)

12. The damping ratio of characteristics equation $s^2+2s+8=0$ is [WBUT 2012]

a) 0.353
b) 0.350
c) 0.30
d) 0.333

Answer: (a)

13. The steady state error of unit ramp input in the type-2 system, is [WBUT 2012]

a) ∞
b) 0
c) 1
d) 5

Answer: (b)

14. Given that $G(s)=\frac{k}{s^2(s+2)(s+3)}$, the type and order of the system is [WBUT 2012]

a) 3 & 3
b) 2 & 4
c) 3 & 1
d) 3 & 0

Answer: (b)

15. The characteristic equation of a system is $s^2+2s+2=0$, the system is [WBUT 2013]

a) critically damped
b) underdamped
c) overdamped
d) none of these

Answer: (b)

16. Addition of a zero to the second order closed loop transfer function [WBUT 2013]

a) improves the transient response characteristics
b) increase effective damping
c) decrease peak overshoot
d) all of these

Answer: (d)

17. The location of the closed loop conjugate poles of poles on $j\omega$ axis indicates that the system is [WBUT 2013]
a) absolutely stable b) conditionally stable
c) marginally stable d) unstable
Answer: (c)

18. The steady state error for a type-3 system in following a unit step input is [WBUT 2014]
a) zero b) infinity c) one d) none of these
Answer: (a)

19. Given that $G(s)=\frac{K}{s^2+(s+2)(s+3)}$ the type and order system is [WBUT 2014]
a) 3 and 3 b) 2 and 4 c) 3 and 1 d) 3 and 0
Answer: (b)

20. A second-order feedback system has two closed loop poles at the same location in the S-plane and has no finite zeros. The nature of unit step response of the system is [WBUT 2015]
a) under damped b) over damped
c) critically damped d) oscillatory
Answer: (c)

21. A negative feedback control system has open loop transfer function $G(s)H(s)=\frac{k}{s^2(s+a)}$. The closed loop system is [WBUT 2015]
a) unstable b) stable
c) marginally stable d) conditionally stable
Answer: (c)

22. As compared to an open loop system, a closed loop system is [WBUT 2015]
a) more stable and more accurate b) more stable and less accurate
c) less stable and more accurate d) less stable and less accurate
Answer: (b)

23. The unit step response of a second-order system is [WBUT 2015]
$$C(t)=1-1.125e^{-3t}\sin(4t-0.927).$$
The damping ratio and the damped frequency in rad/s are respectively
a) 0.5, 3 b) 0.5, 4 c) 0.6, 5 d) none of these
Answer: (c)

24. A system has a single pole of origin. Its impulse response will be [WBUT 2016]
a) constant b) ramp c) decaying in nature d) oscillatory
Answer: (a)

25. The characteristic equation of a system is $s^2+2s+4=0$. The system is [WBUT 2016]

a) critically damped
b) overdamped
c) undamped
d) underdamped

Answer: (d)

26. Addition of zero to the closed loop transfer function [WBUT 2018]

a) increase rise time
b) decrease rise time
c) increase overshoot
d) has no effect

Answer: (b)

27. The value of ξ for a second order system is zero. The step response will be [WBUT 2018]

a) over damped
b) critically damped
c) under damped
d) sustained oscillatory

Answer: (d)

28. If the gain of an open loop system is doubled, the gain margin [WBUT 2019]

a) is not affected
b) gets doubled
c) becomes half
d) become 1/4th

Answer: (c)

29. Feedback control system is basically [WBUT 2019]

a) High pass filter
b) Low pass filter
c) Band pass filter
d) Band stop filter

Answer: (b)

30. For type-1, second order system the resonance peak will occur when the system gain is at the [WBUT 2019]

a) underdamping
b) critical damping value
c) overdamping value
d) none of these

Answer: (b)

Short Answer Type Questions

1. A system has $G(s)=\dfrac{20}{s^2+5s+5}$ & unity feedback. Find [WBUT 2009, 2012]

i) ω_n ii) ξ iii) ω_d iv) M_p v) T_s

Answer:

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)+H(s)}$$

But $H(s)=1$ $\quad G(s)=\dfrac{20}{s^2+5s+5}$

$$\frac{C(s)}{R(s)}=\frac{\frac{20}{s^2+5s+5}}{1+\frac{20}{s^2+5s+5}\cdot 1}=\frac{20}{s^2+5s+5+20}=\frac{20}{s^2+5s+25}$$

Comparing the above equation with the standard second order transfer function, we get

i) $\omega_n^2 = 25$

or, $\omega_n = 5$ rad/sec.

ii) $2\xi\omega_n = 5$

or, $\xi = \frac{5}{2\times 5} = \frac{1}{2} = 0.5$

iii) $\omega_d = \omega_n\sqrt{1-\xi^2} = 25\sqrt{1-0.5^2} = 25\sqrt{1-0.25}$

$= 5\sqrt{0.75} = 4.33$ rad/sec.

iv) $M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} = e^{-\frac{0.5\times\pi}{0.866}} = e^{-\frac{1.57}{0.866}} = e^{-1.8129} = 0.163$

v) $T_s = \frac{4}{\xi\omega_n} = \frac{4}{.5\times 5} = 1.6\,\text{sec}$

2. A unity feedback system has $G(s)=\frac{180}{s(s+6)}$ & $r(t)=4t$.

Determine,

i) the steady state error

ii) the value of k to reduce the error by 6%. [WBUT 2009, 2013, 2018]

Answer:

i) $G(s)H(s)=\frac{k}{s(s+6)}=\frac{180}{s(s+6)}$

and $R(s)=\frac{4}{s^2}\left[R(s)=L\,r(t)\right]$

$\therefore\quad e_{ss} = \underset{s\to 0}{Lt}\frac{s\cdot R(s)}{1+G(s)H(s)}$

$= \underset{s\to 0}{Lt}\, s\cdot\frac{1}{\left[1+\frac{180}{s(s+6)}\right]}\cdot\frac{4}{s^2} = \underset{s\to 0}{Lt}\frac{1\times 4}{s+\frac{180}{(s+6)}} = \frac{4}{0+\frac{180}{6}} = \frac{24}{180} = 0.13$

ii) To have e'_{ss} less by 6% of previous e_{ss}

$\therefore\quad e'_{ss} = 0.94\times e_{ss} = 0.94\times 0.13 = 0.1222$

$\therefore\quad e'_{ss} = \underset{s\to 0}{Lt}\frac{s\cdot 1}{\left[1+\frac{k}{s(s+6)}\right]}\cdot\frac{4}{s^2} = \frac{4}{k/6} = \frac{24}{k} = 0.1222$

$\therefore \quad k=\dfrac{24}{0.1222}=196.40$

3. A system is defined by $G(s)=\dfrac{k}{s\,(Ts+1)}$ **[WBUT 2011]**

Calculate the steady state position and velocity error due to a unit ramp input and hence the loop gain to reduce the error by 10%.

Answer:

$$G(s)=\frac{k}{s(Ts+1)} \qquad R(s)=\frac{1}{s^2}$$

Position error constant (K_p)

$$e_{ss}=\underset{s\to0}{Lt}\frac{s\cdot R(s)}{1+G(s)}=\underset{s\to0}{Lt}\frac{s\cdot\frac{1}{s^2}}{1+\frac{k}{s(Ts+1)}}=\underset{s\to0}{Lt}\frac{\frac{1}{s}}{\frac{s^2T+s+k}{s^2T+s}}=\underset{s\to0}{Lt}\frac{s^2T+s}{s\left(s^2T+s+k\right)}$$

$$\Rightarrow \quad \underset{s\to0}{Lt}\frac{sT+1}{s^2T+s+k}=\frac{1}{k}$$

Using the definition of K_p.

$$e_{ss}=\frac{1}{1+K_p}$$

$$\Rightarrow \quad \frac{1}{k}=\frac{1}{1+K_p}$$

$$\Rightarrow \quad 1+K_p=k$$

$$\Rightarrow \quad K_p=k-1$$

Velocity error constant (K_v)

$$R(s)=\frac{1}{s^2}$$

$$e_{ss}=\underset{S\to0}{Lt}\frac{sR(s)}{1+G(s)H(s)}=\frac{1}{k}$$

$$e_{ss}=\frac{1}{K_v}$$

$$\frac{1}{k}=\frac{1}{k_v} \qquad \Rightarrow k_v=k$$

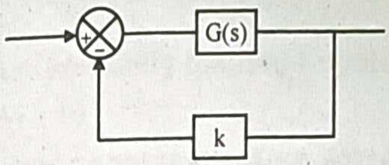
$$e_{ss}=\frac{1}{1+K_p}+\frac{1}{K_v}=\frac{1}{k}+\frac{1}{k}=\frac{2}{k}$$

From the given data

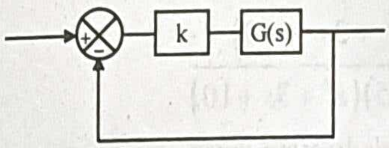
$$0.9 = \frac{2}{k}$$

$$\Rightarrow \quad k = \frac{2}{0.9} = 2.22$$

4. If $G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ is an under damped second order system, what happens to the peak overshoot of the closed loop system shown in figure below, if the gain k is increased. Substantiate your answer. Assume k>0 **[WBUT 2011]**



How does the peak overshoot change for the same value of k if the configuration is as shown in figure



Answer:

1st Part:

Characteristic polynomial

$$1 + kG(s) = 0$$

$$1 + \frac{k\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = 0$$

$$\Rightarrow \quad s^2 + 2\xi\omega_n s + \omega_n^2(1+k) = 0$$

$$\Rightarrow \quad s^2 + 2\xi\frac{\sqrt{1+k}}{\sqrt{1+k}}\omega_n s + \omega_n^2(1+k) = 0$$

The effective natural frequency $= \sqrt{1+k}\,\omega_n$ and effective damping ratio $= \frac{\xi}{\sqrt{1+k}} = \xi'$

As $k\uparrow, \xi'\downarrow$, so overshoot increases.

2nd Part:

Characteristic polynomial is $1 + kG(s) = 0$

$$1 + \frac{k\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = 0$$

$$\Rightarrow \quad s^2 + 2\xi\omega_n s + \omega_n^2(1+k) = 0$$

$$\Rightarrow \qquad s^2 + 2\xi \frac{\sqrt{1+k}}{\sqrt{1+k}} \omega_n s + \omega_n^2 (1+k) = 0$$

The effective natural frequency $= \sqrt{1+k}\,\omega_n$ and effective damping ratio $= \dfrac{\xi}{\sqrt{1+k}} = \xi'$

As $k\uparrow, \xi'\downarrow$, so overshoot increases.
So, overshoots remains same in both of the cases.

5. The forward path transfer function of a unity feedback system is given by $G(s)=\dfrac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$. Determine the steady state error for the input $r(t)=2+3t$.

[WBUT 2012]

Answer:

$$r(t) = 2+3t$$

$$R(s) = L[2+3t] = \left[\frac{2}{s}+\frac{3}{s^2}\right] = \frac{2s+3}{s^2}$$

$$G(s)H(s) = \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$$

Putting the formula for the steady state error

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)} = \lim_{s\to 0} \frac{s\cdot\dfrac{2s+3}{s^2}}{1+\dfrac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}}$$

$$= \lim_{s\to 0} s\cdot \frac{\dfrac{2s+3}{s^2}}{\dfrac{s^2(s+5)(s^2+3s+10)+5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}}$$

$$= \lim_{s\to 0} s\cdot \frac{\dfrac{2s+3}{s^2}}{\dfrac{s^2(s+5)(s^2+3s+10)+5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}}$$

$$= \lim_{s\to 0} s\cdot \frac{\dfrac{2s+3}{s^2}\cdot s^2(s+5)(s^2+3s+10)}{s^2(s+5)(s^2+3s+10)+5s^2+10s+500}$$

$$= \lim_{s\to 0} s \cdot \frac{(2s+3)(s+5)(s^2+3s+10)}{(s^3+5s^2)(s^2+3s+10)+5s^2+10s+500}$$

$$= \lim_{s\to 0} \frac{(2s+3)(s+5)(s^2+3s+10)}{(s^2+5s)(s^2+3s+10)+5s^2+10s+500}$$

$$= \lim_{s\to 0} \frac{(2s+3)(s+5)(s^2+3s+10)}{s^4+8s^3+25s^2+50s+5s^2+10s+500}$$

$$= \lim_{s\to 0} \frac{(2s+3)(s+5)(s^2+3s+10)}{s^4+8s^3+30s^2+60s+500} = \frac{3\cdot 5\cdot 10}{500} = 0.3$$

6. The overall transfer function of a unity feedback system is given by

$$C(s)/R(s) = 10/s^2+6s+10$$

Find the values of the static error constants. Also determine the steady state error for input $r(t)=1+t+t^2/2$. **[WBUT 2013]**

Answer: $G(s) = \dfrac{10}{s^2+6s+10}$

Position error constant

$$K_P = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{10}{s^2+6s+10} = 1$$

Velocity error constant

$$K_V = \lim_{s\to 0} sG(s) = \lim_{s\to 0} s \cdot \frac{10}{s^2+10s+10} = 0$$

Acceleration error constant

$$K_a = \lim_{s\to 0} s^2G(s) = \lim_{s\to 0} s^2 \frac{10}{s^2+10s+10} = 0.$$

$$r(t) = 1+t+\frac{t^2}{2}$$

$$R_{ss} = \frac{R_1}{1+K_P} + \frac{R_2}{K_V} + \frac{R_3}{K_a} = \frac{1}{1+10} + \frac{1}{0} + \frac{1}{0} = \frac{1}{11}$$

7. Determine the damping ratio, undamped natural frequency, delay time, rise time, peak time and maximum overshoot for the second order system whose characteristic equation is given by $s^2+2.5s+10=0$. **[WBUT 2014]**

Answer:

The characteristic equation is $s^2+2.5s+10=0$

$\therefore \omega_n = \sqrt{10} = 3.16$ rad/sec

and $2\xi\omega_n = 2.5$

$$\therefore \quad \xi = \frac{2.5}{2\times 3.16} = 0.4$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 3.16\sqrt{1-(0.4)^2} = 2.66 \text{ rad/sec}$$

$$\therefore \quad \beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}\frac{\sqrt{1-0.4^2}}{0.4} = 66.42 = 1.16 \text{ rad}$$

$$\therefore \text{ Rise time } (t_r) = \frac{\pi-\beta}{\omega_d} = \frac{\pi-1.16}{2.66} = 0.745 \text{ sec}$$

$$\therefore \text{ Peak time } t_p = \frac{\pi}{\omega_d} = \frac{3.14}{2.66} = 1.18 \text{ sec}$$

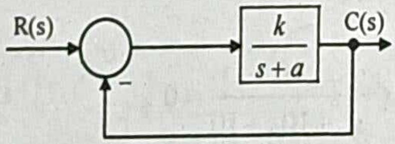
$$\therefore \text{ Peak over shoot } (M_p) = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} = e^{-\frac{0.4\times\pi}{\sqrt{1-0.4^2}}} = 0.2538$$

$\therefore$ % $M_P = 25.38$

$$\therefore \text{ Delay time } (t_d) = (1+0.7\xi)/\omega_n \qquad [\because 0<\xi<1]$$

$$= \frac{(1+0.7\times 0.4)}{3.16} = 0.4050 \text{ sec.}$$

8. For a first order system shown below, find the time constant, rise time and setting time for step response, given $k=12$ and $a=4$ **[WBUT 2015]**



Answer:

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\frac{k}{s+a}}{1+\frac{k}{s+a}\cdot 1} = \frac{\frac{k}{s+a}}{\frac{s+a+k}{s+a}} = \frac{k}{s+a+k}$$

Now, $k=12$

and $a=4$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{12}{s+4+12} = \frac{12}{s+16}$$

For unit step response:

$$R(s) = \frac{1}{s}$$

$$\therefore \quad C(s) = R(s)\cdot\frac{12}{s+16} = \frac{1}{s}\cdot\frac{12}{s+16}$$

$$C(s)=\frac{12}{16}\cdot\frac{16}{s(s+16)}$$

$\therefore$

$$C(t)=\frac{3}{4}-\frac{3}{4}e^{-16t}=\frac{3}{4}\left(1-e^{-16t}\right)$$

$\therefore$

$\therefore$ The time constant $(T)=\frac{1}{16}=0.0625\text{ sec}$

$\therefore$ Rise time $(t_r)=0.566\text{ sec}$

$\therefore$ Settling time $(t_s)=-0.0159\text{ sec}$

9. A unity feedback control system has the open loop transfer function

$$G(s)=\frac{k}{s(s^2+4s+20)}$$

Specify the type of the system.
Find the static error constants and the corresponding steady state errors. Assume that the system is stable. **[WBUT 2015]**

Answer:

Type No. = 1,

$$K_p=\lim_{s\to 0}G(s)H(s)=G(s)=\infty$$

$$e_{ss}=\frac{1}{1+K_p}=0$$

$$K_v=\lim_{s\to 0}G(s)H(s)=\lim_{s\to 0}G(s)=\frac{k}{20}\text{ sec}^{-1}$$

$$e_{ss}=\frac{20}{k}\text{ sec.}$$

$$K_a=\lim_{s\to 0}s^2G(s)H(s)=\lim_{s\to 0}s^2G(s)=0\text{ sec}^{-2}$$

$$e_{ss}=\frac{1}{K_a}=\infty\text{ sec}^2$$

10. Consider the unit step response of a unity feedback system whose open loop transfer function $G(s)=\frac{1}{s(s+1)}$. Obtain the rise time, peak time, maximum overshoot & settling time. **[WBUT 2016]**

Answer:

$$G(S)=\frac{1}{s(s+1)}$$

$$\frac{C(S)}{R(S)}=\frac{G(S)}{1+G(S)H(S)}$$

But $H(S)=1$ and $G(S)=\frac{1}{s(s+1)}$

Therefore $\frac{C(S)}{R(S)}=\frac{1}{s^2+s+1}$ ...(i)

Comparing the equation (i) with the standard second order transfer function, we get

$\therefore \quad \omega_n^2=1$

or $\quad \omega_n=1$ rad/sec

$2\xi\omega_n=1$

or $\quad \xi=\frac{1}{2\omega_n}=\frac{1}{2\times 1}=0.5$

$\therefore \quad \omega_d=\omega_n\sqrt{1-\xi^2}=1\sqrt{1-0.5^2}=.866$ rad/sec

$\beta=\tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}=\tan^{-1}\frac{.866}{0.5}=59.99°=1.04$ rad

1. Rise time $t_r=\frac{\pi-\beta}{\omega_d}=\frac{\pi-1.04}{.866}=2.42$ sec

2. Peak time $t_p=\frac{\pi}{\omega_d}=\frac{3.14}{.866}=3.625$ sec

3. Peak overshoot

$M_p=e\frac{-\xi\pi}{\pi}=e^{\frac{-.5\times 3.14}{.866}}=6.12$

$\therefore \%M_p=61.2$

4. Setting time (with 2% tolerance band)

$t_s=\frac{4}{\xi\omega_n}=\frac{4}{.5\times 1}=\frac{40}{5}=8$ sec

11. Sketch the time-domain response of $C(t)$ of a typical underdamped, second order system to a unit step input $r(t)$. On the above sketch indicate and define the following time domain specifications: **[WBUT 2017]**

i) Maximum peak overshoot
ii) Rise time
iii) Settling time
iv) Steady state error.

Answer:

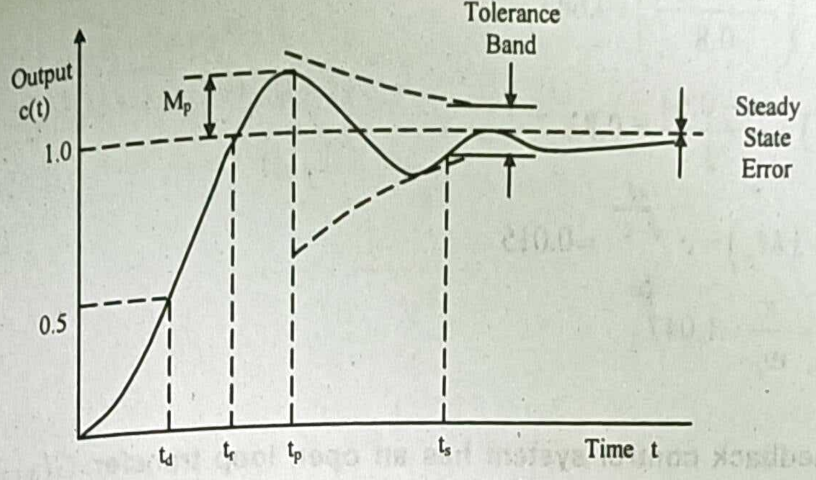


Fig: 1 Time Domain performance parameters

12. A unity feedback system has an open loop transfer function $G(s)=\frac{25}{s(s+8)}$. Find its damping ratio, natural frequency, rise time, over shoot & time required to reach the peak output. **[WBUT 2018]**

Answer:

Open loop transfer function $G(s)=\frac{25}{s(s+8)}$

$\therefore$ As the unity feedback system, so the close loop transfer function will be

$$\text{CLTF}=\frac{25}{s(s+8)+25}=\frac{25}{s^2+8s+25}$$

Comparing with basic 2nd order CLTF,

$$\omega_n^2=25$$

$$\therefore \quad \omega_n=5$$

and $\quad 2\xi\omega_n=8$

$$\xi=\frac{8}{2\times 5}=0.8$$

$\therefore$ Damping ratio $(\xi)=0.8$

Natural frequency $(\omega_n)=5$ rad/sec

Rise time $(t_r)=\frac{\pi-\beta}{\omega_d}$

where $\beta=\tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$; $\omega_d=\omega_n\sqrt{1-\xi^2}=5\left(\sqrt{1-(0.8)^2}\right)$

$$= \tan^{-1}\left(\frac{\sqrt{1-0.8^2}}{0.8}\right) = 0.643$$

$$\therefore \text{ Rise time } (t_r) = \frac{\pi - 0.64}{3} = 0.83$$

$$\text{Peak over sheet } (M_p) = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = 0.015$$

$$\text{Peak time } (t_p) = \frac{\pi}{\omega_d} = 1.047$$

13. A unity feedback control system has an open loop transfer $G(s) = 5/s(s+1)$ Find the rise time, percentage overshoot, peak time & settling time (2% error) for a step input of 10 units. **[WBUT 2019]**

Answer:

Similar to Question No. 10 of Short Answer Type Quesitons.

14. For a closed loop system with $G(s) = 15/(s+2)(s+5)$ and $H(s) = 1$ calculate the generalized error coefficient & find error series for input of $3 + 8t + 5t^2/2$. **[WBUT 2019]**

Answer:

Expression relating error signal and input in s-domain

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)} = \frac{1}{1+G(s)} \quad \text{[For unity feedback]}$$

$$\Rightarrow \quad \frac{E(s)}{R(s)} = \frac{1}{1+\dfrac{15}{(s+2)(s+5)}} = \frac{(s+2)(s+5)}{(s+2)(s+5)+15}$$

$$\Rightarrow \quad \frac{E(s)}{R(s)} = \frac{s^2+7s+1}{s^2+7s+2}$$

Using long division method,

$$\frac{E(s)}{R(s)}=\frac{10+7s+s^2}{25+7s+s^2}$$

$$25+7s+s^2\,\Big)\;10+7s+s^2\;\left(\frac{2}{5}+\frac{21s}{125}-\frac{72s^2}{3125}+\cdots\cdots\right.$$

$$10+\frac{14s}{5}+\frac{2s^2}{5}$$

$$-\quad-\quad-$$

$$\frac{21s}{5}+\frac{3s^2}{5}$$

$$\frac{21s}{5}+\frac{147s^2}{125}+\frac{21s^3}{125}$$

$$-\quad-\quad-$$

$$-\frac{72s^2}{125}-\frac{21s^3}{125}$$

$$-\frac{72s^2}{125}-\frac{504s^3}{3125}-\frac{72s^4}{3125}$$

$$+\quad+\quad+$$

$$E(s)=\frac{2}{5}+\frac{21}{125}s-\frac{72}{3125}s^2+$$

$$\Rightarrow R(s)=\frac{2}{5}R(s)+\frac{21}{125}sR(s)-\frac{72}{3125}s^2R(s)+$$

$\Rightarrow$ Error series

where, generalized coefficients are

$$C_0=\frac{2}{5},\ C_1=\frac{21}{125},\ C_2=\frac{-72}{3125}$$

Taking inverse Laplace of the error series

$$e(t)=\frac{2}{5}r(t)+\frac{21}{125}\dot{r}(t)-\frac{72}{3125}\ddot{r}(t)$$

$$r(t)=3+8t+\frac{5}{2}t^2,\ \dot{r}(t)=8+5t,\ \ddot{r}(t)=5$$

$$\therefore\quad e(t)=\frac{2}{5}\left[3+8t+\frac{5}{2}t^2\right]+\frac{21}{125}(8+5t)-\frac{72}{3125}\times 5\left(\frac{6}{5}+\frac{168}{125}-\frac{360}{3125}\right)+\left(\frac{16}{5}+\frac{105}{125}\right)t+t^2$$

$$\Rightarrow\quad e(t)=2.42+4.04t+t^2$$

Long Answer Type Questions

1. a) Define error coefficients corresponding to step ramp & parabolic inputs.
[WBUT 2006, 2008]

b) A unity feedback closed loop second order system has a transfer function $\frac{81}{s^2+0.6s+9}$ & is excited by a step input of 10 units. Find out its steady state errors.

[WBUT 2006, 2008, 2017]

Answer:

1st Part:

(1) Position Error Constant (K_p)

Position Error Constant is defined for a unit step input i.e. $R(s)=\frac{1}{s}$. The steady state error of the system for a unit step input is called Position Error. K_p is defined as

$$K_p = \underset{s\to 0}{Lt}\, G(s)H(s) \qquad \text{.... (1)}$$

For a unit step input, $R(s)=\frac{1}{s}$

$$\therefore \quad e_{ss} = \underset{s\to 0}{Lt}\frac{s\cdot R(s)}{1+G(s)H(s)} = \underset{s\to 0}{Lt}\frac{1}{1+G(s)H(s)}\cdot s\cdot\frac{1}{s}$$

$$= \underset{s\to 0}{Lt}\frac{1}{1+G(s)H(s)} = \frac{1}{1+\underset{s\to 0}{Lt}\,G(s)H(s)}$$

Using the definition of K_p

$$e_{ss} = \frac{1}{1+K_p} \qquad \text{.... (2)}$$

The equation 2 says

- As K_p increases e_{ss} increases and accuracy of the system increases
- At $K_p=\infty$, $e_{ss}=0$

(2) Velocity Error Constant (K_v)

Velocity Error Constant is defined for unit ramp, i.e. $R(s)=\frac{1}{s^2}$

$$\text{Now,} \quad e_{ss} = \underset{s\to 0}{Lt}\frac{s\cdot R(s)}{1+G(s)H(s)}$$

$$= \underset{s\to 0}{Lt}\left[\frac{s}{1+G(s)H(s)}\right]\cdot\frac{1}{s^2} = \underset{s\to 0}{Lt}\frac{1}{s+sG(s)H(s)} = \frac{1}{\underset{s\to 0}{Lt}\,sG(s)H(s)}$$

$$\Rightarrow \quad e_{ss} = \frac{1}{K_v} \qquad \text{.... (3)}$$

where, $K_v = \underset{s\to 0}{Lt}\, sG(s)H(s)$ is defined as Velocity Error Co-efficient.

We conclude that

- $K_v\uparrow$ $e_{ss}\downarrow$ and accuracy $\uparrow$

$e_{ss} \to 0$ as $K_v \to \infty$

Unit of K_v is $\sec^{-1}$

(3) *Acceleration Error Constant (K_a)*

Acceleration Error Constant is defined for unit parabolic input i.e.,

$$R(s) = \frac{1}{s^3}$$

Now, $$e_{ss} = \underset{s\to 0}{Lt}\frac{s\cdot R(s)}{1+G(s)H(s)}$$

$$= \underset{s\to 0}{Lt}\left[\frac{s}{1+G(s)H(s)}\right]\cdot\frac{1}{s^3} = \underset{s\to 0}{Lt}\frac{1}{\left(s^2+s^2G(s)H(s)\right)} = \frac{1}{\underset{s\to 0}{Lt}\, s^2G(s)H(s)}$$

$$= \frac{1}{K_a} \qquad \text{.... (4)}$$

where, $K_a = \underset{s\to 0}{Lt}\; s^2G(s)H(s)$ is defined as Acceleration Error Co-efficient.

The Unit of K_a is $\sec^{-2}$

2nd part:

Closed loop transfer function is given by

$$\frac{81}{s^2+0.6s+9} = \frac{81}{s^2+0.6s\left(1+\dfrac{9}{s^2+0.6s}\right)}$$

$$= 9\times\left[\frac{\dfrac{9}{s^2+0.6s}}{1+\dfrac{9}{s^2+0.6s}}\right] = G_1(s)\times\left[\frac{G(s)}{1+G(s)H(s)}\right]$$

where, $G_1(s)=9,\; H(s)=1$

and $G(s) = \dfrac{9}{s(s+0.6s)} = G(s)H(s) =$ open loop transfer function.

$\therefore$ Steady state error $= \lim\limits_{s\to 0}\dfrac{1}{1+K_p}$

where, $K_p = \lim\limits_{s\to 0}G(s)H(s) = \infty$

$\therefore \quad e_{ss} = \dfrac{1}{\infty} = 0.$

2. The open loop transfer function of a unity feedback system is given by

$$G(s)=\frac{k}{s(Ts+1)}$$

where k & T are positive constants. By how much should the amplifier gain be reduced so that the peak overshoot of unit step response of the system is reduced from 75% to 25%?

[WBUT 2007, 2018]

Answer:

$$G(s)=\frac{k}{s(Ts+1)}$$

Let the value of damping ratio is ξ_1 when the peak overshoot is 75% and ξ_2 when the peak overshoot is 25%

$$M_P=e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$$

for $M_P=75\%$ $\qquad \xi=\xi_1=0.091$

for $M_P=25\%$ $\qquad \xi=\xi_2=0.4037$

$$\text{Transfer function}=\frac{G(s)}{1+G(s)H(s)}=\frac{k}{Ts^2+s+k}=\frac{C(s)}{R(s)}$$

or, $$\frac{C(s)}{R(s)}=\frac{k/T}{s^2+\frac{1}{T}s+\frac{k}{T}}$$

Therefore $\omega_n=\sqrt{k/T}$ and $2\xi\omega_n=\frac{1}{T}$

Let the value of $k=k_1$ when $\xi=\xi_1$ and $k=k_2$ when $\xi=\xi_2$

Since $$2\xi\omega_n=\frac{1}{T}$$

$$\xi=\frac{1}{2T\omega_n}=\frac{1}{2\sqrt{kT}}$$

$$\frac{\xi_1}{\xi_2}=\sqrt{\frac{k_2T}{k_1T}}=\sqrt{\frac{k_2}{k_1}}$$

or, $$\frac{k_2}{k_1}=\left(\frac{0.091}{0.4031}\right)=0.0508$$

or the amplifier gain has to be reduced by a factor $\frac{1}{0.0508}=20$.

3. Prove that for an under damped second order system defined by $G(s)=\frac{\omega_n^2}{(s^2+2\xi s\omega_n+\omega_n^2)}$, the peak overshoot due to a unit step input depends

on the damping ξ only. What is the locus of constant ω_n roots? Symbols carry usual significance. **[WBUT 2011]**

Answer:

1st Part:

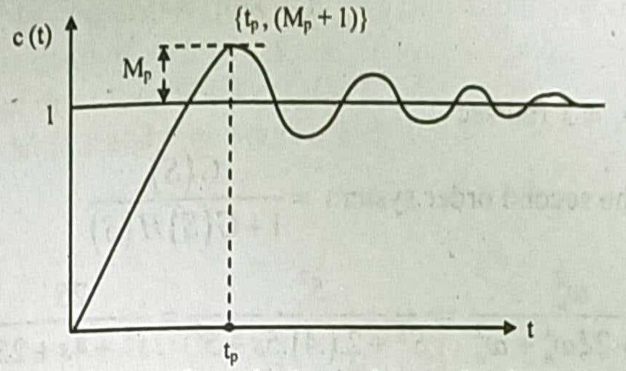


Fig: 1 Step response of a underdamped second order system

From figure 1, at $t = t_p$, i.e., at peak time

$$c(t)\big|_{t=t_p} = 1 + M_P \qquad = 1 - e^{-\frac{\sigma\pi}{\omega_d}}\left(\cos\omega_d\frac{\pi}{\omega_d} + \frac{\sigma}{\omega_d}\sin\pi\right)$$

$$= 1 - e^{-\frac{\sigma\pi}{\omega_d}}\left[\cos\pi + \frac{\sigma}{\omega_d}.0\right] \qquad = 1 + e^{-\frac{\sigma\pi}{\omega_d}} = 1 + M_P$$

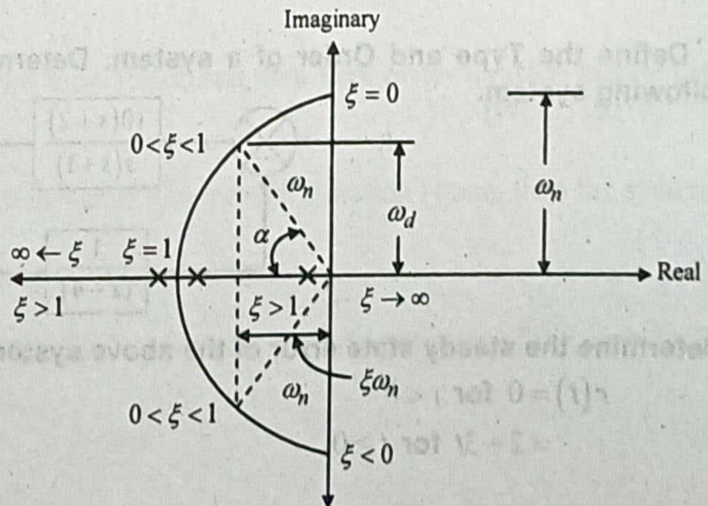
$$M_P = e^{\frac{-\xi\omega_n\cdot\pi}{\omega_n\sqrt{1-\xi^2}}}$$

$$M_P = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}, \text{ for } 0 \le \xi < 1 \qquad \text{......(1)}$$

$$\text{Percent peak overshoot} = \%M_P = \left[e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}\right] \times 100 \qquad \text{......(2)}$$

Hence peak overshoot depends on ξ only.

2nd Part: Figure below shows the general pole locus for a second order system with fixed ω_n and variable damping ratio/factor. It can be seen that as ξ increase the poles sketch a circular locus of radius ω_n and move away from imaginary axis. The locus meets the negative real axis at ω_n. At this point it separates but travels along the real axis, one travels towards zero and other towards infinity.



4. A second order control system, having $\xi = 0.4$ & $W_n = 5$ rad/sec, is subject to a step input. Determine (i) transfer function, (ii) t_r, (iii) t_p, (iv) t_s for 2% tolerance, (v) M_p. **[WBUT 2016]**

Answer:

i) $\xi = 0.4$ $\quad\quad \omega_n = 5$ rad/sec

Closed loop T.F of the second order system $= \dfrac{G(S)}{1+G(S)H(S)}$

or, $$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n^s + \omega_n^2} = \frac{5^2}{S^2 + 2.(.4).5s + 5^2} = \frac{25}{s^2 + 4s + 25}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 5\sqrt{1-.4^2} = 5\times.916 = 4.58 \text{ rad/sec}$$

$$\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}\frac{\sqrt{1-.42}}{.4} = \tan^{-1}\frac{.916}{.4} = 66.41° = .368 \text{ rad}$$

ii) Rise time $t_r = \dfrac{\pi - \beta}{\omega_d} = \dfrac{\pi - .368}{4.58} = .605$ sec

iii) Peak time $t_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{4.58} = .685$ sec

iv) Setting time t_s for 2% tolerance

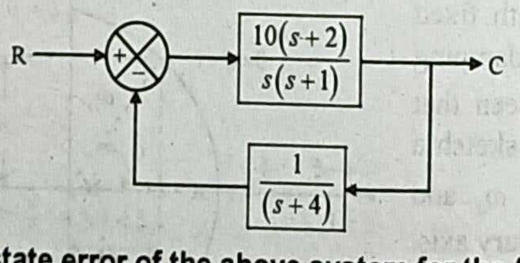
$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{.4\times5} = 2 \text{ sec}$$

v) Peak overshoot

$$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = e^{-\frac{\pi\times.4}{.916}} = e^{-1.37} = 3.93$$

$\% M_p = 39.3$

5. Define the Type and Order of a system. Determine the type and order of the following system.



Determine the steady state error of the above system for the following input:

$r(t) = 0$ for $t < 0$

$= 2 + 3t$ for $t \geq 0$. **[WBUT 2017]**

Answer:

1st Part: The order of the system is defined by the number of independent energy storage elements in the system, and is the highest order of the linear differential equation that describes the system. In a transfer function representation, the order is the highest exponent in the transfer function.

Type says the number of open loop poles at origin.

For the said system, order is 3 and type number is 1.

2nd Part:

For the given system:

$$\text{Open loop transfer system} = G(s)H(s) = \frac{10(s+2)}{s(s+1)(s+4)}$$

Input has two sections

i) Step function of magnitude 2.

ii) Ramp function with a slope of 3.

For step function:

$$K_P = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0}\frac{10(s+2)}{s(s+1)(s+4)} = \infty$$

$$\therefore \quad e_{ss} = \frac{1}{1+K_P} = 0$$

For ramp function:

$$K_V = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} s\cdot\frac{10(s+2)}{s(s+1)(s+4)} = 5/\text{sec}$$

$$\therefore \quad e_{ss} = \frac{1}{K_V} = \frac{1}{5} = 0.2\ \text{sec.}$$

$\therefore$ The effective $e_{ss} = 0 + 0.2 = 0.2$ sec.

$e_{ss} \Rightarrow$ steady state error.

6. a) Derive the expression for the time response of a first order system subjected to unit step input. **[WBUT 2018]**

Answer:

If the highest power of s in the denominator of a transfer function is one, then the system is called a first order system.

A simple block diagram of a first order system is shown in Fig. 1 where forward path transfer function is $G(s) = \frac{1}{Ts}$. T in $G(s)$ represents the time constant of the open loop system.

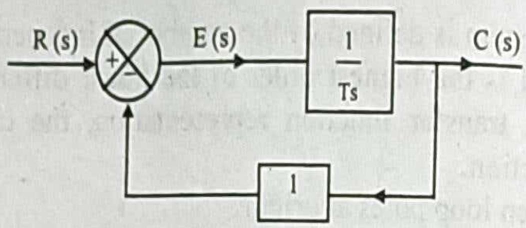


Fig: 1 Block diagram of a first order system

The input-output relationship is (assuming the feedback is unity)

$$\frac{C(s)}{R(s)}=\frac{\frac{1}{Ts}}{1+\left(\frac{1}{Ts}\right)\times 1}=\frac{\frac{1}{Ts}}{\frac{(1+Ts)}{Ts}}=\frac{1}{1+sT}$$

For unit step excitation to the system Fig. 2

$$R(s)=\frac{1}{s}$$

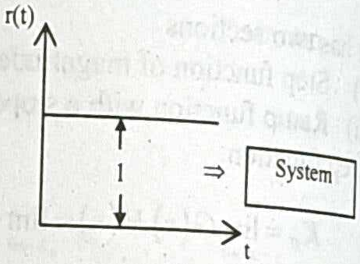


Fig: 2 Step excitation to the system

∴ From equations we have,

$$C(s)=\left(\frac{1}{1+Ts}\right).\ R(s)=\frac{1}{1+Ts}.\frac{1}{s}$$

Applying partial-fraction approach, we have,

$$C(s)=\left(\frac{1}{s}-\frac{T}{Ts+1}\right)=\frac{1}{s}-\frac{1}{\left(s+\frac{1}{T}\right)} \quad \text{.... (1)}$$

Taking inverse Laplace transformation of Eqn. 1, we have

$$C(t)=L^{-1}\left[C(s)\right]=L^{-1}\left[\frac{1}{s}-\frac{1}{s+\frac{1}{T}}\right]=1-e^{\frac{-t}{T}} \quad \text{.... (2)}$$

Eqn. 2 represents the response of the first order system. We then form a table

Table:

t	0	T	$2T$	$3T$	$4T$	∞
$C(t)$	0	0.632	0.865	0.95	0.982	1

From the table, we can say

i) Initially ($t=0$) output is zero.

ii) finally ($t\to\infty$) output is one.

iii) At $t=T$, output is 63.2% of final output. T is called the time constant of the system.

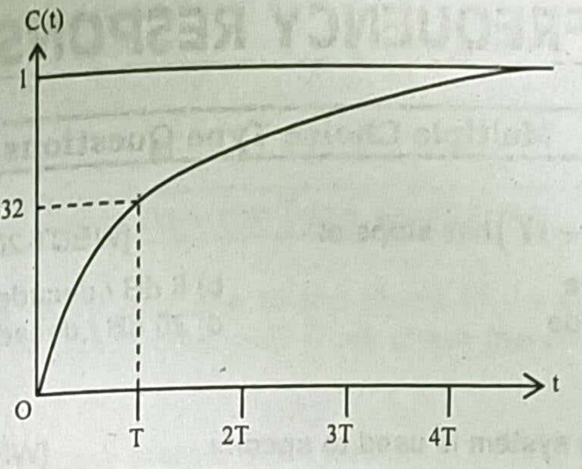


Fig: Response of a first order system

b) Define position, velocity and acceleration error constants. **[WBUT 2018]**

Answer:

Depending upon the nature of excitation signal, error constants / co-efficients are classified as

- Position Error Constant
- Velocity Error Constant
- Acceleration Error Constant

(1) Position Error Constant (K_p)

Position Error Constant is defined for a unit step input i.e. $R(s)=\frac{1}{s}$.

The steady state error of the system for a unit step input is called Position Error Co-efficient. K_p is defined as $K_p = \lim_{s\to 0} G(s)H(s)$

(2) Velocity Error Constant (K_v)

Velocity Error Constant is defined for unit ramp, i.e. $R(s)=\frac{1}{s^2}$

The steady state error of the system for a unit ramp input is called Velocity Error Co-efficient (K_v). It is defined as

$$K_v = \lim_{s\to 0} sG(s)H(s)$$

(3) Acceleration Error Constant (K_a)

Acceleration Error Constant is defined for unit parabolic input i.e.,

$$R(s)=\frac{1}{s^3}$$

The steady state error of the system for a unit parabolic input is called Acceleration Error Co-efficient (K_a). It is defined as

$$K_a = \lim_{s\to 0} s^2 G(s)H(s)$$

FREQUENCY RESPONSE

Multiple Choice Type Questions

1. The function $1/(1+sT)$ has slope of [WBUT 2006, 2010, 2013, 2017]
   a) – 6 dB / decade
   b) 6 dB / decade
   c) – 20 dB / decade
   d) 20 dB / decade

Answer: (c)

2. Phase margin of a system is used to specify [WBUT 2006, 2010, 2017]
   a) time response
   b) frequency response
   c) absolute stability
   d) relative stability

Answer: (d)

3. If the gain of a third order system (all poles) is increased, then the phase margin [WBUT 2011]
   a) increases
   b) decreases
   c) remains same
   d) it is not possible to predict

Answer: (c)

4. A closed loop system is unstable if [WBUT 2014]
   a) both gain margin and phase margin are negative
   b) gain margin is positive and phase margin is negative
   c) gain margin is negative and phase margin is positive
   d) both gain margin and phase margin are positive

Answer: (a,b & c)

5. A system has the transfer function $\frac{(1-s)}{(1+s)}$. What is its gain at 1 rad/sec. [WBUT 2014]
   a) 1
   b) 0
   c) –1
   d) 0.5

Answer: (a)

6. The gain margin in dB for a system with open-loop transfer function $G(s)=\frac{2\sqrt{2}}{s(s+2)^2}$ is [WBUT 2015]
   a) 0
   b) 6 dB
   c) 9 dB
   d) 12 dB

Answer: (c)

7. The open loop transfer function of a feedback control system is $\frac{1}{(s+1)^3}$. The gain margin of the system is [WBUT 2017]
   a) 16
   b) 8
   c) 4
   d) 2

Answer: (b)

8. If the gain of the open loop system is doubled, the gain margin **[WBUT 2017]**
a) is not affected
b) gets doubled
c) becomes half
d) becomes 1/4th

Answer: (c)

Short Answer Type Questions

1. Define gain margin. What happens to the phase of the system at a particular frequency if the system gain is changed? Does phase margin change with system gain? [WBUT 2011]

OR,

Define gain margin and phase margin of a system. [WBUT 2014]

Answer:

1st Part:

Gain margin:

It is the reciprocal of the magnitude of $[G(j\omega).H(j\omega)]$, i.e., $|G(j\omega).H(j\omega)|$, at the frequency at which the phase angle is $-180°$.

Mathematically, Gain Margin = $K_{GM} = \dfrac{1}{|G(j\omega).H(j\omega)|}$ ...(i)

In terms of decibels, K_{GM} (dB) = $20\log_{10} K_{GM}$

$= -20\log_{10}|G(j\omega)H(j\omega)|$. ...(ii)

It is a measure of the relative stability. It says how much gain can be increased to cause system instability.

Phase Margin:

It is that amount of additional phase lag at the gain crossover frequency required to bring the system to the verge of instability.

It is a measure of the relative stability. It says the phase angle can be increased to make the system unstable from a stable condition.

2nd Part: Gain margin is the reciprocal of the gain at the frequency at which the phase angle is -180°.So Phase reduces if the value of System gain is increased.

3rd Part: The phase margin measures the system's tolerance to time delay. If there is a time delay greater than $180/\omega_{pc}$ in the loop (where ω_{pc} is the frequency where the phase shift is 180 deg), the system will become unstable in closed loop. The time delay can be thought of as an extra block in the forward path of the block diagram that adds phase to the system but has no effect the gain.

Long Answer Type Questions

1. Write the advantages of frequency response. Define Cut-off frequency (ω_c) & Cut-off rate. [WBUT 2012]

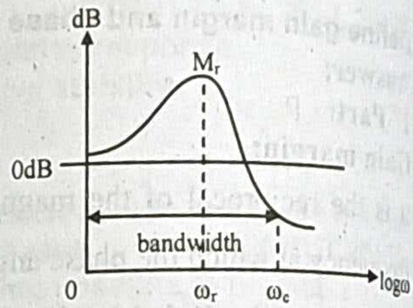
Answer:

Advantages of Frequency Domain Analysis

1. The ease and accuracy of measurements.
2. When it is difficult to derive the transfer function of a system through analytical technique, one may get the necessary information to compute transfer function by undergoing frequency response test on the system / component.
3. Easy approach for design of a system having specified closed loop performance.
4. Easy approach for compensation of a system.

Cut off frequency:

It is donated by ω_c. The frequency at which the magnitude of the closed loop response is 3dB down from its zero frequency value is called cut off frequency. The range to ω_c is nothing but bandwidth of the system whose frequency is shown. Bandwidth indicates the speed of the response. It indicates the ability to reproduce the input signal. It is inversely proportional to the rise time. Large bandwidth means small rise time means fast response.



Cut-off Rate:

It is the slope of the magnitude curve near the cut-off frequency.
It indicates the ability of the system to differentiate the signal from noise.
Higher the cut-off rate better is the rejection of noise.

2. A second order system is described by the differential equation.

$$\frac{d^2y(t)}{dt^2}+0.8\frac{dy(t)}{dt}+y(t)=x(t).$$

When $x(t)$ is the input and $y(t)$ is the output. Determine resonance frequency, peak resonance, cut off frequency and band width. **[WBUT 2015]**

Answer:

The system is described by, $\frac{d^2y(t)}{dt^2}+0.8\frac{dy(t)}{dt}+y(t)=x(t)$

Taking Laplace transform of both sides,

$$\left(s^2+0.8s+1\right)Y(s)=X(s)$$

The transfer function is, $M(s)=\frac{Y(s)}{X(s)}=\frac{1}{s^2 0.8s+1}$

The characteristic equation is, $s^2+0.8s+1=0$

$\omega_n=1$ rad/s and $2\delta\omega_n=0.8$

$\delta=0.4$

Since $\omega<0.707$, peak resonance occurs.

Resonance frequency, $\omega_r=\omega_n\sqrt{1-2\delta^2}=\sqrt{1-2(0.4)^2}=0.825$ rad/s

Peak resonance, $M_r = \frac{1}{2\delta\sqrt{1-\delta^2}} = \frac{1}{2\times0.4\sqrt{1-(0.4)^2}} = 1.364$

Cut-off frequency, $\omega_c = \omega_n\sqrt{1-2\delta^2+\sqrt{2-4\delta^2+4\delta^4}}$

$$= \sqrt{1-0.32+\sqrt{2-4(0.4)^2+4(0.4)^4}} \doteq 1.375 \text{ rad/s}$$

Bandwidth $= 1.375$ rad/s

3. Explain what is meant by relative stability of a system. How do we specify relative stability in terms of (i) closed loop pole locations, (ii) gain margin and phase margin? **[WBUT 2017]**

Answer:

1st Part: Relative Stability of a system is an indicator of the degree of stability. It says how close the system is to instability. It is measure of how fast the transient dies out in the system. It is related to the time taken by the system to get settled when the system is excited.

2nd Part:

In terms of pole locations: a system having poles away from the left half of imaginary axis is considered to be relatively more stable compared to a **system** having poles closed to imaginary axis.

In terms of gain margin (GM) and phase margin (PM): GM and PM tells you how much uncertainty one can tolerate in the open loop system before the closed loop system goes to instability. Higher the values of the margins (GM and PM) better is the stability of the system.

Gain margin: It is the reciprocal of the magnitude of $[G(j\omega).H(j\omega)]$, i.e., $|G(j\omega).H(j\omega)|$, at the frequency at which the phase angle is $-180°$.

Mathematically, Gain Margin = $K_{GM} = \frac{1}{|G(j\omega).H(j\omega)|}$ .... (i)

In terms of decibels, K_{GM} (dB) = $20\log_{10} K_{GM}$

$$= -20\log_{10}|G(j\omega).H(j\omega)| \quad \text{.... (ii)}$$

It is a measure of the relative stability. It says how much gain can be increased to cause system instability.

Phase Margin: It is that amount of additional phase lag at the gain crossover frequency required to bring the system to the verge of instability.

It is a measure of the relative stability. It says the phase angle can be increased to make the system unstable from a stable condition.

ROOT LOCUS

Multiple Choice Type Questions

1. The centre of asymptotes for an open-loop transfer function $G(s) = \frac{2(s+2)}{s^2(s+4)}$ is [WBUT 2011]

a) –1 b) –3 c) –4 d) 0

Answer: (a)

2. Which of the following effects are correct in respect of addition of a pole to the open loop transfer function? [WBUT 2013]

I) The root locus is pulled to the right
II) The system becomes more oscillatory
III) The system stability relatively reduces
IV) The range gain for stability reduces

Of these statements:

a) I and II are correct b) I and IV are correct
c) I, III and IV are correct d) All are correct

Answer: (c)

3. The open loop transfer function of a unity feedback system is given by $G(s)=\frac{K}{s(s+1)(s+2)}$ the location of the centroid of the root locus is [WBUT 2014]

a) –1 b) –2 c) 0 d) –0.5

Answer: (a)

4. The open-loop transfer function of a negative feedback system is $G(s)H(s)=\frac{k}{s(s+1)(s+2)}$. The root crosses the imaginary axis when the value of k is [WBUT 2015]

a) 8 b) 6 c) 4 d) 2

Answer: (c)

5. The root locus diagram is [WBUT 2016]

a) always symmetric about the real axis
b) always symmetric about the imaginary axis
c) never symmetric about the real axis
d) always asymmetric about both the real & imaginary axes

Answer: (a)

6. A unity feedback system has three open loop poles at $(-2 \pm j2)$ and 0. It has a single zero at (-4 ± 0). The angle of departure of the root locus branch starting from the pole $(-2-j2)$ is [WBUT 2017]

a) 135° b) 225° c) 0° d) –45°

Answer: (c)

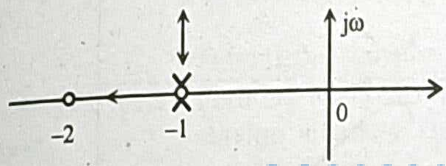
7. The root loci of a system have three asymptotes. The system can have [WBUT 2018]

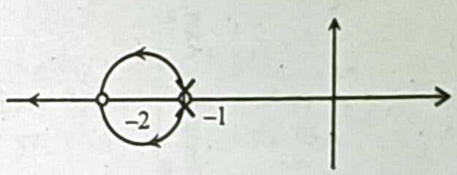
a) five poles & two zeros
b) three poles & one zero
c) four poles & two zeros
d) six poles & two zeros

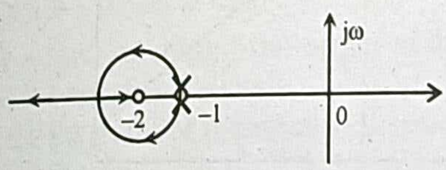
Answer: (a)

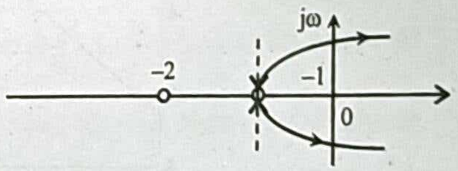
8. Given a unity feedback system with open-loop transfer function $(s) = \frac{K(s+2)}{(s+1)^2}$.

The correct root-locus plot of the system is [WBUT 2018]

a) 

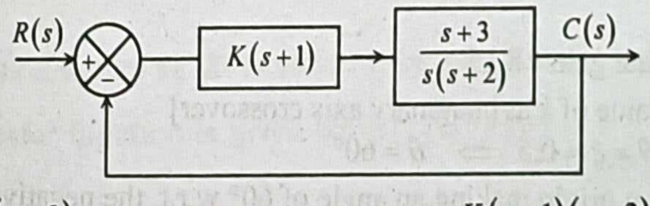
b) 

c) 

d) 

Answer: (c)

9. For the system in the given figure the characteristic equation is [WBUT 2018]



a) $1+\frac{K(s+1)(s+3)}{s(s+2)}=0$
b) $1+\frac{K(s-1)(s-3)}{s(s-2)}=0$
c) $K(s+1)(s+3)=0$
d) $s(s+2)=0$

Answer: (a)

Long Answer Type Questions

1. A feedback control system has an open-loop transfer function

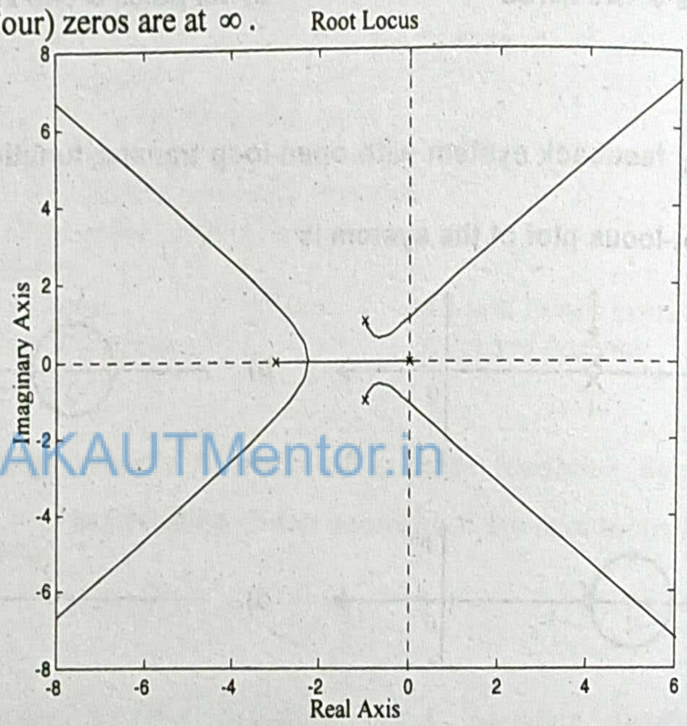
$$G(s)H(s)=\frac{k}{s(s+3)(s^2+2s+2)}$$

a) Find the root loci as k is varied from 0 to α. **[WBUT 2012, 2016]**

Answer:

$$GH=\frac{k}{s(s+3)(s+1+j1)(s+1-j1)}$$

No. of loci = 4, 4 (Four) zeros are at ∞.



b) Determine the value of k where damping coefficient $\xi=0.5$ and gain margin at this point. **[WBUT 2012]**

Answer:

k_u = Ultimate gain = 8.15

[This value is the value of k at imaginary axis crossover]

We know that $\cos\theta=\xi=0.5 \Rightarrow \theta=60°$

A line is drawn from origin making an angle of 60° w.r.t. the negative real-axis to meet a point $s=-0.414+j0.707$ on the locus. The value of k at that point is 2.59 (evaluated from Evan's magnitude condition)

So, gain margin $=\frac{k_u}{k\big|_{\text{at } s=-0.414+j0.707}}=\frac{8.15}{2.59}=3.15$

$\therefore$ Gain Margin in dB $=20\log_{10}3.15=9.966$

2. Sketch the root locus for $G(s)H(s)=\frac{K}{s(s+2)(s+4)}$. Evaluate the value of K at the point where the root loci crosses the imaginary axis. Also determine the frequency at this point. Determine the value of K such that the dominant pair of complex poles of the system has a damping ratio of 0.5. **[WBUT 2013]**

Answer:

Information from transfer function	m = 3 n = 0 (m – n) = 3 , saying number of loci = 3 and 3 zeros will be infinity
Information about asymptotes	Number of asymptotes = 3 Asymptotic angles are 60°, 180° and 300°, Centroid at –2
Break away point	$K=-(s(s+2)(s+4))$ $\Rightarrow K=-s^3-6s^2-8s$ $\frac{dK}{ds}=-3s^2-12s-8=0$ $\Rightarrow \frac{dK}{ds}=3s^2+12s+8=0$ Solving the equation we get break away point is -0.845
Imaginary axis cross over point	Forming Routh array and auxiliary equation followed by solving auxiliary equation we get cross over points as ± 2.9

Step 1: $\xi=\cos\theta=0.5\Rightarrow\theta=60^0$

Step 2: Draw a line initiating from origin in the root locus plot that makes an angle of 60° relative to the negative real axis.

Step 3: Observe the point of intersection between the line and the root locus and evaluate the point of intersection.

Step 4: Use Evan's Magnitude criterion

$$\left|\frac{K}{s(s+2)(s+4)}\right|=1$$

3. Sketch the root locus diagram as K is varied from zero to infinity for the system whose open loop transfer function is given by $G(s)H(s)=\frac{K}{s(s+6)(s^2+4s+13)}$. Evaluate the value of K at the point where the root locus crosses the imaginary axis. Also determine the frequency at this point. **[WBUT 2014]**

Answer:

For the given open-loop transfer function $G(s)H(s)$:

The open-loop poles are at $s=0, s=-6, s=\frac{-4\pm\sqrt{16-52}}{2}=-2\pm j3$. Therefore, $n=4$.

There are no open-loop zeros. Therefore, $m=0$.
Hence the number of branches of root locus $=n=4$ and the number asymptotes $=n-m=4-0=4$.
The complete root locus is drawn as shown in Fig. 1, as per the rules given as follows:

1. The root locus will be symmetrical about the real axis because the pole-zeros location is symmetrical with respect to the real axis.
2. The four branches of the root locus start at the open-loop poles $s=0, s=-6, s=-2+j3$ and $s=-2-j3$, where $K=0$ and terminate at the open-loop zeros at infinity, where $k=\infty$.
3. The four branches of the root locus go to the zeros at infinity along asymptotes making angles of $\theta_q=\frac{(2q+1)\pi}{n-m}$, $q=0,1,2,3$ with the real axis, i.e.,
$$\theta_0=\frac{\pi}{4},\ \theta_1=\frac{3\pi}{4},\ \theta_2=\frac{5\pi}{4},\ \theta_3=\frac{7\pi}{4}$$
4. The point of intersection of the asymptotes on the real axis (centroid) is given by
$$-\sigma=\frac{\text{sum of real parts of poles} - \text{sum of real parts of zeros}}{\text{number of poles} - \text{number of zeros}}$$
$$=\frac{(0-6-2-2)-(0)}{4-0}=-2.5$$
5. The root locus exists on the real axis from $s=0$ to $s=-6$.
6. The break points are given by the solution of the equation $\frac{dK}{ds}=0$.
$$|G(s)H(s)|=\left|\frac{K}{s(s+6)(s^2+4s+13)}\right|=1$$
$$\therefore \quad K=s^4+10s^3+37s^2+78s$$
$$\frac{dK}{ds}=4s^3+30s^2+74s+78=0$$
i.e., $s^3+7.5s^2+18.5s+19.5=0$
i.e., $(s+4.1)(s+1.7+j2.72)(s+1.7-j2.72)=0$
Therefore, the break points are $s=-4.1, s=-1.7+j2.72$ and $s=-1.7-j2.72$.
Out of these three break points, the actual break point is $s=-4.1$, because this point lies on the root locus. The other two are not actual break points, because the root locus does not exist there. They can be ignored.
The break angles at $s=-4.1$ are $\pm\frac{\pi}{r}=\pm\frac{180°}{2}=\pm 90°$
7. The angle of departure of the root locus branch from the pole at $s=-2+j3$ us given by
$$\theta_d=\pm(2q+1)\pi+\phi$$

where $\phi = -(\theta_1 + \theta_2 + \theta_3) = -(122° + 90° + 38°) = -250°$

$\therefore \quad \theta_d = \pi - 250° = -70°$

Therefore, the angle of departure of the root locus branch from the pole at $s = -2 - j3$ is $\theta_d = +70°$.

8. The point of intersection of the root locus with the imaginary axis and the critical value of K are obtained using the Routh criterion. The characteristic equation is

$$1 + G(s)H(s) = 0$$

i.e., $$1 + \frac{K}{s(s+6)(s^2+4s+13)} = 0$$

i.e., $$s^4 + 10s^3 + 37s^2 + 78s + K = 0$$

The Routh table is as follows:

s^4	1	37	K
s^3	10	78	
s^2	$\frac{370-78}{10} = 29.2$	K	
s^1	$\frac{29.2 \times 78 - 10K}{29.2}$		
s^0	K		

For stability, all the elements in the first column of the Routh array must be positive.

Therefore, $K > 0$

and $29.2 \times 78 - 10K > 0$

i.e., $$K < \frac{29.2 \times 78}{10} = 227.76$$

So the range of values of K for stability is $0 < K < 227.76$.

The marginal value of K for stability is $K_m = 227.76$. For $K > 227.76$, the system has two closed-loop poles in the right half of the s-plane and is thus unstable.

For that value of K, the frequency of sustained oscillations is given by the solution of the auxiliary equation, $29.2s^2 + K = 0$

i.e., $29.2s^2 + K_m = 0$

i.e., $29.2s^2 + 227.76 = 0$

$$\therefore \quad s^2 = -\frac{227.76}{29.2} = -7.8$$

or, $s = \pm j2.8$

$\therefore \quad \omega = 2.8$ rad/sec.

Hence the frequency of sustained oscillations is $\omega = 2.8$ rad/sec.

The complete root locus is drawn as shown in Fig. 1.

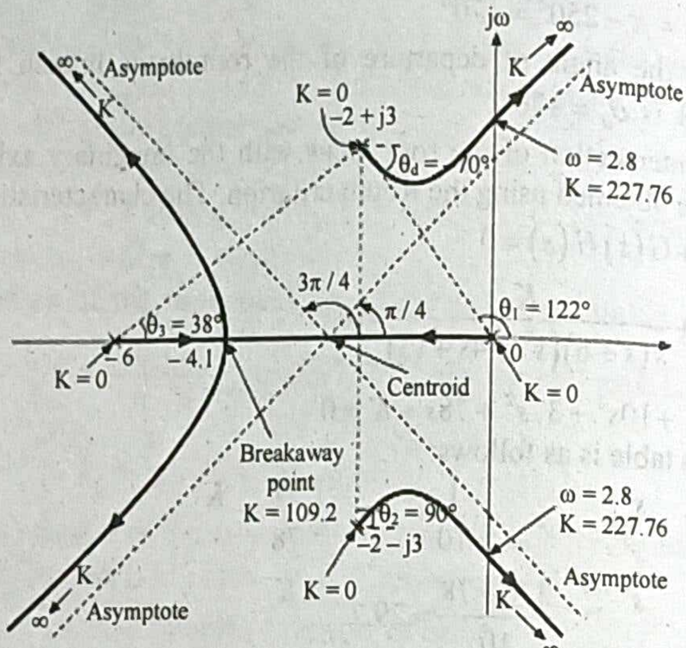


Fig: 1 Root locus

4. The forward path and feedback path transfer functions of a negative feedback system are $G(s)=\frac{5}{s^2(s+2)}$ and $H(s)=(s+a)$ respectively. Sketch the root contour for the system with respect to the parameter. For what range of value of 'a' does the system remain stable? **[WBUT 2015]**

Answer:

The characteristic equation is given by

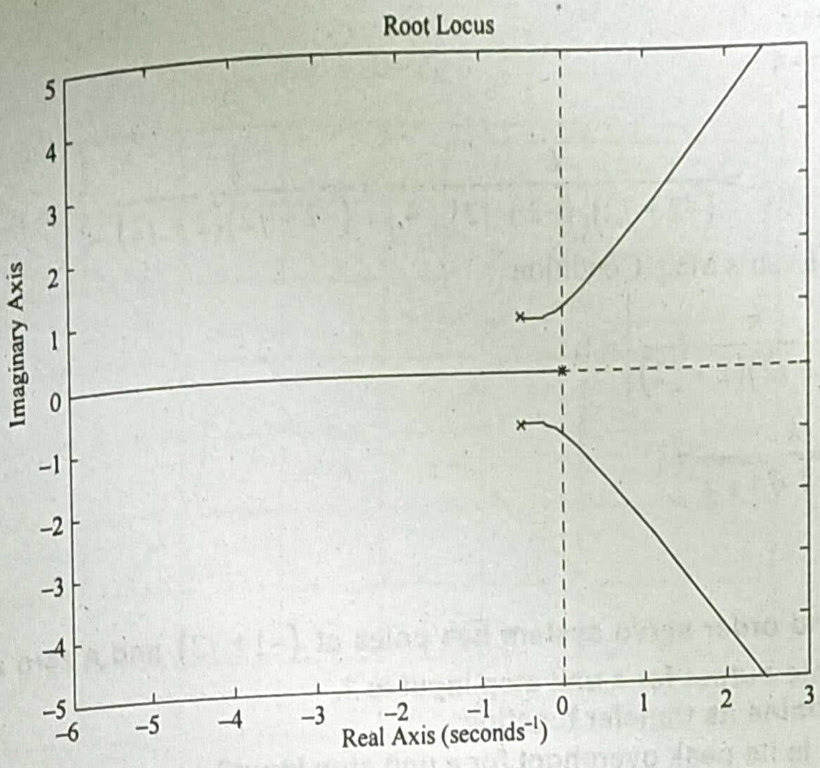
$$1+\frac{5(s+a)}{s^2(s+2)}=0=s^3+s^2+5s+5a$$

$$\Rightarrow \quad 1+\frac{5a}{s^3+s^2+5s}=0$$

We have to draw the rot locus of the open loop transfer function where the parameter a is varied from 0 to + infinity

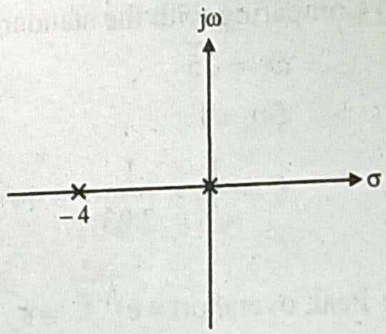
$$\frac{5a}{s^3+s^2+5s}$$

No. of loci	3
Three open loop zeros are at	∞, ∞ and ∞
Three open loop poles are at	$0,\ -1+j2.18,\ -1-j2.18$
No of asymptotes	3
Angles of asymptotes are	60°, 180° and 300°
Break-away point	Nil
Finite imaginary axis crossover point	$\pm j2.25$



5. Open loop transfer function of a system is given by $G(s)H(s)=\frac{k}{s(s+4)}$, check whether $s=-2+j2$ lies on root locus. If so, find system gain, k at given point. **[WBUT 2016]**

Answer:



Pole zero plot on s-plane

$n=2$ and located at $0, -4$

$m=0$ and no finite zero is there.

$\therefore$ Number of loci $=n=2$

Number of loci ending at $\alpha=n-m=2$

$n-m=2$

Hence no. of asymptotes $=2$

Angle of asymtotes $=\frac{(2q+1)180°}{2}$ $[q=0, 1]$

$\theta=60°, 180°$

$$\sigma_c=\frac{\sum\text{real part of OLTF poles}-\sum\text{real part of OLTF zero}}{n-m}$$

$$=\frac{\Sigma(0-4)-\Sigma(0)}{2}=\frac{-4}{2}=-2$$

For $\sigma_b=k=\frac{-s(s+4)}{1}=-(s^2+4s)$

$$\frac{dk}{ds}=0=-2s+4$$

Equation gives

$$-2s=-4$$
$$s=2$$

$$GH\big|_{s=-2+j2}=\frac{k}{(-2+j2)\left[(-2+j2)+4\right]}=\frac{k}{(-2+j2)(2+j2)}$$

According to Evan's Mag. Condition

$$\left|\frac{k}{(-2+j2)(2+j2)}\right|=1$$

$$\frac{k}{\sqrt{4+4}\,\sqrt{4+4}}=1$$

$$k=8$$

6. a) A second order servo system has poles at $(-1\pm j2)$ and a zero at $(-1+j0)$. Its steady state output for a unit step input is 2.
  i) Determine its transfer function.
  ii) What is its peak overshoot for a unit step input? **[WBUT 2017]**

Answer:

i) Transfer function of the servo system $=\dfrac{10}{(s+1+j2)(s+1-j2)}=\dfrac{10}{s^2+2s+5}$

[As steady state output is 2 for a unit step]

ii) Comparing with the standard second order system's characteristics equation

$$\omega_n=\sqrt{5}$$
$$\xi\omega_n=1$$
$$\therefore \quad \xi=\frac{1}{\sqrt{5}}=\frac{1}{2.23}=0.44<1$$

$\therefore$ Peak overshort $=e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}=e^{\frac{-0.44\times 3.14\times 2.24}{2}}=e^{-1.54}=0.214$

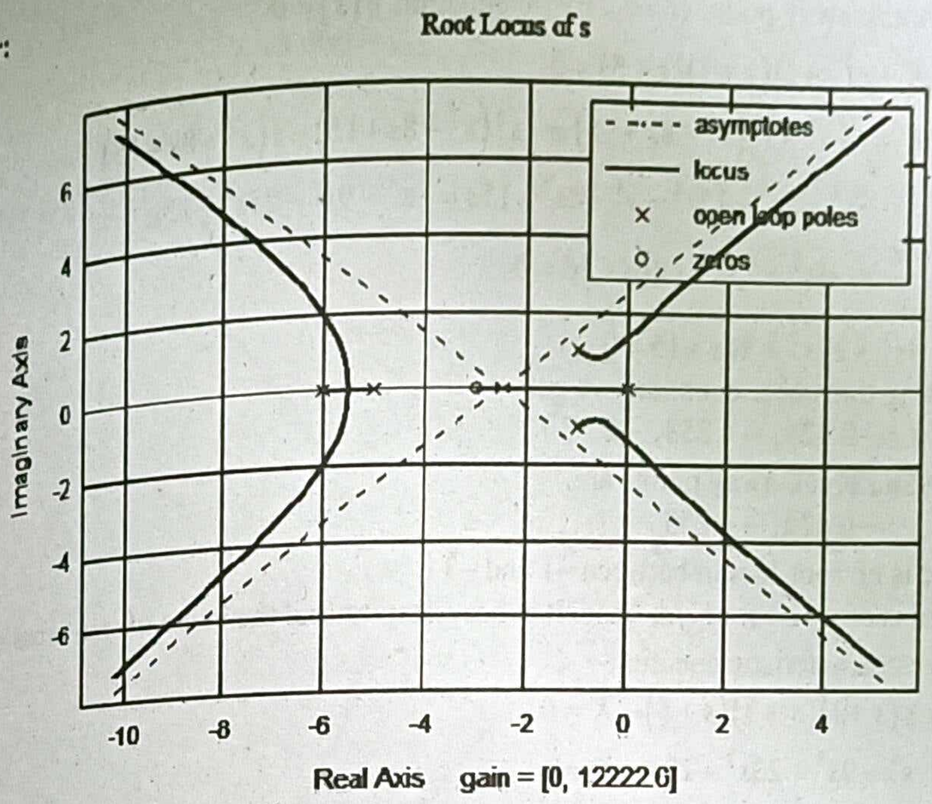
b) Consider the open loop transfer function of a unity feedback system

$$G(s)=\frac{K(s+3)}{s(s^2+2s+2)(s+5)(s+6)}.$$

Draw the root locus diagram of the system on a graph paper and indicate on that diagram,
  i) At what points will the imaginary axis be crossed by the root loci and what is the corresponding value of K?
  ii) Is $(-10+j0)$ a point on the root loci? Explain with valid reasons.

[WBUT 2017]

Answer:



7. a) The open loop transfer function of a unity feedback control system is given by

$$G(s)=\frac{K}{s(s+1)(s+3)(s+5)}$$

Sketch the root locus plot of the system by finding the following:

i) angle of asymptotes, centroid and breakaway points
ii) angle of departure
iii) the value of K and frequency at which the root loci cross the imaginary axis. [WBUT 2018]

Answer:

$$G(s)=\frac{K}{s(s+1)(s+3)(s+5)}$$

Open loop pole location are at $s=0,\ -1,\ -3,\ -5$

$\therefore$ No. of pole $(n)=4$ and no. of zero $(m)=0$

$\therefore$ No. of root loci $=4$ ends at ∞

$$\text{Centroid}=\frac{0-1-3-5}{4}=\frac{-9}{4}=-2.25$$

No. of asymptotes $=n-m=4$

$\therefore$ Angle of asymptotes $\beta=\frac{(2q+1)180°}{n\ \ m}$; $q=0,1,2,3$

$\therefore\quad \beta=45°, 135°, 225°, 315°$

To find break-away point, characteristic equation $q(s)=0$

$$K+s(s+1)(s+3)(s+5)=0$$

$$K=-(s^2+s)(s^2+8s+15)=-s^2(s^2+8s+15)-s(s^2+8s+15)$$

$$=-s^4-8s^3-15s^2-s^3-8s^2-15s=-s^4-9s^3-23s^2-15s$$

$\therefore \quad \frac{dk}{ds}=-4s^3-27s^2-46s-15=0$

or, $\quad 4s^3+27s^2+46s+15=0$

Calculating this cubic equation, we get

$$s=-0.425,\ -4.253,\ -2.07$$

So, the valid break-away points are,

$$s=-0.425,\ -4.253$$

As there is no root-loci in between –1 and –3

As, there is cross-over of $j\omega$-axis. It is necessary to find the value of $j\omega$ cross-over

From the characteristic equation

$$s(s+1)(s+3)(s+5)+K=0$$

$$s^4+9s^3+23s^2+15s+K=0$$

The Routh array is

s^4	1	23	K
s^3	9	15	
s^2	21.33	K	
s^1	$\frac{320-9k}{21.33}$	0	
s^0	K		

For cross-over the term,

$$\frac{320-9K}{21.33}=0$$

$$320=9K$$

$$K=\frac{320}{9}$$

$$K=35.55$$

The auxiliary equation is

$$21.33s^2+K=0$$

$$21.33s^2=-K$$

$$21.33s^2=-35.55$$

$$s^2 = -\frac{35.55}{21.33} = -1.6667$$

$\therefore \quad s = \pm j1.29$

The Root Locus Plot:

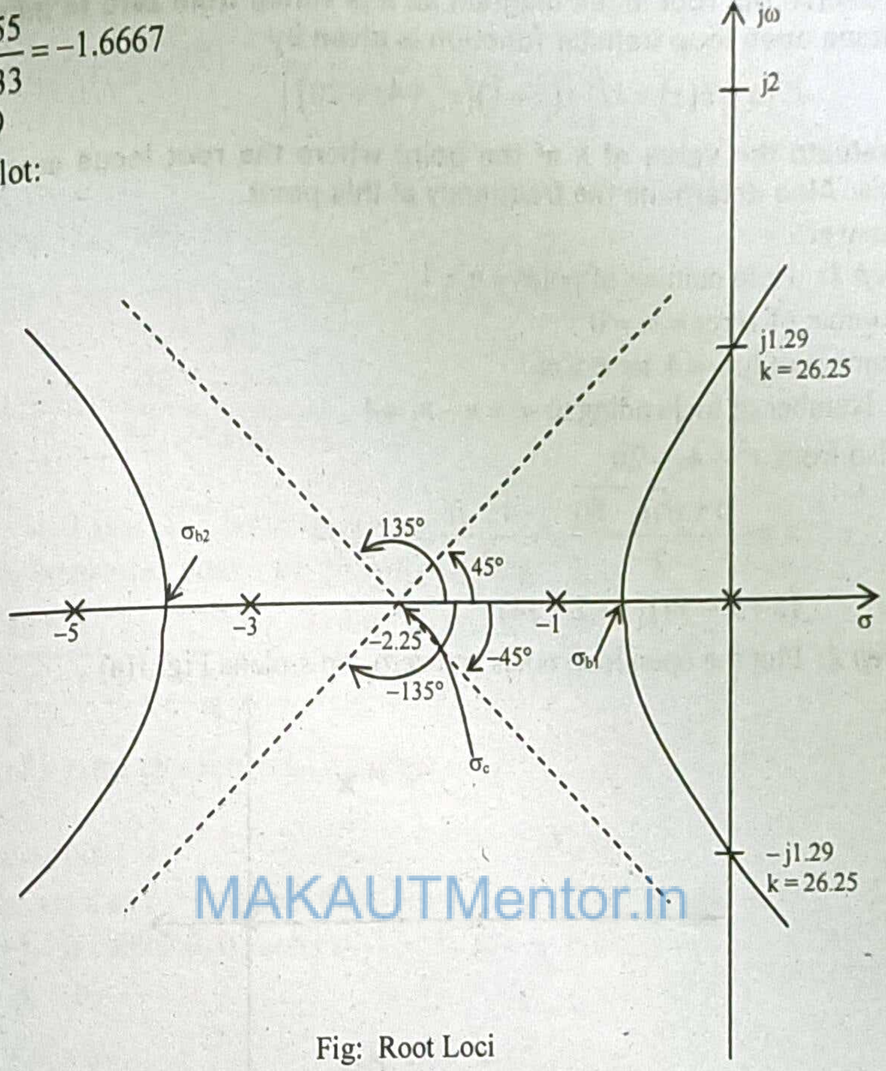


Fig: Root Loci

b) Explain what is meant by stability of the system. How root locus plot of a system is useful to find stability? [WBUT 2018]

Answer:

1st Part:

A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable. A **stable system** produces a bounded output for a given bounded input.

2nd Part:

In addition to determining the stability of the system, the root locus can be used to design the damping ratio (ζ) and natural frequency (ω_n) of a feedback system. Lines of constant natural frequency can be drawn radially from the origin and lines of constant damping ratio can be drawn as arccosine whose center points coincide with the origin. By selecting a point along the root locus that coincides with a desired damping ratio and natural frequency, a gain K can be calculated and implemented in the controller. More elaborate techniques of controller design using the root locus are available in most control textbooks: for instance, lag, lead, PI, PD and PID controllers can be designed approximately with this technique.

8. Sketch the root locus diagram as *k* is varied from zero to infinity for the system whose open loop transfer function is given by

$$G(s)H(s) = k/\left[s(s+4)\left(s^2+4s+20\right)\right]$$

Evaluate the value of *k* at the point where the root locus crosses the imaginary axis. Also determine the frequency at this point. **[WBUT 2019]**

Answer:

Step 1: Here number of poles $= n = 4$

Number of zeros $= m = 0$

Number of loci $= 4$ as $n > m$

$\therefore$ Number of loci ending at $\infty = n - m = 4$

Also from $s^2 + 4s + 20$

$$s = \frac{-4 \pm \sqrt{16-80}}{2} = \frac{-4 \pm j8}{2} = -2 \pm j4$$

i.e., $(s+2-j4)(s+2+j4)$

Step 2: Plot the open loop poles and zeros on s-plane Fig. 1(a)

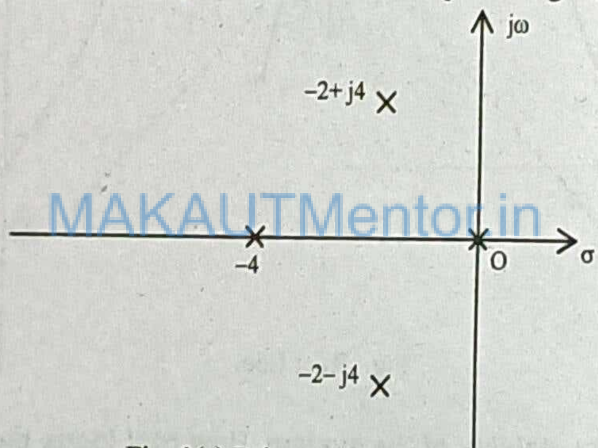


Fig: 1(a) Pole-zero map

Step 3: Real axis loci are

(i) Present $-4 \le \sigma \le 0$

(ii) Absent for $-\infty < \sigma < -4$

Complex factors are not taken, as they do not affect real axis loci.

Step 4: The number of asymptotes $= n - m$

Angles of asymptotes,

$$\therefore \quad \theta = \frac{(2q+1)180}{n-m}, \; q = 0, 1, 2, 3$$

$\therefore \quad \theta = 45°, 135°, 225°, 315°$ are the angles of the asymptotes.

Step 5: Centroid

$$= \sigma_c = \frac{\sum \text{R.P. of poles} - \sum \text{R.P. of zeros}}{n-m} = \frac{(0-2-2-4)-0}{4} = -\frac{8}{4} = -2$$

Step 6: To evaluate breakaway point.

$$s(s+4)(s^2+4s+20)+k=0$$

$$\therefore \quad k=-s(s+4)(s^2+4s+20) \;=-\left[s^4+8s^3+36s^2+80s\right]$$

$$\therefore \quad \frac{dk}{ds}=0 \text{ gives}$$

$$-(4s^3+24s^2+72s+80)=0$$

Since breakaway point is between 0 and –4, try $s=-2$ as starting point.

–2	4	24	72	80
		–8	–32	–80
	4	16	40	(0)

So, $s=-2$ is a valid breakaway point.
Other breakaway points are given by solving

$$4s^2+16s+40=0$$

i.e., $$s=\frac{-16\pm\sqrt{256-640}}{8}=\frac{-16\pm 19.59j}{8}=-2\pm j2.48$$

Putting these values of s in the characteristic equation.
For $s=-2$, $k=64$
For $s=-2\pm j2.45, k=100$.
Hence these points are valid as k > 0 in both these cases.

Step 7: ϕ_D at s = –2 + j 4 is calculated graphically. Refer to Fig. 1(b)

$$\sum\phi_{zeros}=\sum\phi_Z=0$$

$$\sum\phi_{poles}=\theta_1+\theta_2+\theta_3$$

To find, θ_1:

$$\angle BOC=\tan^{-1}\frac{4}{2}=63.43°$$

$$\therefore \quad \angle AOC=180°-\angle BOC=116.57°=\theta_1$$

To find θ_2:

$$\angle FEC=90°$$

To find θ_3:

$$\angle BDC=\tan^{-1}\frac{4}{2}=63.43°$$

$$\therefore \quad \sum\phi_Z-\sum\phi_P=0-270°=-270°$$

$$\therefore \quad \phi_D=180-270°=-90° \text{ at } s=-2+j4$$

$$\therefore \quad \phi_D \text{ at } s=-2-j4=90°$$

Step 8: Cross-over at jω- axis is determined as

$$1+G(s)H(s)=0 \text{ gives}$$

$$1+\frac{k}{s(s+4)(s^2+4s+20)}=0$$

$$\therefore \quad s^4+8s^3+36s^2+80s+k=0$$

s^4	1	36 k
s^3	8	80
s^2	26	k
s^1	$\frac{2080-8k}{26}$	0
s^0	k	

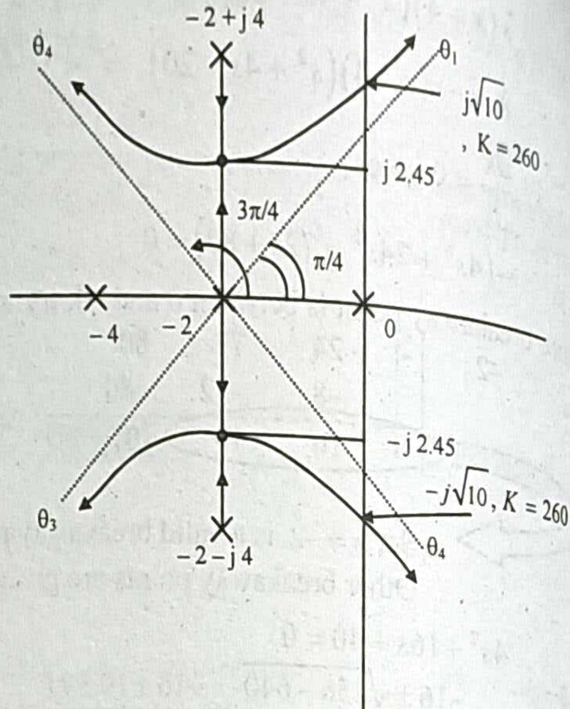


Fig: 1(c) Root Loci

Cross over at jω- axis is at

$$\frac{2080-8k}{26}=0$$

$$\therefore \quad 8k=2080$$

$$\therefore \quad k=260$$

The auxiliary equation at s^2 row is

$$26s^2+k=0$$

$$\therefore \quad 26s^2+260=0$$

$$\therefore \quad s^2=-10$$

$$\therefore \quad s=\pm j3.16$$

The Root Locus is drawn in the Fig. 1(c).

BODE PLOT

Multiple Choice Type Questions

1. A system has 14 poles & 2 zeros. The slope of its highest frequency asymptote in its magnitude plot is [WBUT 2009, 2012]
   a) – 40 dB/decade
   b) – 240 dB/decade
   c) – 280 dB/decade
   d) – 320 dB/decade

Answer: (b)

2. For a type-3 system, the asymptote at a lower frequency will have a slope of [WBUT 2009]
   a) – 6 dB / octave
   b) – 12 dB / octave
   c) – 24 dB / octave
   d) – 40 dB / octave

Answer: (b)

3. A slope of 20 dB/decade corresponds to [WBUT 2011]
   a) 3 dB/octave
   b) 6 dB/octave
   c) 9 dB/octave
   d) 20 dB/octave

Answer: (b)

4. The initial slope of Bode plot for a transfer function having single poles at origin is [WBUT 2016]
   a) 20 db/decade
   b) – 40 db/decade
   c) 40 db/decade
   d) – 20 db/decade

Answer: (d)

5. The Bode Plot of $(1+JWT)$ has [WBUT 2018]
   a) Slope of 20 dB/decade and phase angle $+\tan^{-1}(WT)$
   b) Slope of –20 DB/decade & phase angle $+\tan^{-1}(WT)$
   c) Slope of 20 dB/decade & phase angle $-\tan^{-1}(WT)$
   d) Slope of –40/dB decade & phase angle $-\tan^{-1}(WT)$

Answer: (a)

Short Answer Type Questions

1. A system is defined by the $G(s)=\frac{5(1-0.1s)}{(0.2s+1)(s+1)}$. Calculate the corner frequencies and the slopes of the asymptotes. [WBUT 2011]

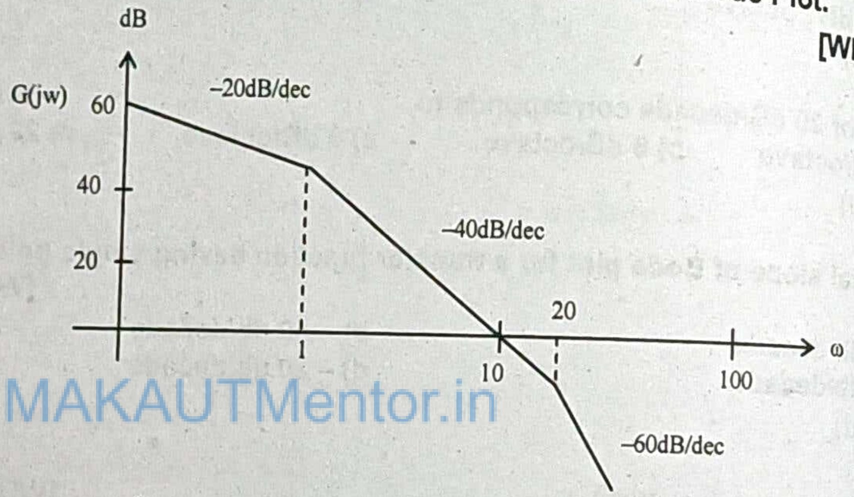
Answer:

$$G(s)=\frac{5(1-0.1s)}{(0.2s+1)(s+1)}$$

Factors	Corner frequency	Slope contributed by each factor
5	None	0
$(1-0.1s)$	$\frac{1}{0.1}=10$	+20 dB
$\frac{1}{0.2s+1}$	$\frac{1}{0.2}=5$	–20 dB
$\frac{1}{s+1}$	1	–20 dB

2. The asymptotic Bode Plot of a transfer function is as shown in the figure. Determine the transfer function $G(s)$ corresponding to this Bode Plot.

[WBUT 2012]



Answer:

$$\text{T.F.}=\frac{k}{s(1+s)\left(1+\frac{s}{20}\right)}$$

To find K

$$\text{T.F.}=G(s)=\frac{k}{j\omega} \quad \Rightarrow \quad |G(j\omega)|=M=\frac{k}{\omega}$$

$$M\big|_{dB}=20\log_{10}k-20\log_{10}\omega$$

$$\Rightarrow \quad 40=20\log_{10}k-20\log_{10}1$$

$$\Rightarrow \quad k=100$$

$$\therefore \quad \text{T.F.}=GH=\frac{100}{s(1+s)(1+0.05s)}.$$

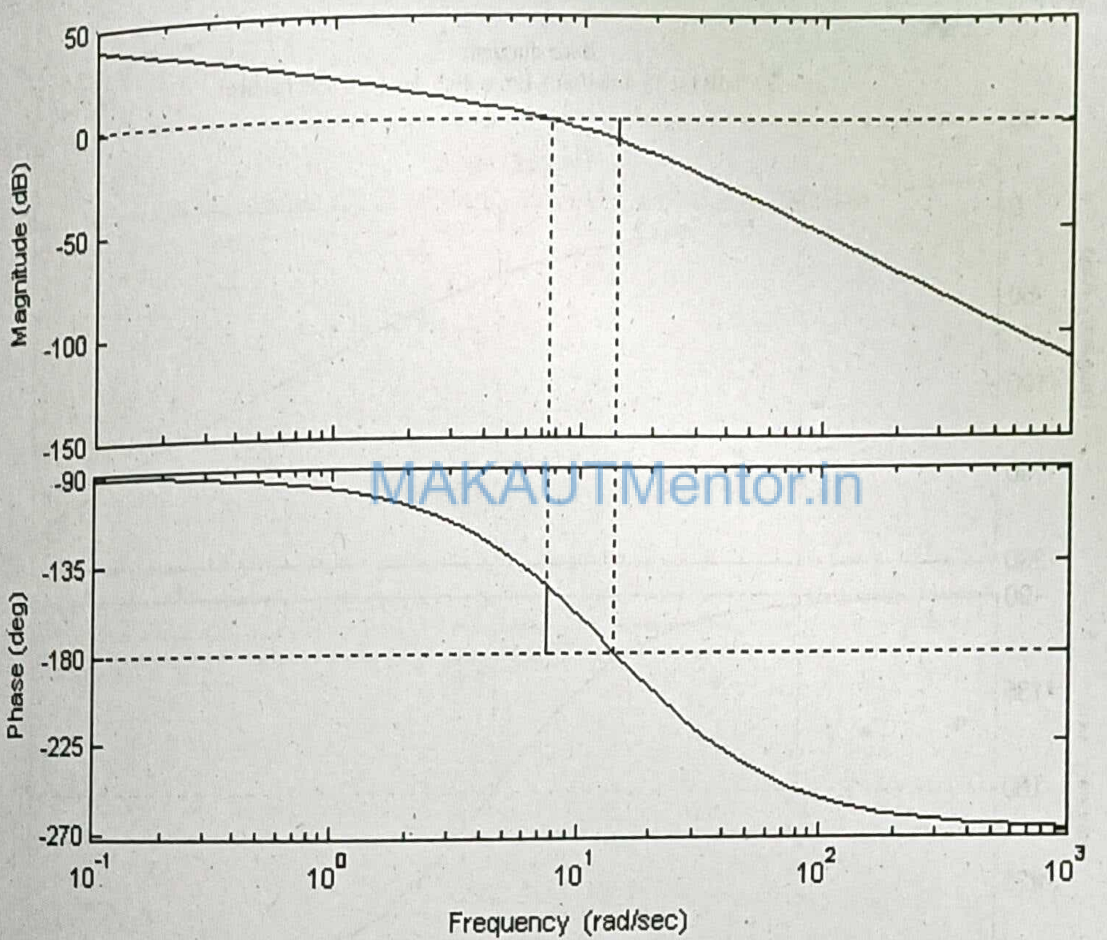
Long Answer Type Questions

1. For the system defined by $G(s)=\dfrac{10}{s\,(0.1s+1)\,(0.05s+1)}$ draw the asymptotic gain and phase plots and calculate the gain and phase margins. What is the output of the system at steady - state if a step of 2 units is applied as an input at $t=0$?

[WBUT 2011]

Answer:

1st Part:



From the Bode plots we evaluate

GM=9.54 dB

PM=32.6°

2nd Part:

The output in s-domain is given by

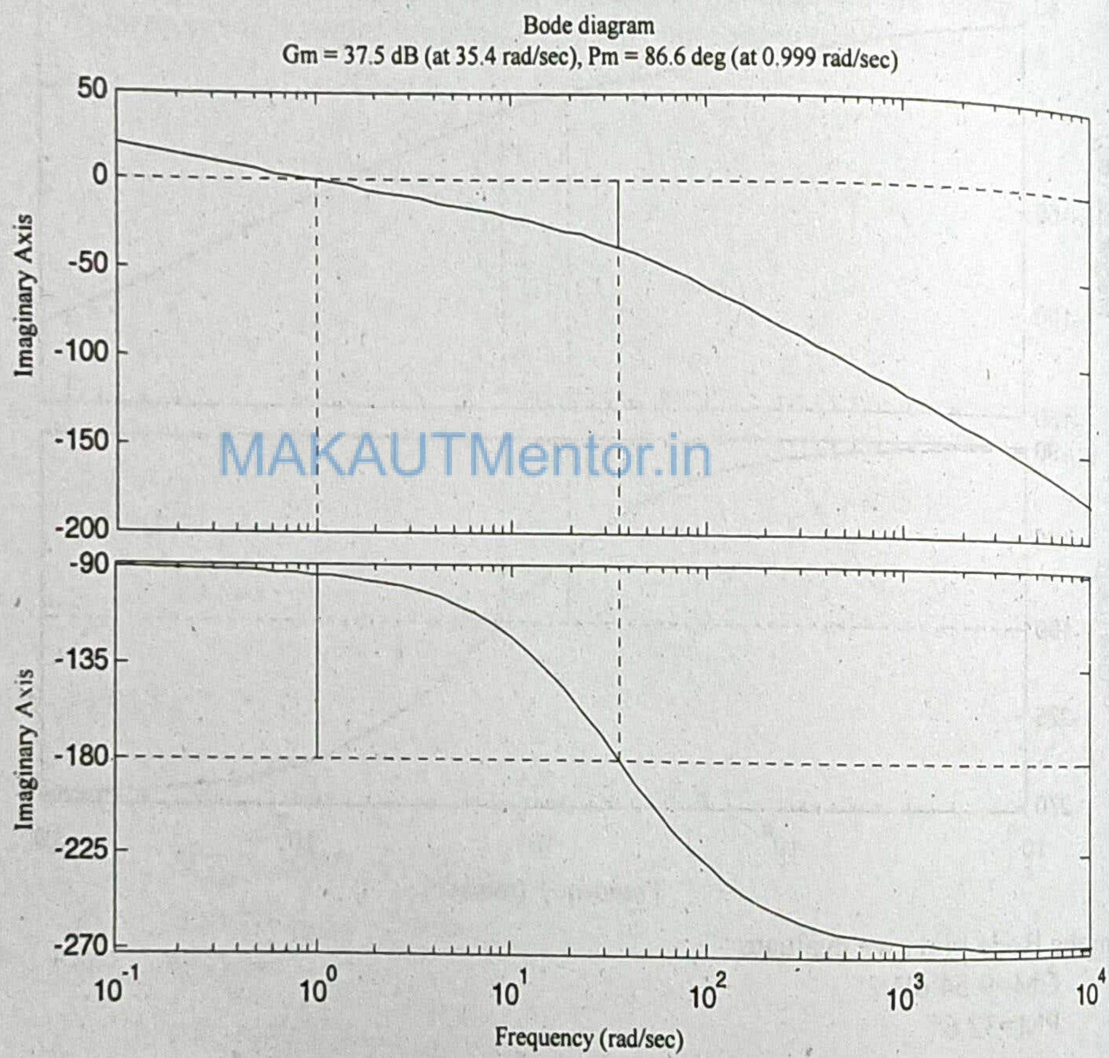
$$C(s)=\frac{10}{s(0.1s+1)(0.05s+1)}\times R(s)$$

The steady-state output is

$$\lim_{s\to 0} s\cdot C(s) = \lim_{s\to 0} s\cdot\frac{10}{s(0.1s+1)(0.05s+1)}\times\frac{2}{s} = \infty$$

2. Draw the bode plot of the open loop transfer function of a unity feedback system is given by $G(s) = \frac{k}{s(1+0.02s)(1+0.04s)}$. Find the gain margin and phase margin. Hence find the value of open loop gain so that the closed loop system has a phase margin of 45°. **[WBUT 2012]**

Answer:



Gain margin $= 37.5$ dB

Phase margin $= 86.6$ degrees.

To have a Pm of 45°

$$\phi = \text{Pm} - 180° = -135°$$

$\therefore \quad -135° = -90° - \tan^{-1} 0.02\omega - \tan^{-1} 0.04\omega$

$\Rightarrow \quad 45° = \tan^{-1} 0.02\omega + \tan^{-1} 0.04$

$$1=\frac{0.02\omega+0.04\omega}{1-0.0008\omega^2}=\frac{0.06\omega}{1-0.0008\omega^2}=\frac{600\omega}{10000-8\omega^2}$$

$\Rightarrow$ $8\omega^2+600\omega-10000=0$

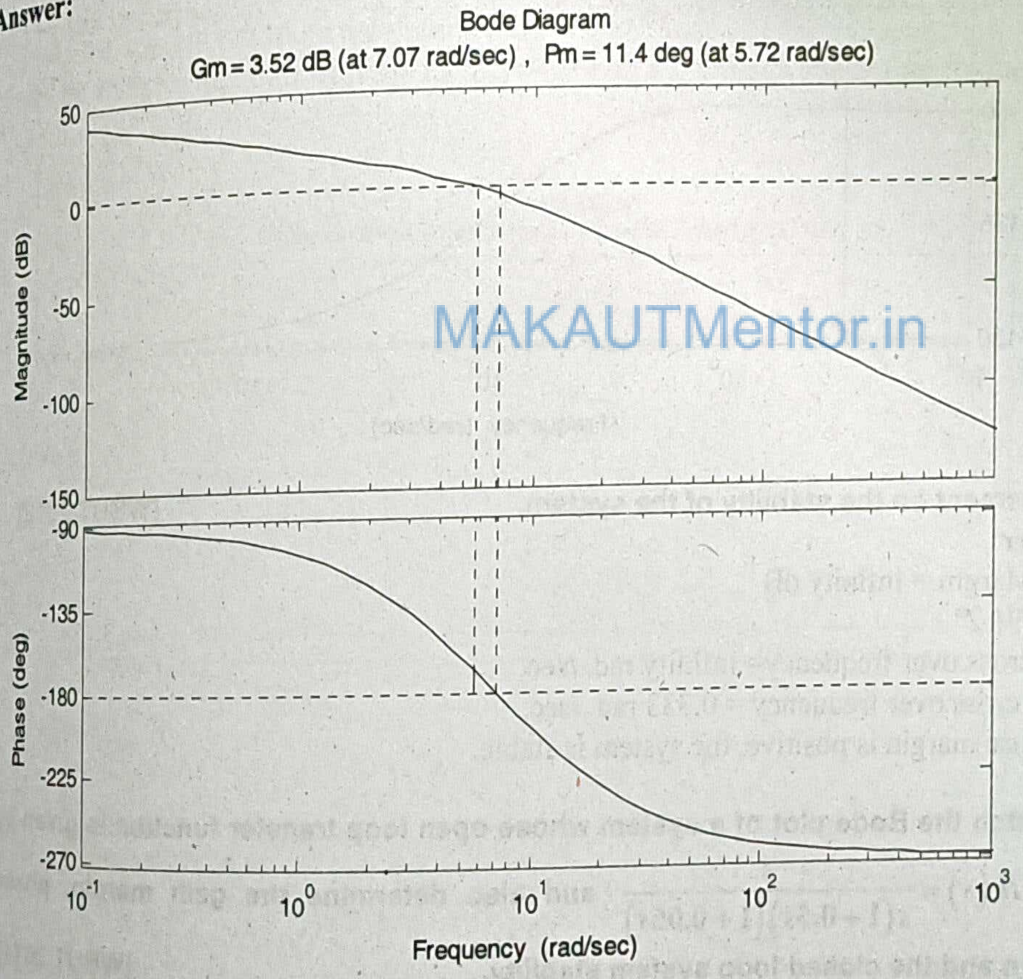
$\Rightarrow$ $\omega^2+75\omega-1250=0$

$\Rightarrow$ $\omega=\frac{-75\pm\sqrt{625}}{2}=\frac{-75\pm103}{2}$

$\Rightarrow$ Discussing negative value of ω, $\omega=14$ rad/sec.

3. Sketch the bode plot and determine the GCF and PCF of the following $G(s)=\frac{10}{s(1+0.5s)(1+0.1s)}$. **[WBUT 2013]**

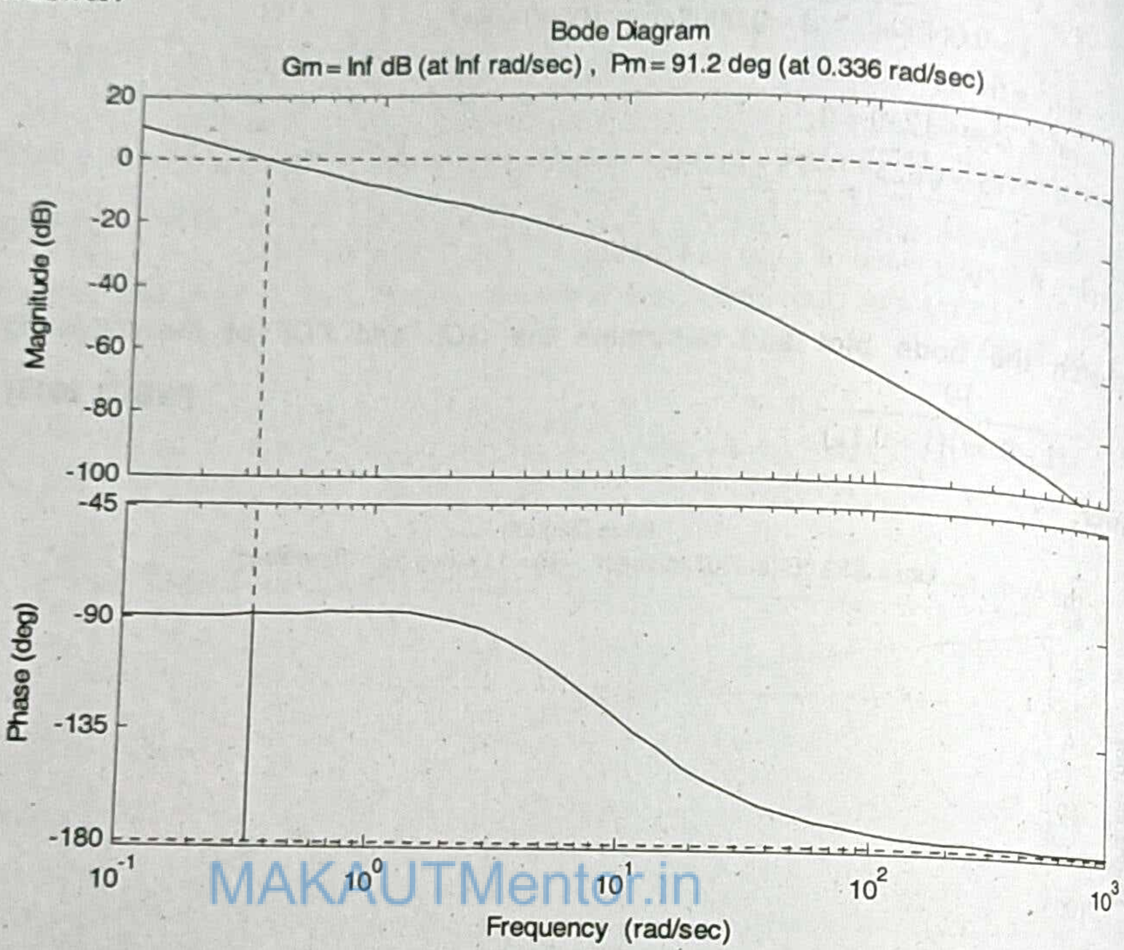
Answer:



4. a) Sketch the Bode plot of a unity negative feedback closed loop system of which open loop transfer function is given by $\frac{5(s+2)}{s(s+3)(s+10)}$.

Determine gain margin, phase margin, gain cross-over frequency & phase cross-over frequency. [WBUT 2013]

Answer:



b) Comment on the stability of the system. **[WBUT 2013]**

Answer:

Gain Margin = Infinity dB

PM = 91.2°

Gain cross over frequency= Infinity rad. /sec.

Phase cross over frequency = 0.333 rad. /sec.

The phse margin is positive, the system is stable.

5. Sketch the Bode plot of a system whose open loop transfer function is given by $G(s)H(s)=\dfrac{4}{s(1+0.5s)(1+0.05s)}$ **and also determine the gain margin, phase margin and the closed loop system stability.** **[WBUT 2014]**

Answer:

Getting the magnitude plot:

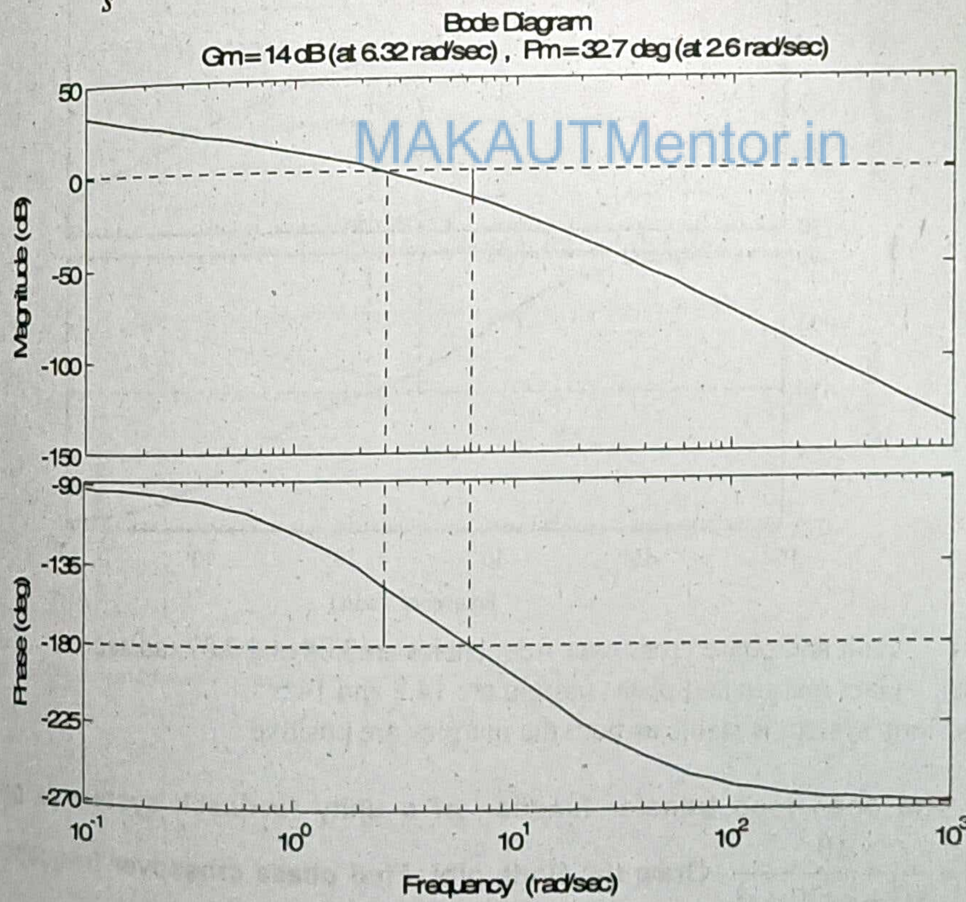
Forming the table

Factors	Corner frequency (rad/sec)	Slope contributed by each factor (dB/sec)	Cumulative slope (dB/sec)	The straight line	
				Starts at ω	Ends at ω
4	None	0			
$\frac{1}{s}$	None	–20	–20	$\frac{1}{4}=0.25$	2
$\frac{1}{1+0.5s}$	$\frac{1}{0.5}=2$	–20	–40	2	20
$\frac{1}{1+0.05s}$	$\frac{1}{0.05}=20$	–20	–60	20	Extend the line

Getting the starting point the magnitude plot.

From the transfer function with the factors which do not contain any corner frequency

$$\text{T.F.} = \frac{4}{s}$$



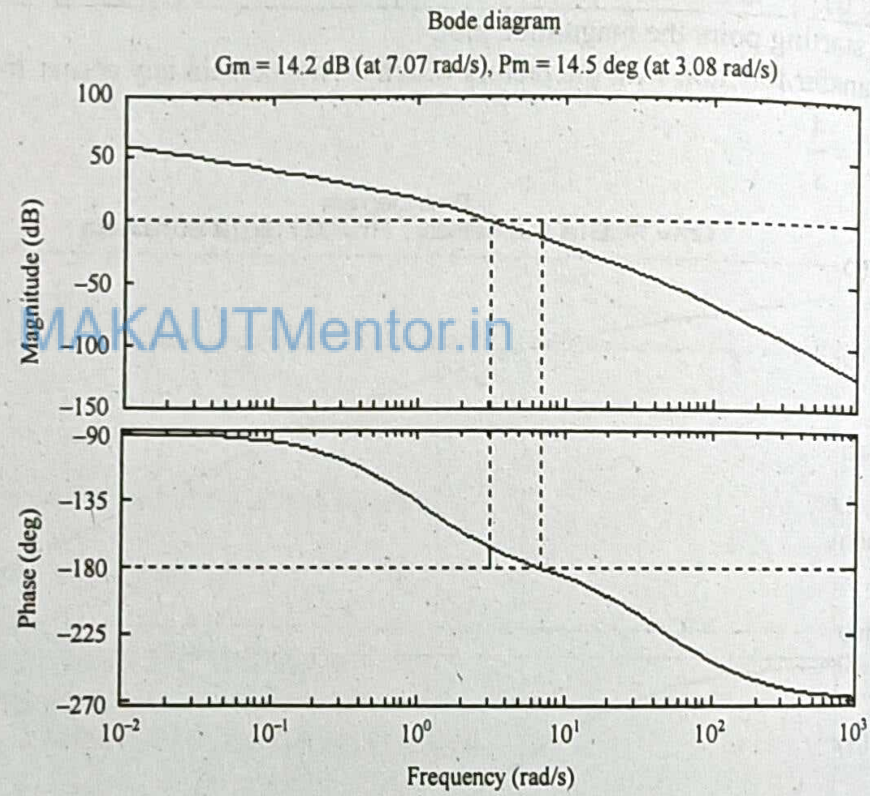
Gain margin=14 dB
Gain cross over frequency=6.32 rad/sec
Phase margin=32.7°
Phase cross over frequency=2.6 rad/sec
Both gain margin & Phase margin are positive so the system is stable.

6. Construct the Bode plots for a unity feedback system whose open loop transfer function is given by $G(s)=\dfrac{10}{s(s+1)(1+0.025)}$. From the Bode plot determine:

(i) Gain and phase cross over frequencies
(ii) Gain margin and phase margin
(iii) Stability of the closed loop system.

[WBUT 2015]

Answer:



(i) Gain and phase cross over frequencies are3.08 and 7.07 rad/sec
(ii) Gain margin and phase margin are 14.2 and 14.5

Closed loop system is stable as both the margins are positive

7. a) The open loop transfer function of a unity feedback system is given by $G(s)=\dfrac{10}{s(1+s)(10+s)}$. Draw the Bode plot. Find phase crossover frequency, gain crossover frequency, gain margin & phase margin.

b) What is minimum phase system & non-minimum phase system? Give example.

[WBUT 2016]

Answer:

a) $$G(s)=\frac{10}{s(1+s)(10+s)}=\frac{10}{10s(1+s)(1+.1s)}=\frac{1}{s(1+s)(1+.1s)}$$

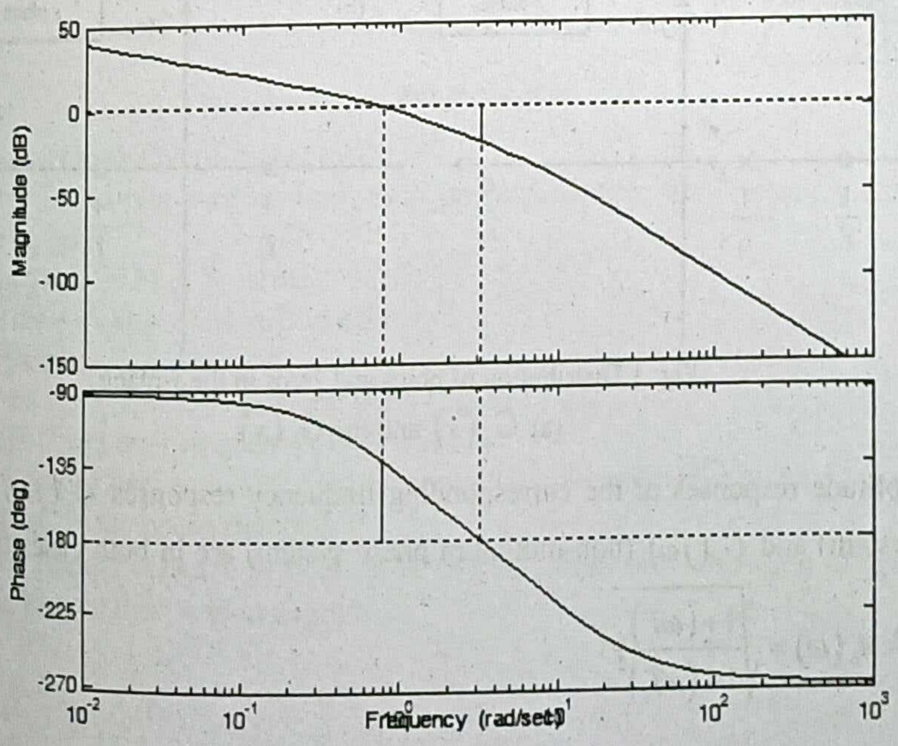
Magnitude Plot

S.No.	Factor	Corner frequency rad/sec	Asymptotic log-magnitude characteristics
1.	$\frac{1}{s}$	None	Straight line of –20 dB/dec passing through $\omega=1$
2.	$\frac{1}{1+s}$	1	Straight line of –20dB/dec originating from $\omega=1$
3.	$\frac{1}{1+0.1s}$	10	Straight line of –20 dB/dec originating from $\omega=10$

Phase Plot: $\phi=-90°-\tan^{-1}\omega-\tan^{-1}0.1\omega$

S. No	ω rad/sec	Asymptotic log-magnitude characteristics
1.	0	–90°
2.	0.1	–96.3°
3.	0.5	–119.4°
4.	1	–140.7°
5.	3	–178.3°
6.	5	–195°
7.	10	–219°

Bode Diagram
Gm = 20.8 dB (at 3.16 rad/sec), Pm = 47.4 deg (at 0.784 rad/sec)



Gain Margin = 20.8 dB
Gain Crossover Frequency = 3.16 rad/sec
Phase Margin = 47.4°
Phase Crossover Frequency = 0.784 rad/sec.

b) Minimum and Non-minimum Phase System

A stable system having no dead time transfer function and the transfer function does not carry zeros in the right half of s-plane, is called ***minimum phase*** system. Minimum-phase systems are what one commonly builds from passive components like resistor (R), inductor (L) and capacitor (C) in electrical systems and mass (M), spring (K) and dashpot (B) in mechanical system. If a transfer function has a zero / zeros in the right half of s-plane then this system shows non-minimum phase behaviour.

We know that for the stability of a system the poles should lie in the left half of the s-plane, but the zeros can be either side of the s-plane. If the zeros are reflected to the right half of s-plane, the magnitude response is the same but the phase shift is more than what would be the case if all of the zeros were in the left half-plane.

In order to illustrate the non-minimum phase behaviour, two systems will be considered, which have in fact the same amplitude response $A(\omega)$ but differ considerably in the phase response. The transfer functions of the two systems are $G_a(s)=\frac{1+sT}{1+sT_1}$ and $G_b(s)=\frac{1-sT}{1+sT_1}$ with $0<T<T_1$. The distributions of the poles and zeros of $G_a(s)$ and $G_b(s)$ in the s-plane are shown in Figs. 1(a) and (b).

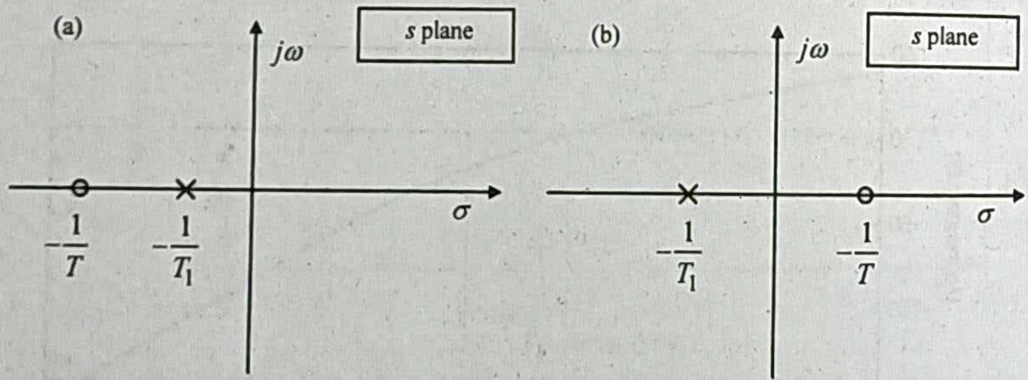


Fig: 1 Distribution of poles and zeros in the s-plane
(a) $G_a(s)$ and (b) $G_b(s)$

The amplitude responses of the corresponding frequency responses $G_a(j\omega)$ (minimum phase system) and $G_b(j\omega)$ (non-minimum phase system) are in both cases the same, as

$$A_a(\omega)=A_b(\omega)=\sqrt{\frac{1+(\omega T)^2}{1+(\omega T_1)^2}}.$$

For the phase responses one finds

$$\phi_a(\omega) = -\arctan\frac{\omega(T_1 - T)}{1+\omega^2 T_1 T} \text{ and } \phi_b(\omega) = -\arctan\frac{\omega(T_1 + T)}{1-\omega^2 T_1 T}.$$

8. a) Explain the meaning and significance of phase margin and gain margins of a control system. How will you obtain the values of these margins from Bode plots?
b) Sketch the Bode plot for the following function and find out the approximate values of the gain margin and the phase margin

$$G(s) = \frac{10(s+2)}{s(s+6)(s+10)}.$$

[WBUT 2017]

Answer:

To get Gain Margin

Step 1: First obtain the Magnitude and Phase plots (Fig. 1)

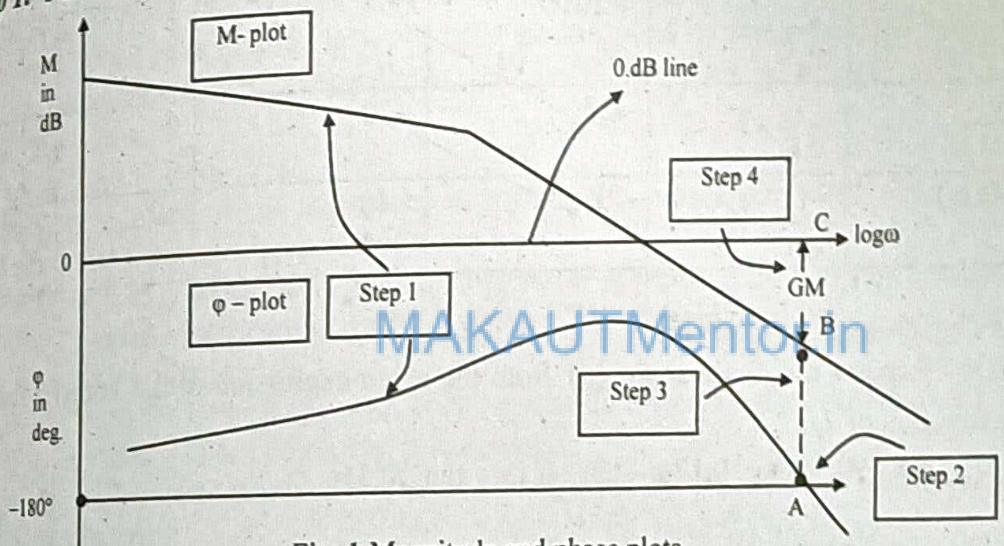


Fig: 1 Magnitude and phase plots

Step 2: Find Phase Cross-Over Frequency (PCF).

Step 3: Draw a dotted line starting from PCF perpendicular to the Frequency axis to meet a point on M-plot.

Step 4: Find the value of M at B, in dB

Step 5: GM = 0 dB – (Value of M at B in dB)

To get Phase Margin

Step 1: First draw the Magnitude & Phase plots (figure 11.12)

Step 2: Find Gain Cross over frequency (GCF), say X.

Step 3: Draw a dotted line, say XY, starting from GCF, perpendicular to the frequency axis to meet a point on the phase-plot (say Y).

Step 4: Get the shortest distance of point Y in degrees, from the –180° line to get the phase margin as, PM = (180° + ϕ) in degrees.

b) ***Step A:*** $G(s)=\dfrac{10(s+2)}{s(s+6)(s+10)}=\dfrac{\frac{1}{3}(1+0.5s)}{s(1+0.17s)(1+0.1s)}$

Step B: Forming the table

Factors	Corner frequency (rad/sec)	Slope contributed by each factor (dB/dec)	Cumulative slope (dB/dec)	The straight line Starts at ω	Ends at ω
1/3	None	0	0		
$\frac{1}{s}$	None	–20	–20	0.1 rad/sec	2 rad/sec
$\frac{1}{(1+0.5s)}$	$\frac{1}{0.5}=2$	+20	0	2	6
$\frac{1}{(1+0.17s)}$	$\frac{1}{0.17}=6$	–20	–20	6	10
$(1+0.1s)$	$\frac{1}{0.1}=10$	-20	–40	10	Extend the line

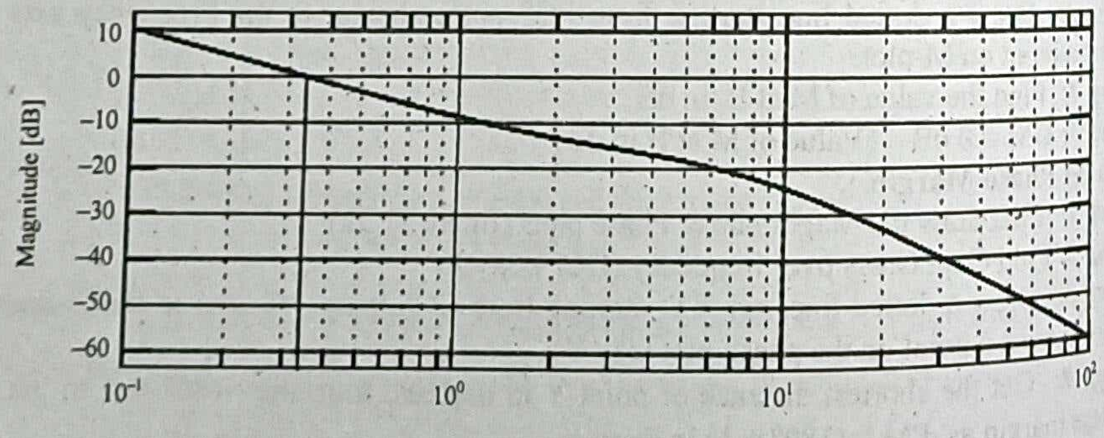
Step C: Getting the Phase plot

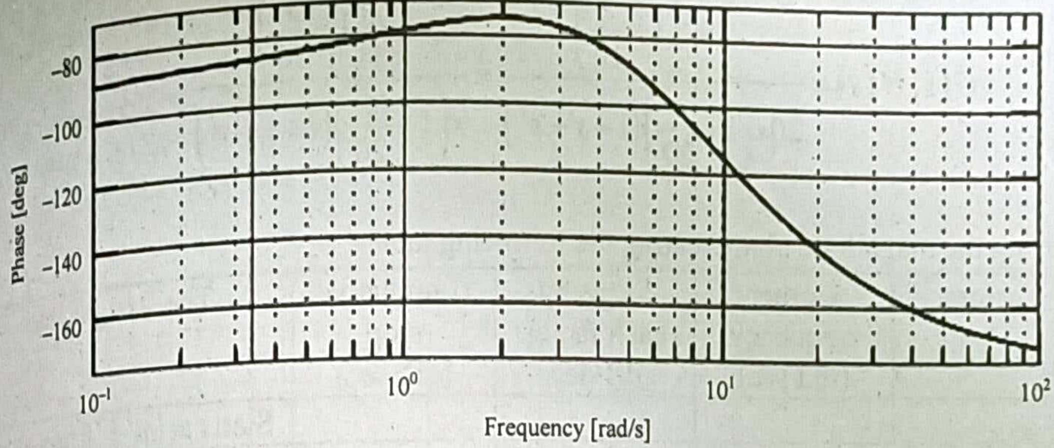
Step A: Express the phase angle (ϕ) from the given expression of the Transfer function as a function of ω.

$$\phi=-90^\circ-\tan^{-1}0.17\omega-\tan^{-1}0.1\omega+\tan^{-1}0.5\omega$$

Step D: After Plotting

GM = inf dB (at NaN rad/s), PM = 94.4236 deg (at 0.337316 rad/s)





9. a) What are the advantages of Bode diagram? **[WBUT 2018]**

Answer:

Bode plots were first introduced by H.W. Bode, when he was working at Bell labs in the United States. Now before I describe what are this plots it is very essential here to discuss a few advantages over other stability criteria. Some of the advantages of this plot are written below:

Advantages of Bode Plot:

1. It is based on the asymptotic approximation, which provides a simple method to plot the logarithmic magnitude curve.
2. The multiplication of various magnitude appears in the transfer function can be treated as an addition, while division can be treated as subtraction as we are using a logarithmic scale.
3. With the help of this plot only we can directly comment on the stability of the system without doing any calculations.
4. **Bode plots** provides relative stability in terms of **gain margin** and **phase margin.**
5. It also covers from low frequency to high frequency range.

b) Sketch the asymptotic Bode plot for the following open loop transfer function with unit feedback.

$$G(s)H(s)=\frac{20(s+10)}{s(s+20)(s^2+s+1)}$$

Calculate the gain and phase cross-over frequency, gain margin and phase margin of the Bode plot. Also determine the closed loop stability of the system. **[WBUT 2018]**

Answer:

$$G(s)H(s)=\frac{20(s+10)}{s(s+20)(s^2+s+1)}$$

Converting the T.F. into the time constant form.

$$G(s)H(s)=\frac{200\left(1+\frac{s}{10}\right)}{20s\left(1+\frac{s}{20}\right)\left(1+s+s^2\right)}=\frac{10\left(1+\frac{s}{10}\right)}{s\left(1+\frac{s}{20}\right)\left(1+s+s^2\right)}$$

Getting the magnitude plot, creating the following table

Factors	Corner frequency (rad/sec)	Slope by each factor dB/dec	Cumulative slop (dB/dec)	The straight line	
				Starts at ω	Ends at ω
10	NONE	0	(+) 0		
$\frac{1}{s}$	NONE	−20	(+) −20	0.1	1
$\frac{1}{1+s+s^2}$	1	−40	(+) −60	1	10
$\left(1+\frac{s}{10}\right)$	10	+20	(+) −40	10	20
$\frac{1}{\left(1+\frac{s}{20}\right)}$	20	−20	−60	20	Extend the line

Getting the starting point of the magnitude

$$G(j\omega)=\frac{10}{s}=\frac{10}{j\omega}$$

$$M=\left|\frac{10}{j\omega}\right|=\frac{10}{\omega}$$

$$M \text{ in dB}=20\log_{10}10-20\log_{10}\omega$$

$$M \text{ in dB}\Big|_{\omega=0.1}=20\log_{10}10=20\text{ dB}$$

Getting the phase plot:

$$G(j\omega)=\frac{20(j\omega+10)}{j\omega(j\omega+20)\left(1-\omega^2+j\omega\right)}$$

$$\varphi=\angle G(j\omega)=-90^\circ+\tan^{-1}\frac{\omega}{10}-\tan^{-1}\frac{\omega}{20}-\tan^{-1}\frac{\omega}{1-\omega^2}$$

ω (rad/sec)	1	10	20	50
φ degree	−177.15°	−77.27°	−74.425°	−72.339°

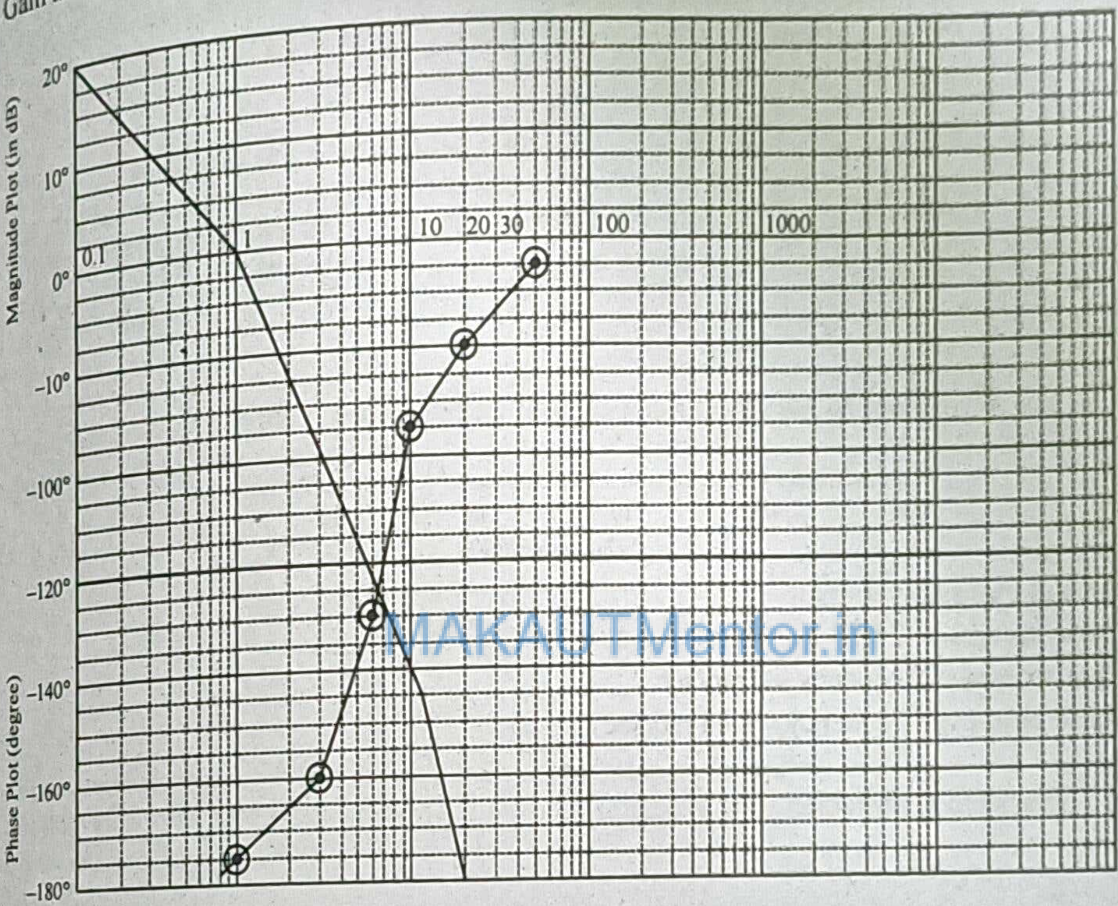
From the Bode Plot:

Gain cross-over frequency = 1 rad/sec

Phase cross-over frequency = 24 rad/sec

Phase margin = $180° - 93.4° = 86.6°$

Gain margin = $0 - (-34) = 34$ dB



10. a) Construct the Bode plot for a unity feedback control system having

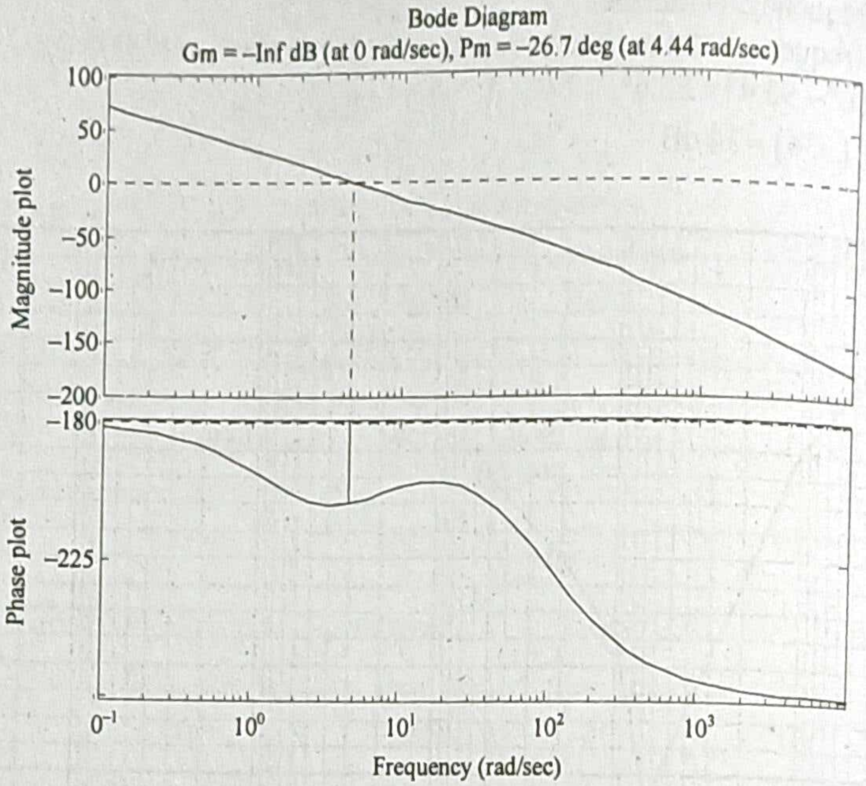
$$G(s) = 36(0.2s+1)/s^2(0.05s+1)(0.01s+1)$$

b) From the plot obtain the gain margin, phase margin, gain cross over frequency, phase cross over frequency.

c) Comment on the closed loop stability of the system. **[WBUT 2019]**

Answer:

a)



b) Gain margin = $-\infty$ dB

Phase margin = $-26.7°$

Gain crossover frequency = 0 rad/sec

Phase crossover frequency = 4.44 rad/sec

c) The system is unstable.

NYQUIST PLOT

Multiple Choice Type Questions

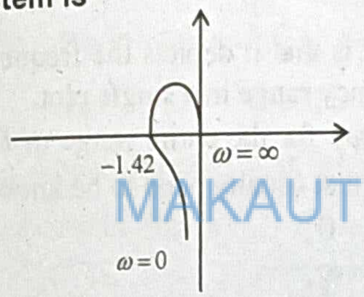
1. If the Nyquist plot of a certain feedback system crosses the negative real axis at –0.1 point, the gain margin of the system is given by [WBUT 2008, 2011]
   a) 0.1 b) 10 c) 100 d) none of these

Answer: (b)

2. The disadvantage(s) of polar plot is (are) [WBUT 2009, 2010]
   a) plot is cramped of high frequencies
   b) the calculations are time consuming for exact plot
   c) it is very difficult to calculate gain & phase margin
   d) all of these

Answer: (d)

3. The polar plot of a type-1, 3-pole, open loop system is shown in the figure given below. The closed loop system is [WBUT 2012]



   a) always stable b) marginally stable
   c) unstable with one RHS pole d) unstable with two RHS pole

Answer: (d)

4. The Nyquist plot cuts the negative real axis at a distance of 0.4, then the gain margin of the system is [WBUT 2013]
   a) 0.4 b) –0.4 c) 4 d) 2.5

Answer: (d)

5. The frequency at which Nyquist diagram crosses the negative real axis is known as [WBUT 2014, 2018]
   a) gain crossover frequency b) phase crossover frequency
   c) natural frequency d) breakaway point

Answer: (b)

6. The Nyquist plot of a system encloses the point (–1, 0). The gain margin of the system is [WBUT 2015]
   a) less than zero b) greater than zero
   c) zero d) infinite

Answer: (b)

7. The centre of the constant M-circles are defined by [WBUT 2016]

a) $\left[\dfrac{M^2}{1+M^2},0\right]$ b) $\left[\dfrac{M^2}{1-M^2},0\right]$ c) $\left[0,\dfrac{M^2}{1+M^2}\right]$ d) $\left[0,\dfrac{M^2}{1-M^2}\right]$

Answer: (b)

8. If the gain margin of a certain feedback system is given as 20 db, the Nyquist plot will cross the negative real axis at the point [WBUT 2016]

a) $S=-0.05$ b) $S=-0.2$ c) $S=0.5$ d) $S=-0.1$

Answer: (d)

Short Answer Type Questions

1. What is polar plot? [WBUT 2007, 2009, 2009-Short Note]

Answer:

The polar plot of a sinusoidal transfer function $G(j\omega)$ is a plot of the magnitude of $G(j\omega)$ versus the phase angle of $G(j\omega)$ on polar coordinates as ω is varied from zero to infinity.

An advantage of using polar plot is that it depicts the frequency response characteristics of a system over the entire frequency range in a single plot.

To sketch the polar plot of $G(j\omega)$ for the entire range of frequency ω, i.e., from 0 to infinity, there are four key points that usually need to be known:

(1) the start of plot where $\omega=0$,
(2) the end of plot where $\omega=\infty$,
(3) where the plot crosses the real axis, i.e., $\text{Im}(G(j\omega))=0$, and
(4) where the plot crosses the imaginary axis, i.e., $\text{Re}(G(j\omega))=0$.

2. Sketch polar plot for the unity feedback system with open loop transfer function $G(s)=\dfrac{1}{s(s+2)}$. [WBUT 2013]

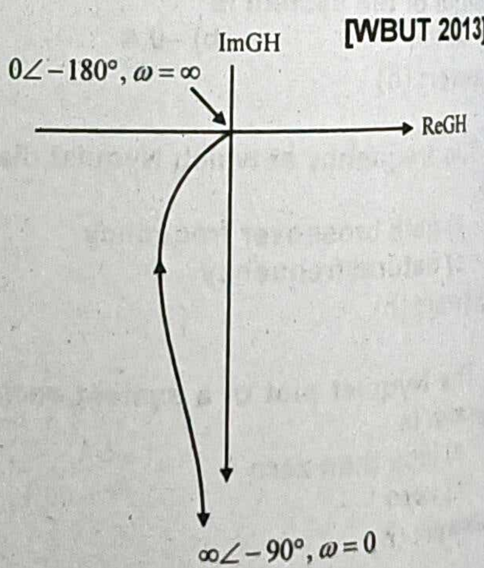
Answer:

$$G(j\omega)=\frac{1}{j\omega(j\omega+2)}$$

$$M=\frac{1}{\omega\sqrt{\omega^2+4}},\ \phi=-90^\circ-\tan^{-1}\frac{\omega}{2}$$

Now forming a table as shown below:

ω	M	ϕ
0	∞	$\angle -90^\circ$
∞	0	-180°



1	$\frac{1}{\sqrt{s}}$	$-116°$
2	$\frac{1}{4\sqrt{2}}$	$-135°$

Polar plot may be constructed with the data in the above table

Long Answer Type Questions

1. a) State and explain the Nyquist criterion for studying of a control system. **[WBUT 2006, 2007]**

OR,

What do you mean by Nyquist criterion? **[WBUT 2009]**

OR,

State & explain Nyquist stability criterion. **[WBUT 2010, 2014]**

OR,

State Nyquist stability criteria. From which principle is Nyquist stability criterion originated? State the principle. **[WBUT 2013]**

OR,

State and explain Nyquist criterion for studying stability of a control system. **[WBUT 2018]**

Answer:

Let us consider a system whose forward path transfer function is $G(s)$ and that of feedback path is $H(s)$ as shown in Fig: (a).

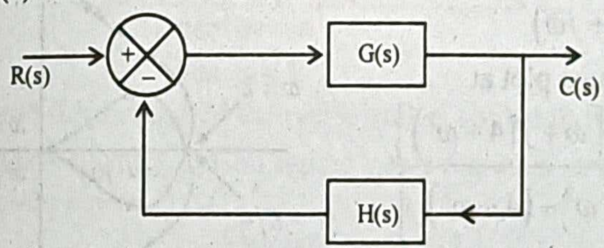


Fig: (a) A closed loop system with forward path T.F = G(s) and feed back path T.F =H(s)

The closed loop transfer function, $T.F.=\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}$

and the characteristic polynomial $= F(s)=1+G(s)H(s)$

From our basic knowledge, if all the roots of the characteristic equation lie in the left half of s-plane then the system in stable.

Now, assume

$$G(s)H(s)=\frac{K(s+z_1)(s+z_2)\cdots\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots\cdots(s+p_n)};\quad n\geq m \quad \text{.... (i)}$$

and $$F(s)=\frac{(s+z_1')(s+z_2')\cdots\cdots(s+z_n')}{(s+p_1)(s+p_2)\cdots\cdots(s+p_n)} \quad \text{.... (ii)}$$

From the equation (i) and (ii) we can say,

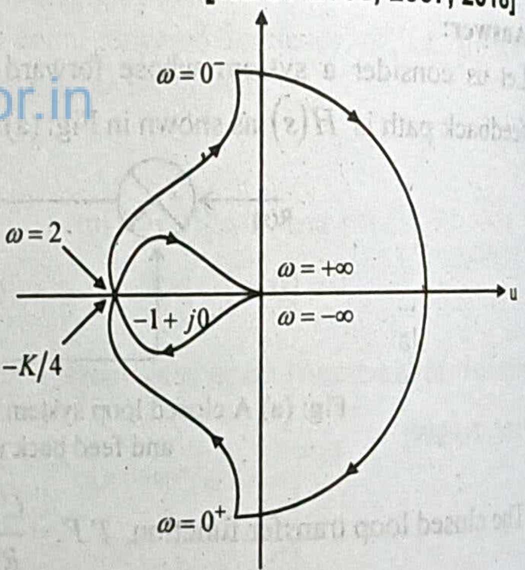
Open loop poles are same. As the closed loop poles i.e. poles of $G(s)\,H(s)$ are same as that of $F(s)$. Zeros of the characteristic polynomial $F(s)$ are the roots of the characteristic equation (i.e. $F(s)=0$. For stability, the roots of characteristic equation or zeros of $F(s)$ should lie in left half of s-plane. If a zero of $F(s)$ is found in right half side (with positive real part) then the system is stable. With this fact and taking the help of Cauchy's principle of argument Nyquist constructed a closed path or contour in s-plane such that entire right half of s-plane is encircled to find the presence of poles and zeros of $F(s)$. For stability, $F(s)$ will not encircle the origin.

b) A unity feedback control system has open loop transfer function

$$G(S)=\frac{K}{S(S^2+S+4)}.$$

Draw the Nyquist plot & hence investigate the stability of the system for various value of K. **[WBUT 2006, 2007, 2018]**

Answer:



$$GH(j\omega)=\frac{K}{j\omega(4-\omega^2+j\omega)}$$

Real axis crossing Nyquist plot at

$$GH(j\omega)=\frac{-k\left[\omega+j\left(4-\omega^2\right)\right]}{\omega\left[\omega^2+\left(4-\omega^2\right)^2\right]}$$

Equating imaginary part to zero

$$4-\omega^2=0 \quad \text{or} \quad \omega^2=4 \quad \text{or} \quad \omega=\pm 2$$

$$\left|GH(j\omega)\right|_{\omega^2=4}=\frac{-K}{4}$$

So the Nyquist plot crosses the axis of real at $\omega=\pm 2$ with an intercept of $-K/4$.

From the Nyquist plot $K/4>1$ or $K>4$

For this value of K

$$N=P-Z$$

$$-2=0-Z \quad \text{or} \quad Z=2$$

So the system is unstable. It can be shown to be stable for $K<4$.

c) What are the advantages of Nyquist plot? **[WBUT 2006, 2007, 2008]**

Answer:

1) The stability of the closed-loop system may be determined from open loop frequency response.
2) Knowledge of roots of the closed loop system is not needed.
3) Often we do not get any mathematical model of the physical system. We often get the frequency response.

2. For the system defined by $G(s)\,H(s)=\dfrac{(1+4s)}{s(s+1)\,(2s+1)}$ draw the Nyquist plot and hence comment on the stability of the closed loop system. [WBUT 2011]

Answer:

$$M=\frac{\sqrt{1+16\omega^2}}{\omega\sqrt{1+\omega^2}\,\sqrt{1+4\omega^2}}$$

$$\varphi=\tan^{-1}4\omega-90^\circ-\tan^{-1}\omega-\tan^{-1}2\omega$$

The angle φ for different values of 'ω' is tabulated below.

ω	$\tan^{-1}4\omega$	-90°	$-\tan^{-1}\omega$	$-\tan^{-1}2\omega$	ω
0	0	−90°	0	0	−90°
∞	90°	−90°	−90°	90°	0°
0.1	21.8°	−90°	−5.7°	−11.3°	−85.2°
1	75.96°	−90°	−45°	−63.43°	−122.47°

The value of $M=0$ when $\omega=\infty$

$M=\infty$ when $\omega=0$

The Nyquist's plot from the calculated data of M and φ shown in fig

The Nyquists plot makes an intercept on the negative real axis at point X. The intercept OX is calculated as given below.

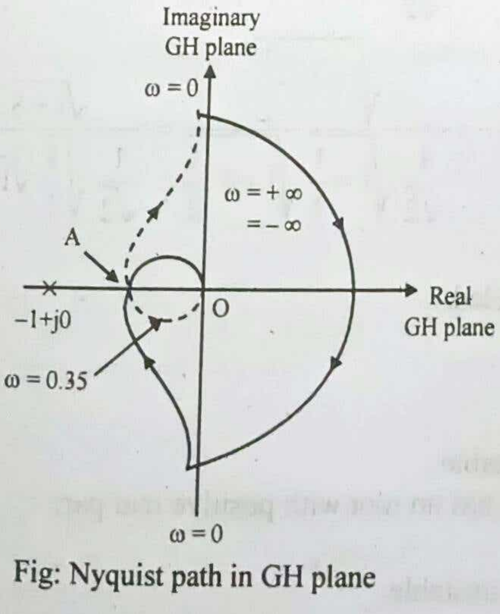


Fig: Nyquist path in GH plane

Separating into real & imaginary parts

$$G(j\omega)H(j\omega)=\frac{1+4j\omega}{j\omega(1+j\omega)(1+2j\omega)}=\frac{1+4j\omega}{\left(j\omega+j^2\omega^2\right)(1+2j\omega)}$$

$$=\frac{1+4j\omega}{\left(j\omega+\omega^2\right)(1+2j\omega)}=\frac{1+4j\omega}{j\omega-\omega^2+2j^2\omega^2-2j\omega^3}$$

$$=\frac{1+4j\omega}{j\omega-\omega^2-2\omega^2-2j\omega^3}=\frac{1+4j\omega}{j\omega-3\omega^2-2j\omega^3}$$

$\varphi=-180°$

$\Rightarrow \quad \tan^{-1}4\omega-90°-\tan^{-1}\omega-\tan^{-1}2\omega=-180°$

$\Rightarrow \quad \tan^{-1}4\omega-\tan^{-1}\omega-\tan^{-1}2\omega=-90°$

$\Rightarrow \quad \tan^{-1}4\omega-\left[\tan^{-1}\omega+\tan^{-1}2\omega\right]=-90°$

$\Rightarrow \quad \tan\left[\tan^{-1}4\omega-\tan^{-1}\omega-\tan^{-1}2\omega\right]=\tan(-90°)$

$\Rightarrow \quad \tan^{-1}4\omega-\tan^{-1}\omega-\tan^{-1}2\omega=-90°$

$\Rightarrow \quad -\left(\tan^{-1}\omega+\tan^{-1}2\omega\right)=-90°-\tan^{-1}4\omega$

$\Rightarrow \quad \tan^{-1}\omega+\tan^{-1}2\omega=90°+\tan^{-1}4\omega$

$\Rightarrow \quad \dfrac{\omega+2\omega}{1-2\omega^2}=\infty+4\omega$

$\Rightarrow \quad \dfrac{3\omega}{1-2\omega^2}=\infty=\dfrac{1}{0}$

$\Rightarrow \quad 1-2\omega^2=0$

$\Rightarrow \quad \omega^2=\dfrac{1}{2} \quad \Rightarrow \omega=\pm\dfrac{1}{\sqrt{2}}$ rad./sec.

$$M=\frac{\sqrt{1+16\omega^2}}{\omega\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}=\frac{\sqrt{1+16\cdot\frac{1}{2}}}{\frac{1}{\sqrt{2}}\sqrt{1+\frac{1}{2}}\sqrt{1+4\cdot\frac{1}{2}}}=\frac{\sqrt{1+8}}{\frac{1}{\sqrt{2}}\sqrt{\frac{3}{2}}\sqrt{1+2}}=\frac{\sqrt{9}}{\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}\sqrt{3}}=2$$

$OA=2$

Point $-1+j0$ is not encircled.

$N=0$

$Z=N+P$

$0+0=0$

Closed loop system is stable.

The closed loop system has no root with positive real part.

As $\quad P=0$

$\therefore$ Open loop system is unstable.

3. a) State Nyquist stability criterion. **[WBUT 2012, 2013, 2014]**

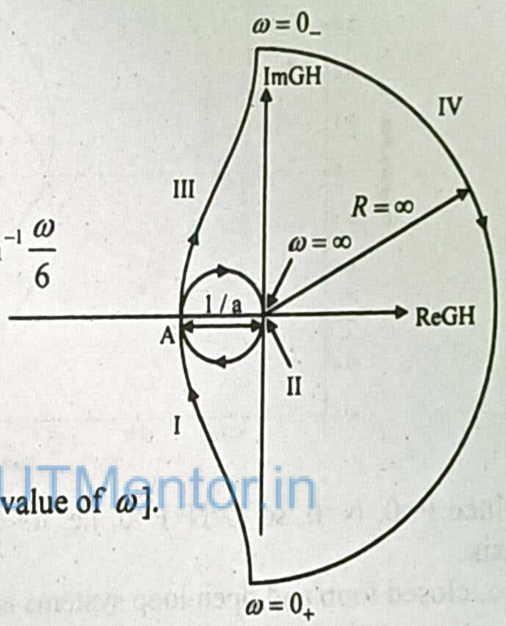
Answer:

Refer to Question No. 1(a) of Long Answer Type Questions.

b) Using Nyquist stability criterion determine whether the unity feedback close loop system having open loop transfer function $G(s)=\dfrac{120}{s(s+4)(s+6)}$ is stable or not. **[WBUT 2012]**

Answer:

The point A is defined by a $\angle -180°$, phase crossover frequency is ω_{pc}



Now, $M=\left|G(j\omega)\right|=\dfrac{120}{\omega\sqrt{\omega^2+16}\,\sqrt{\omega^2+36}}$

and $\phi=-90°-\tan^{-1}\dfrac{\omega}{4}-\tan^{-1}\dfrac{\omega}{6}$

At phase crossover $-180°=-90°-\tan^{-1}\dfrac{\omega}{4}-\tan^{-1}\dfrac{\omega}{6}$

$$\Rightarrow \quad \frac{\dfrac{\omega}{4}+\dfrac{\omega}{6}}{1-\dfrac{\omega^2}{24}}=\frac{1}{0}$$

$\Rightarrow \quad \omega = 4.9$ rad/sec [Discarding negative value of ω].

$$\therefore \quad M=a=\frac{120}{\sqrt{24}\sqrt{28}\sqrt{60}}\approx 200$$

From the plot, we get $N=2$

Since, $P=0$, so, as per Nyquist $Z=N+P=2+0=2$.

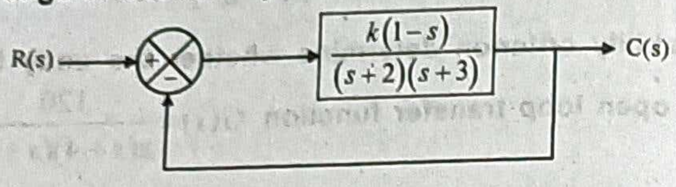
So, closed loop system is unstable.

4. a) How is Nyquist criterion different from R-H criterion? **[WBUT 2013]**

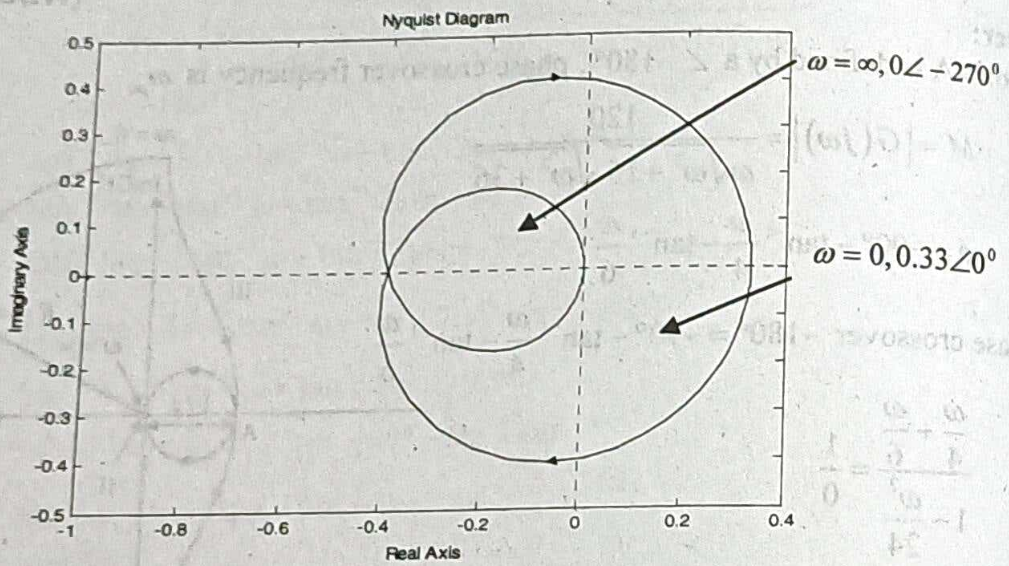
Answer:

Factors	R-H Criterion	Nyquist criterion
Analysis of Stability	Absolute stability of a LTI system.	Both absolute and relative stability of a LTI system.
System Loop	Stability is derived from closed loop system.	Open loop system.
Presentation	Algebraic method forming an array of Routh elements.	Graphical technique for determining the Stability (derived from Nyquist plot).
Frequency domain specifications	Cannot be derived.	Can be derived.
Approach	It finds the number of closed loop poles lying right side of imaginary axis in s-plane.	It finds P, N and Z.

b) For the system shown in figure below, sketch the Nyquist plot for $k=2$ & comment for the stability of the system using Nyquist stability criterion. Also find the range of k for the system to be stable. **[WBUT 2013]**



Answer:



Since P=0, N=0, so Z=N+P=0, i.e. no closed loop pole is lying right side of imaginary axis.

So, closed loop and open loop systems are stable.

2nd Part:

From the plot, the Nyquist plot intersects the negative real axis at -0.396 with k=2, so with k=1 the cross over will be at -0.198. Since P=0 and since close loop system has to be stable, so, N=0 this is possible if 0.198k <1 i.e. k<5.0505 . Thus the range of k for stability

0<k<5.0505

5. Sketch the Nyquist plot of a system whose open loop transfer function is given by $G(s)H(s)=\dfrac{K}{s(1+s)(2+s)}$. Find the values of K for which the system becomes stable. **[WBUT 2014]**

Answer:

$$G(s)H(s)=\frac{K}{s(1+s)(2+s)}$$

$$\phi=-90°-\tan^{-1}\omega-\tan^{-1}0.5\omega$$

$$M = \frac{K}{2\omega\sqrt{1+\omega^2}\sqrt{1+0.25\omega^2}}$$

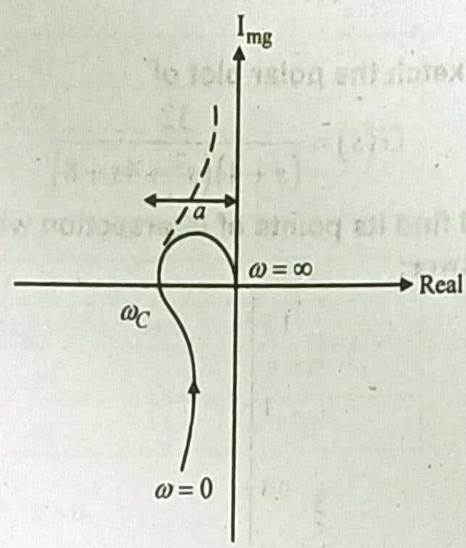
When $\omega = 0$ $\quad GH = \infty\angle -90^\circ$

$\omega = \infty$ $\quad GH = 0\angle -270^\circ$

$\omega = 1$ $\quad GH = \frac{K}{\sqrt{10}}\angle -162^\circ$

$\omega = 2$ $\quad GH = \frac{K}{2\sqrt{5}\sqrt{10}}\angle -198^\circ$

Nyquist plot is shown in Fig.



$$\omega_C = \frac{1}{\sqrt{\tau_1\tau_2}}$$

$$\tau_1 = 1$$

$$\tau_2 = \frac{1}{2} = 0.5$$

$$\therefore \quad \omega_C = \frac{1}{\sqrt{1\times 0.5}} = \sqrt{2} \qquad \textbf{(Ans.)}$$

$$a = K\left(\frac{\tau_1\tau_2}{\tau_1+\tau_2}\right) = K\left(\frac{1\times 0.5}{1+0.5}\right) = \frac{K}{6}$$

$\therefore$ For stability $\frac{K}{6} < 1$

or, $\quad K < 6$

Gain margin $= 20\log\frac{1}{a}$

i.e., $\quad 3 = 20\log\frac{K}{6}$

or, $\quad K = 4.25$

With $K = 4.25$ the transfer function is

$$GH = \frac{4.25}{s(1+s)(2+s)}$$

$$M = \frac{4.25}{\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$$

Frequency at which magnitude will become unity is given by

$$\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2} = 4.25$$

or, $\quad \omega^2(1+\omega^2)(1+4\omega^2) = 18.1$

Putting $x = \omega^2$ we get

$$x(1+x)(x+4) = 18.1$$

By hit and trial method

$$x = 1.4$$

$$\therefore \quad \omega = \sqrt{1.4} = 1.18$$

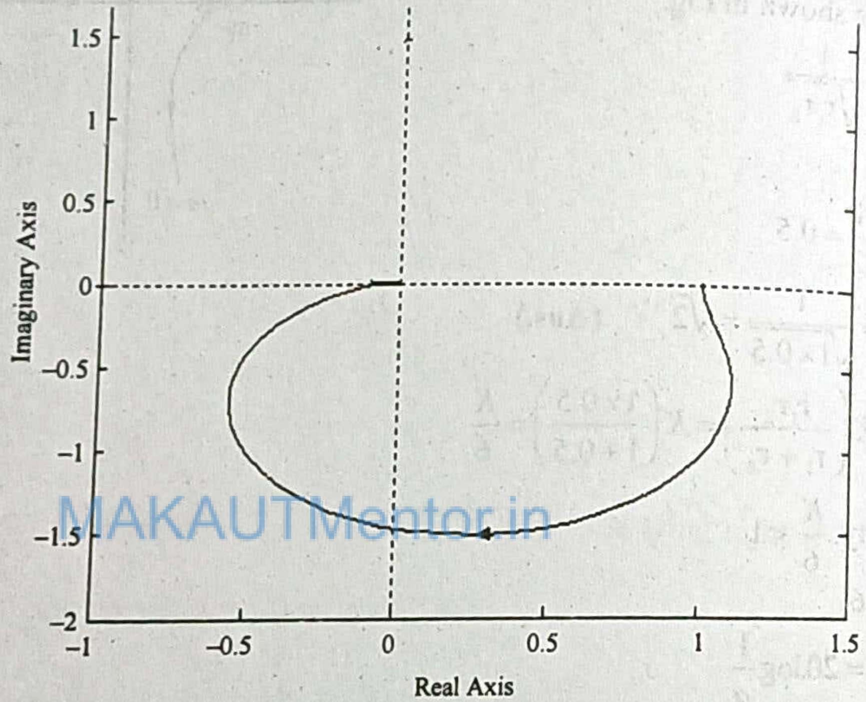
6. Sketch the polar plot of

$$G(s) = \frac{32}{(s+4)(s^2+4s+8)}$$

[WBUT 2015]

And find its points of intersection with real and imaginary axes.

Answer:



Negative imaginary axis intersection= -1.47 at frequency 0.943 rad./sec.

Polar plot meets the negative real axis at the point $0\langle -180^0$ at frequency infinity rad./sec.

7. A feedback control system has forward path gain $G(s) = \frac{2}{s(s-1)}$ **and feedback path gain** $H(s) = (s+1)$.

Draw the Nyquist diagram for the system and assess the stability of the closed loop system. **[WBUT 2015]**

Answer:

$$GH = \frac{2(s+1)}{s(s-1)}$$

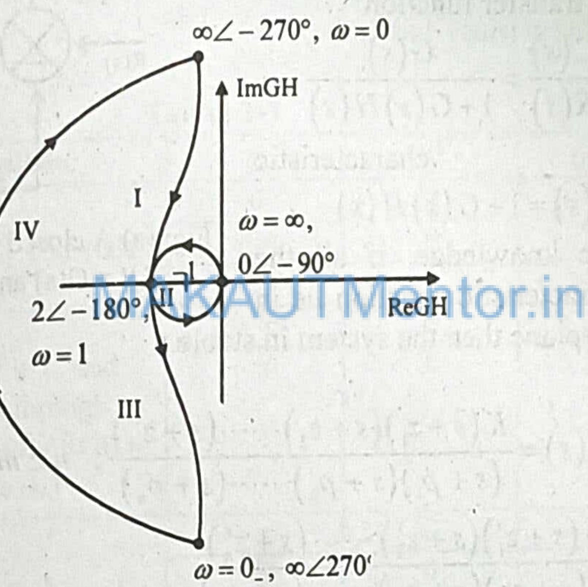
$$M = \frac{2\sqrt{\omega^2-1}}{\omega\sqrt{\omega^2+1}} = \frac{2}{\omega}$$

$$\phi = -90° + \tan^{-1}\omega - \tan^{-1}\frac{\omega}{-1}$$

$$= -90° + \tan^{-1}\omega - 180° + \tan^{-1}\omega = -270° + 2\tan^{-1}\omega$$

Now, forming a table (shown below)

Section	ω	M	ϕ
I	0_+	∞	−270°
	∞	0	−90°
	1	2	−180°
II	$\omega = \infty$	0	−90°
	$\omega = -\infty$	0	+90°
III	Mirror image of section I		
IV	$\omega = 0_-$	∞	270°
	$\omega = 0_+$	∞	−270°



$P = 1$

$N = -1$

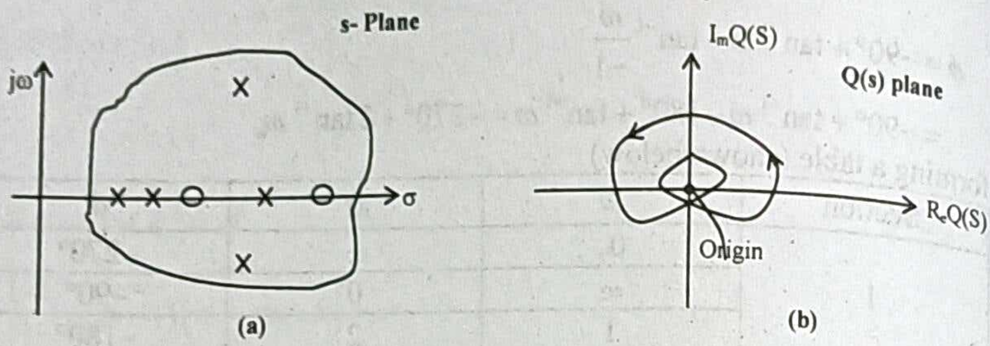
$\therefore \quad Z = N + P = 0$

So, closed loop system is stable, though open loop system is unstable.

8. a) State the 'Principle of argument' & its extension to Nyquist criterion.
b) Draw the Nyquist plot & determine the stability condition for the open loop transfer function of the system $G(s)H(s) = \dfrac{(s+2)}{(s+1)(s-1)}$. **[WBUT 2016]**

Answer:

a) Consider the s-plane contour encloses P poles and Z zeros of Q(s) in s-plane as shown in the figure (a). Then, corresponding Q(s) plane contour must encircle the origin (z+p) times in the anti-clockwise direction or (z−p) times in the clockwise direction as shown in the figure (b).



This relation between the enclosure of poles and zeros of Q(s) in the s-plane contour to the encirclement of the origin by the Q(s)-plane contour is known as 'Principle of Argument'.

Let us consider a system whose forward path transfer function is $G(s)$ and that of feedback path is $H(s)$ as shown in Fig: (a).

The closed loop transfer function

$$T.F. = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

and the characteristic polynomial $= F(s) = 1 + G(s)H(s)$

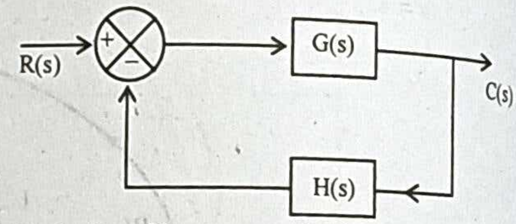


Fig: (a) A closed loop system with forward path T.F = G(s) and feedback path T.F =H(s)

From our basic knowledge, if all the roots of the characteristic equation lie in the left half of s-plane then the system in stable.

Now, assume

$$G(s)H(s) = \frac{K(s+z_1)(s+z_2)\cdots\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots\cdots(s+p_n)};\quad n \ge m \qquad \ldots \text{(i)}$$

and
$$F(s) = \frac{(s+z_1')(s+z_2')\cdots\cdots(s+z_n')}{(s+p_1)(s+p_2)\cdots\cdots(s+p_n)} \qquad \ldots \text{(ii)}$$

From the equation (i) and (ii)

We can say,

Open loop poles are same. As the closed loop poles i.e. poles of $G(s)\,H(s)$ are same as that of $F(s)$. Zeros of the characteristic polynomial $F(s)$ are the roots of the characteristic equation (i.e. $F(s)=0$. For stability, the roots of characteristic equation of or zeros of $F(s)$ should lie in left half of s-plane. If a zero of $F(s)$ is found in right half side (with positive real part) then the system is stable. With this fact and taking the help of Cauchy's principle of argument Nyquist constructed a closed path or contour in s-plane such that entire right half of s-plane is encircled to find the presence of poles and zeros of $F(s)$. For stability, $F(s)$ will not encircle the origin.

b) ***Step I:*** To determine P

$P = 1$

Step II: The Nyquist Contour in s-plane is as shown in (Fig. 1a)

Step III: Put $s = j\omega$ in O.L.T.F.

$$G(j\omega) = \frac{(j\omega+2)}{(j\omega+1)(j\omega-1)}$$

Step IV: Evaluate $|G(j\omega)| = M$

$$M = \frac{2\sqrt{1+(0.5\omega)^2}}{\sqrt{1+\omega^2}\sqrt{1+\omega^2}}$$

Step V: Evaluate $\phi = \angle\{G(j\omega)H(j\omega)\}$

$$\phi = \tan^{-1}0.5\omega - \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega}{-1}\right)$$

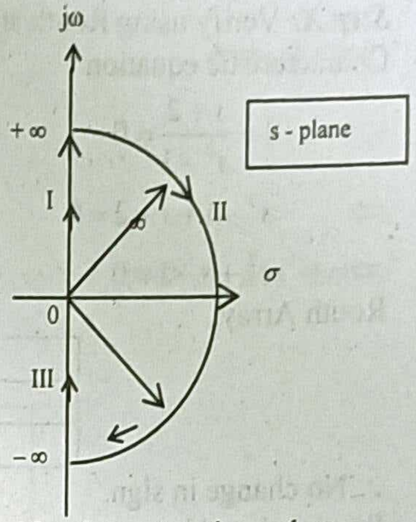


Fig: 1a Nyquist contour in s--plane

Step VI: Formation of table (T-1)

Table: T-1

Section in s-plane	**ω**	**M**	**ϕ**
I	0	2	+180°
begins from $\omega = 0$ and ends	$+\infty$	0	+270°
at $\omega = +\infty$	1	1	+206.57°
II	$+\infty$	0	+270°
begins from $\omega = +\infty$ and terminates at $-\infty$ through a semicircular path of infinite radius $(R \to \infty)$	$-\infty$	0	−270°
III	$-\infty$	0	−270°
$-\infty \le \omega \le 0$	$+\infty$	0	+270°

Step VII: To evaluate Real axis intersection of Nyquist path.

For $\phi = 180°,\ M = 2$

∴ The path intersects the negative real axis at the point (2, 180°)

Step VIII: To draw the Nyquist path (Fig. 1b)

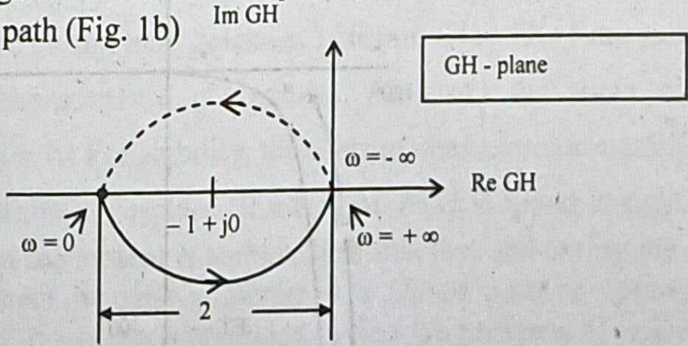


Fig: 1b Nyquist path in GH-plane

Step IX: Conclusion:
From the Nyquist path, we have N = –1,

$\therefore \quad Z = N + P = -1 + 1 = 0$

Step X: Verify using Routh stability criterion.
Characteristic equation

$$1+\frac{s+2}{s^2-1}=0$$

$$\Rightarrow \quad s^2-1+s+2=0$$

$$\Rightarrow \quad s^2+s+1=0$$

Routh Array:

s^2	1	1
s^1	1	0
s^0	1	

$\therefore$ No change in sign.
So, the closed loop system is stable.
$\therefore$ Closed loop system has no root with the +ve real part.

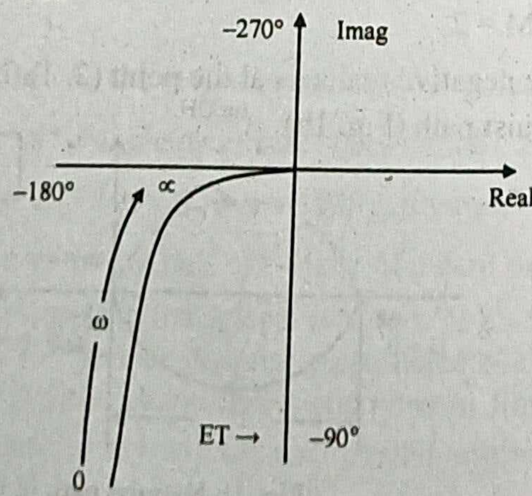
9. Consider the transfer function $G(j\omega)=\dfrac{1}{j\omega(1+j\omega T)}$. Draw the polar plot of the function. Find the gain crossover frequency & phase margin of the transfer function. **[WBUT 2016]**

Answer:

$$G(J\omega)=\frac{1}{J\omega(1+j\omega T)}$$

Now $\quad M=\dfrac{1}{\omega\sqrt{1+\omega^2T^2}} \qquad \phi=-90^\circ-\tan^{-1}\omega T$

when ω approaches zero, $M=\infty$ and $\phi=90^\circ$ and when ω approaches infinity $M=0$ and $\phi=-90^\circ-90^\circ=-180^\circ$



10. State and explain the Nyquist stability criteria. Explain how gain and phase margins can be obtained from the Nyquist plot of a system. Sketch the Nyquist plot on a plain paper for the following transfer function and hence comment on the stability of the system:

$$G(s)=\frac{10}{s(1+s)(1+0.5s)}$$

[WBUT 2017]

Answer:

1st Part:

Nyquist stability criteria:

Let us consider a system whose forward path transfer function is $G(s)$ and that of feedback path is $H(s)$ as shown in Fig: (1).

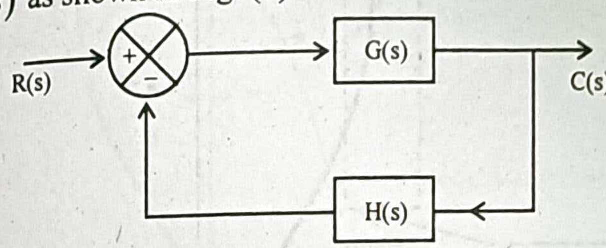


Fig: (1) A closed loop system with forward path T.F = G(s) and feedback path T.F =H(s)

The closed loop transfer function

$$T.F.=\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}$$

and the characteristic polynomial $=F(s)=1+G(s)H(s)$

From our basic knowledge, if all the roots of the characteristic equation lie in the left half of s-plane then the system in stable.

Now, assume

$$G(s)H(s)=\frac{K(s+z_1)(s+z_2)\cdots\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots\cdots(s+p_n)};\quad n\geq m \qquad \text{.... (i)}$$

and $$F(s)=\frac{(s+z_1')(s+z_2')\cdots\cdots(s+z_n')}{(s+p_1)(s+p_2)\cdots\cdots(s+p_n)} \qquad \text{.... (ii)}$$

From the Eqns. (i) and (ii) we can say,

Open loop poles are same. As the closed loop poles i.e. poles of $G(s)\,H(s)$ are same as that of $F(s)$. Zeros of the characteristic polynomial $F(s)$ are the roots of the characteristic equation (i.e. $F(s)=0$. For stability, the roots of characteristic equation of or zeros of $F(s)$ should lie in left half of s-plane. If a zero of $F(s)$ is found in right half side (with positive real part) then the system is stable. With this fact and taking the help of Cauchy's principle of argument Nyquist constructed a closed path or contour in s-plane such that entire right half of s-plane is encircled to find the presence of poles and zeros of $F(s)$. For stability, $F(s)$ will not encircle the origin.

2nd Part:

Getting GM from Nyquist Plot

The steps for evaluating the gain margin from the obtained Nyquist plot are as follows (Refer to Fig. (2).

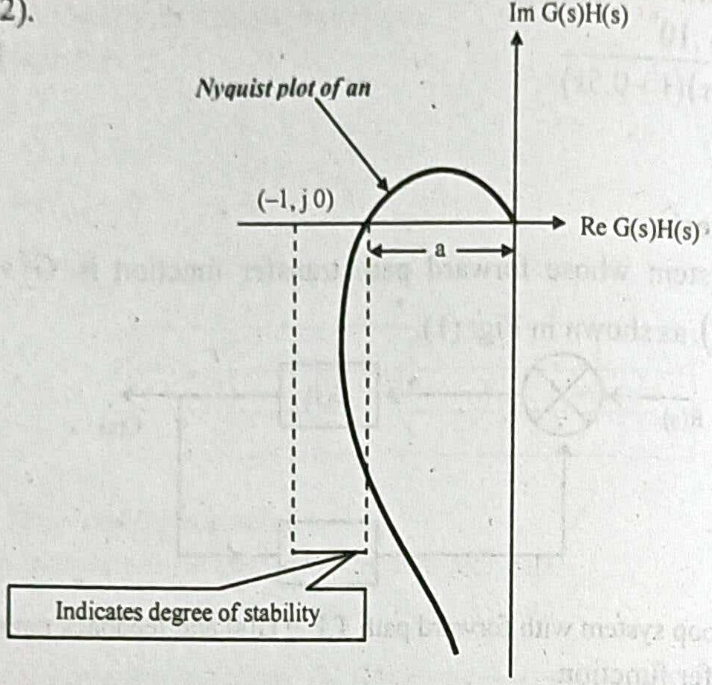


Fig: (2) Evaluating gain margin

Step1: First derive the magnitude of the real axis intersection (using the rule VII). Let this value is '**a**'

Step2: GM is given by

$GM = \frac{1}{a}$, but GM is generally expressed in terms of dB

$$\therefore \quad GM \text{ in dB} = 20\log_{10}\left(\frac{1}{a}\right) = 20\log_{10}1 - 20\log_{10}a = -20\log_{10}a$$

So three cases are supposed to come:

Case 1: If a <1, then GM will be positive. The closed loop system will be a stable one

Case 2: If a =1, then the system is said to be marginally stable as for that case $GM = 0$ dB.

Case 3: If a >, then the system is unstable as GM is negative in that case.

N.B.

1. Lower the value of 'a' better will be the relative stability.
2. If the Nyquist plot pass through the origin then $a = 0$ and the GM = infinity. The system will be absolutely stable.

Getting Phase Margin (PM) from Nyquist Plot

The steps for evaluating the phase margin are as follows (Fig.3):

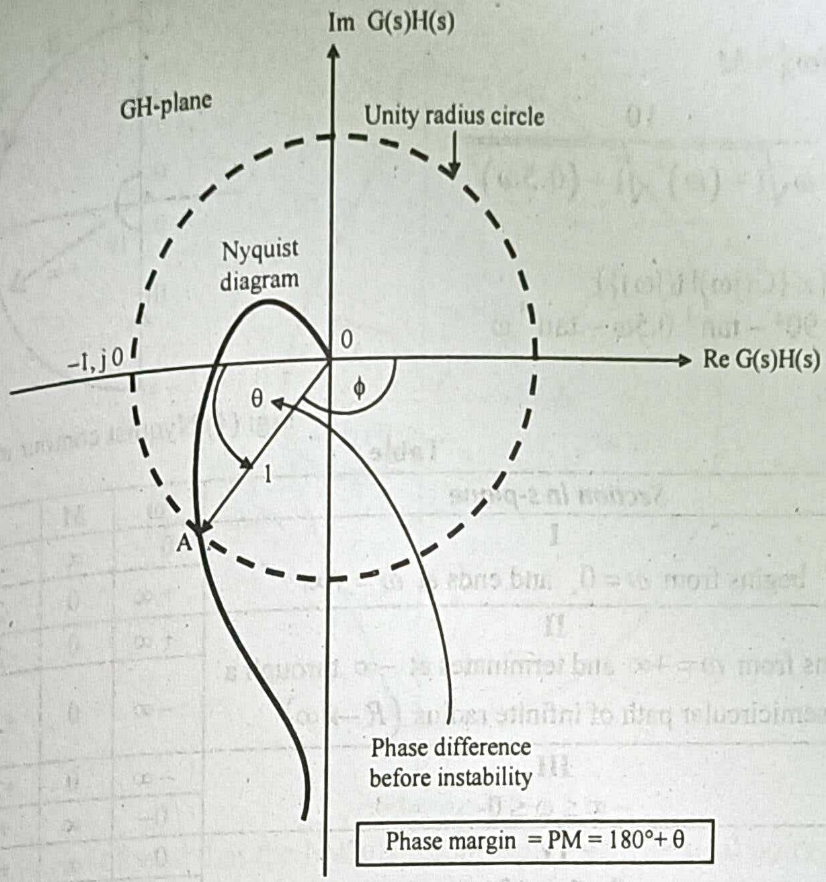


Fig: (3) Evaluating Phase Margin PM from

Step 1: draw a circle of unit radius (taking origin as its center) on the obtained Nyquist plot.

Step 2: Find the point of intersection of the circle on the Nyquist plot.

Step 3: Connect the origin and the point of intersection by a straight line (OA).

Step 4: Find the angle subtended by the line OA with respect to the positive direction of the real axis in clockwise direction.

Step 5: Evaluate the phase margin using the relation $PM = 180° + \phi$.

Three case may be supposed to have

Case1: If PM is positive, then the closed loop system may be a stable one

Cae 2: If PM is 0°, then the closed loop system is marginally stable.

Case 3: If PM is negative then the system is unstable.

3rd part: *Step I:* To determine P, $P = 0$

Step II: The Nyquist contour in s-plane having detour at $s = 0$ (Fig. 4)

Step III:

Put s = jω in O.L.T.F.

$$G(j\omega)H(j\omega) = \frac{10}{j\omega(j\omega+1)(j0.5\omega+1)}$$

Step IV:

Evaluate $|G(j\omega)| = M$

$$M = \frac{10}{\omega\sqrt{1+(\omega)^2}\sqrt{1+(0.5\omega)^2}}$$

Step V:

Evaluate $\phi = [\angle\{G(j\omega)H(j\omega)\}]$

$$\phi = -90° - \tan^{-1} 0.5\omega - \tan^{-1}\omega$$

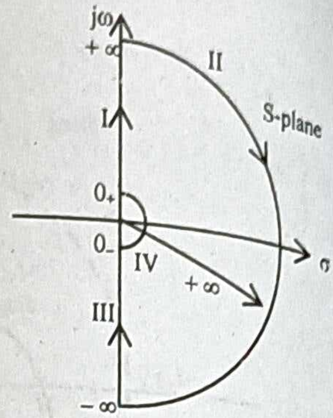


Fig: (4) Nyquist contour in s-plane

Step VI:

Table

Section in s-plane	ω	M	ϕ
I begins from $\omega = 0_+$ and ends at $\omega = +\infty$	0 +	∞	−90°
	+ ∞	0	−270°
II begins from $\omega = +\infty$ and terminates at $-\infty$ through a semicircular path of infinite radius $(R \to \infty)$	+ ∞	0	−270°
	− ∞	0	+270°
III $-\infty \le \omega \le 0-$	− ∞	0	+270°
	0−	∞	+90°
IV $0- \le \omega \le 0+$	0−	∞	+90°
	0+	∞	−90°

Step VII:

To evaluate Real axis intersection of Nyquist path.

At the point of Real axis intersection, $\phi = -180°$. So putting $\phi = -180°$ in the phase equation,

$$-180° = -90° - \tan^{-1} 0.5\,\omega - \tan^{-1}\omega$$

$$\Rightarrow \quad \tan^{-1} 0.5\,\omega + \tan^{-1} 0.1\omega = 90°$$

$$\tan^{-1}\left[\frac{\omega + 0.1\omega}{1-(0.5\omega).\omega}\right] = 90°$$

$$\frac{1.1\omega}{1-0.5\omega^2} = \tan 90° = \frac{1}{0}$$

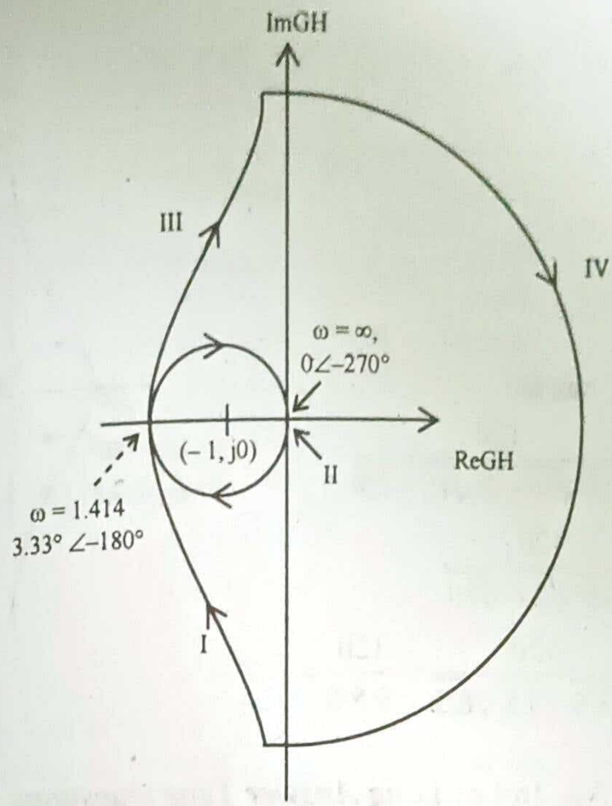
$$\Rightarrow \quad 1-0.5\omega^2 = 0$$

$$\Rightarrow \quad \omega^2 = \frac{10}{5} = 2$$

$$\omega = 1.414 \text{ rad/sec}$$

$$\therefore \quad M\Big|_{\omega=1.414} = \frac{10}{3} = 3.\dot{3}\dot{3}.$$

At the phase cross over frequency = 1.414 rad./sec, M = 3.33



From the plot, it is found that the Nyquist plot encircles the critical point $(-1+j0)$ twice. Since, $P=0$, $N=2$, therefore $Z=N+P=2+0=2$ saying the two closed loop poles are right handed. Hence the closed loop system is unstable. However, the open loop system is stable.

11. a) State and explain Nyquist criteria for study of control system.
b) The open loop transfer function of closed loop system is

$$G(s)H(s)=120/[s(s+3)(s+5)]$$

Draw the Nyquist plot and hence find out whether the system is stable or not.
c) What are the advantages of Nyquist plot? **[WBUT 2019]**

Answer:

a) *Refer to Question No. 1(a) of Long Answer Type Questions.*

b) Step I: $P=0$

Step II: $M=\frac{120}{\omega\sqrt{\omega^2+9}}\cdot\frac{1}{\sqrt{\omega^2+25}}$

Step III: $\phi=-90°-\tan^{-1}\frac{\omega}{3}-\tan^{-1}\frac{\omega}{5}$

Step IV: At negative real axis crossover

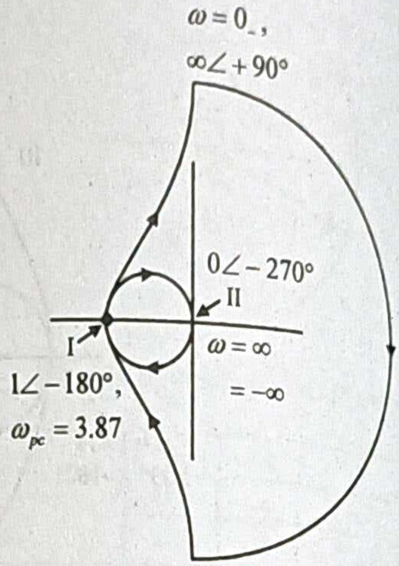
$$-180°=-90°-\tan^{-1}\frac{\omega}{3}-\tan^{-1}\frac{\omega}{5}$$

$\Rightarrow \qquad \tan^{-1}\frac{\omega}{3}+\tan^{-1}\frac{\omega}{5}=90°$

Taking tan both sides

$$\frac{\frac{\omega}{3}+\frac{\omega}{5}}{1-\frac{\omega^2}{15}}=\frac{1}{0}$$

$\Rightarrow \qquad \omega=\sqrt{15}=3.87$ rad/sec



$$M\big|_{at\ \phi=-180°}=\frac{120}{\sqrt{15}\sqrt{15+9}\sqrt{15+25}}$$

Step V: $$=\frac{120}{\sqrt{15}\cdot\sqrt{24}\cdot\sqrt{40}}$$

$$=\frac{120}{\sqrt{3.5}\sqrt{3.8}\sqrt{8.5}}=\frac{120}{3\cdot5\cdot8}=1$$

c) ***Refer to Question No. 1(c) of Long Answer Type Questions.***

12. Write short note on Polar plot. **[WBUT 2014]**

Answer:

Polar plot is a tool in frequency domain for assessing feedback control system stability. It is constructed on GH –plane from open loop transfer function (GH), when frequency of the excitation signal is varied from zero to infinity. To sketch the polar plot of GH for the entire range of frequency ω, i.e., from 0 to infinity one has to follow the steps as said below:

Step1: The start of plot where ω= 0,

Step2: the end of plot where ω= ∞,

Step3:The quadrant or quadrants where the polar plot has to lie. The shape of the polar plot depends on the type no . of the system.

Type- 0 system: Polar plot starts from a finite point on the positive real axis and the terminal point is the origin at the tangent to one of the four axes.

Type -1 system: starting at infinity asymptotically parallel to negative imaginary axis and the curve converges to zero at the tangent to one of the axes.

Type 2 system: the starting point is infinity and asymptotic to -180°.

The end point is origin and tangent to one of the axes.

CONTROL ACTION

Multiple Choice Type Questions

1. By the use of PD control to the second order system, the rise time [WBUT 2007, 2008, 2012, 2019]
a) decreases
b) increases
c) remains same
d) has no effect

Answer: (a)

2. The transfer function of a basic PD controller is given by (all k's are real constant) [WBUT 2010, 2013]
a) $k_0 + \frac{k_1}{s} + k_2 s$
b) $k_2 s + k_3 s$
c) $k_0 + k_2 s$
d) $k_0 + \frac{k_1}{s}$

Answer: (d)

3. A D only Controller will produce [WBUT 2011]
a) always zero steady state error
b) always infinite steady state error
c) output only if error changes
d) none of these

Answer: (c)

4. A PID controller can [WBUT 2014]
a) speed up response
b) reduce overshoot
c) provide good tracking performance
d) all of these

Answer: (d)

5. The addition of a PD controller in cascade with the plant [WBUT 2015]
a) improves damping
b) reduces the steady state error
c) increases undamped frequency
d) increase the order of the system

Answer: (a)

6. For eliminating the steady state error, the control action required is [WBUT 2016]
a) proportional control
b) proportional plus derivative control
c) proportional plus integral control
d) proportional, derivative & integral control

Answer: (d)

7. For a unit step input, a system with closed loop transfer function $\frac{20}{s^2+2s+5}$ has a steady state output of [WBUT 2017]
a) 10
b) 5
c) 2
d) 4

Answer: (d)

8. The transfer function $G(s)$ of a PID controller is [WBUT 2018]

a) $k\left[1+\frac{1}{T_i s}+T_d s\right]$
b) $k\left[1+T_i s+T_d s\right]$
c) $k\left[1+\frac{1}{T_i s}+\frac{1}{T_d s}\right]$
d) $k\left[1+T_i s+\frac{1}{T_d s}\right]$

Answer: (a)

9. For eliminating steady state error, the control action required is [WBUT 2018]

a) Proportional control
b) Proportional plus derivative control
c) Proportional plus integram control
d) Proportional, derivative & integral control

Answer: (c)

Short Answer Type Questions

1. For the system defined by $G(s)=\frac{k}{(Ts+1)}$ calculate the steady state error due to a unit step input with a PI controller. [WBUT 2011]

Answer:

Block diagram of P-I Controller

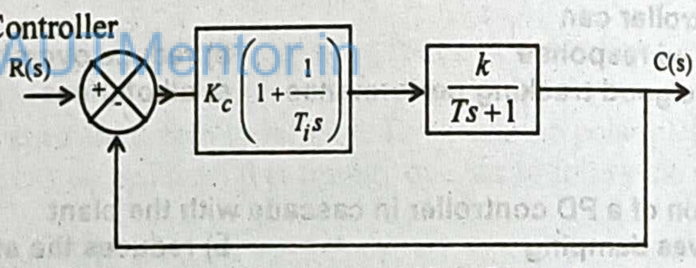


Fig: 1 PI-controller introduced in a first order process

From the block diagram of figure 1 we have forward path

$$T.F.=G(s)=K_c\left(1+\frac{1}{T_i s}\right).\left(\frac{k}{Ts+1}\right) \quad \cdots(1)$$

- Feedback transfer function $H(s)=1 \quad \cdots(2)$
- $\frac{E(s)}{R(s)}=\frac{R(s)-C(s)}{R(s)}=\left[1-\frac{C(s)}{R(s)}\right] \cdots(3)$

$$\frac{C(s)}{R(s)}=\frac{K_c\left(1+\frac{1}{T_i}.s\right)\left(\frac{k}{Ts+1}\right)}{1+K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{k}{Ts+1}\right)} \quad \ldots(4)$$

$\therefore E(s) = R(s) - \frac{C(s)}{R(s)}.R(s)$ [Combining equations (3) and (4)]

$$= \frac{1}{s} - \frac{K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{k}{Ts+1}\right)}{1+K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{k}{Ts+1}\right)}.\frac{1}{s} \quad \cdots(5)$$

$\therefore$ Steady state error $= e_{ss} = \lim_{S\to 0} sE(s)$

$$= \lim_{s\to 0} s.\left[\frac{1}{s} - \frac{K_c\left(\frac{1+T_i s}{T_i s}\right)\left(\frac{k}{Ts+1}\right)}{\frac{sT_i s(Ts+1)+K_c(T_i s+1)}{T_i s(Ts+1)}}\right]$$

$$= 1 - k\frac{K_c}{K_c} = 1-k \Rightarrow e_{ss} = 1-k \quad \cdots(6)$$

Long Answer Type Questions

1. Draw the response characteristic curves of the following controlling actions: *P, I, D, PI, PD & PID*. Discuss salient features. **[WBUT 2010]**

Answer:

Integral Control (I):

In integral control action the rate of change of the controller's output is proportional to the error signal i.e. the output $m(t)$ depends on the integral of the error signal $e(t)$.

Mathematically,

$$\frac{dm(t)}{dt} \propto e(t)$$

$$\Rightarrow \frac{dm(t)}{dt} = K_I e(t) \quad \text{.... (i)}$$

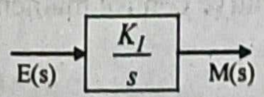
$$\Rightarrow m(t) = \int K_I e(t)dt = K_I \int e(t)dt \quad \text{.... (ii)}$$

where, K_I is the integral constant.

Taking Laplace transform at both sides of the equation (ii) we have,

$$M(s) = \frac{K_I E(s)}{s}.$$

So, the block diagram of the integral controller is as shown below:



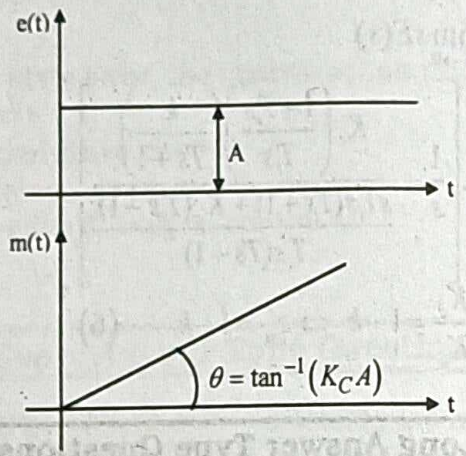
Characteristic curve of I-action

Let error signal $e(t)$ follows the step function, i.e. $e(t) = A$, so,

$$m(t) = K_I \int A\,dt$$

$$\Rightarrow \quad m(t) = K_I At \qquad \text{.... (iii)}$$

So, response of the integral controller against the step excitation will be ramp in nature as shown in the figure below.



Derivative Control:

A control system is said to have a derivative control action if the output of the controller $m(t)$ is proportional to the rate of change of error signal $e(t)$.

Mathematically, $m(t) \propto \frac{d}{dt}e(t)$

$$\Rightarrow m(t) = K_D \frac{d}{dt}e(t) \qquad \text{.... (i)}$$

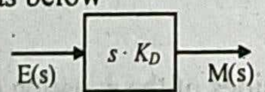
where, K_D is an adjustable constant and is called variable of derivative control action.

Taking Laplace transform of equation (i) we have,

$$M(s) = K_D \cdot s \cdot E(s) \qquad \text{.... (ii)}$$

Hence, the transfer function of a derivation control action is $\frac{M(s)}{E(s)} = s \cdot K_D$

The block diagram can be shown as below



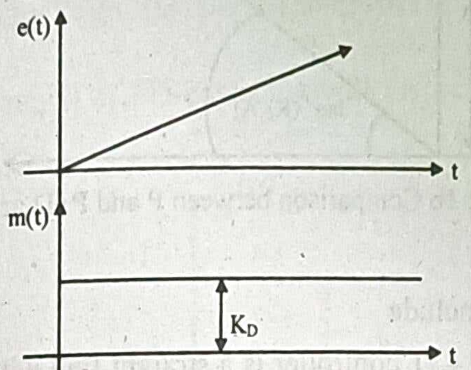
Characteristics curve of a derivative control action

Let the error signal be a step function i.e. $e(t) = A$.

∴ Response, $m(t)$, from the derivative control mathematically is given by

$$m(t) = K_D \frac{d}{dt} A = 0,$$

so, output from controller will be zero. So, to have an effective derivative control action error signal should always be time varying.
So, if error signal varies with time always we will not get any effective control action from a derivative controller. So, derivative control action is not used alone.
Figure below shows the response curve of a D-controller against a ramp excitation $e(t)=t$, hence, $m(t)=K_D\frac{d}{dt}t=K_D$.



Characteristic Curve: ***P, PI, PD & PID refer to Question No. 2. of Long Answer Type Questions.***

2. Briefly discuss the necessity of PID controller in minimizing errors of a dynamic system response under step input. Show relevant graphical & mathematical expression. [WBUT 2016]

Answer:

a) 1st Part:

Let the error signal be defined as

$$e(t) = At \quad \text{for } t \geq 0 \qquad \ldots (1)$$

Putting this error signal in equation

$$m(t) = K_c(At) + K_cT_D\frac{d}{dt}At$$

$$\Rightarrow m(t) = (K_CA)t + K_CT_DA \qquad \cdots (2)$$

Equation (2) satisfies the equation of a straight line.

Figures 1a and 1b show the graphical presentation of e(t) and m(t).

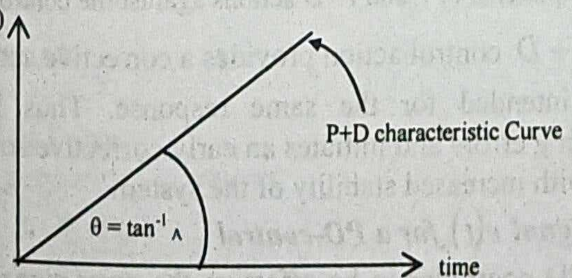


Fig: 1a Ramp signal

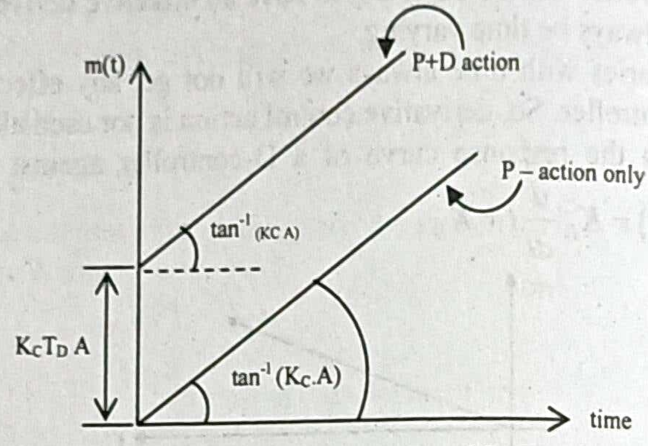


Fig: 1b Comparison between P and P+D action

Conclusions:

From figure 1b we may conclude

1) ramp response of a $P+D$ controller is a straight line with positive slope $K_C A$ and the line intersects the $m(t)$ at $(O, K_C T_D A)$ point.
2) ramp response of the P-action only is also a straight line passing through the origin, with the same slope as $P+D$ action.

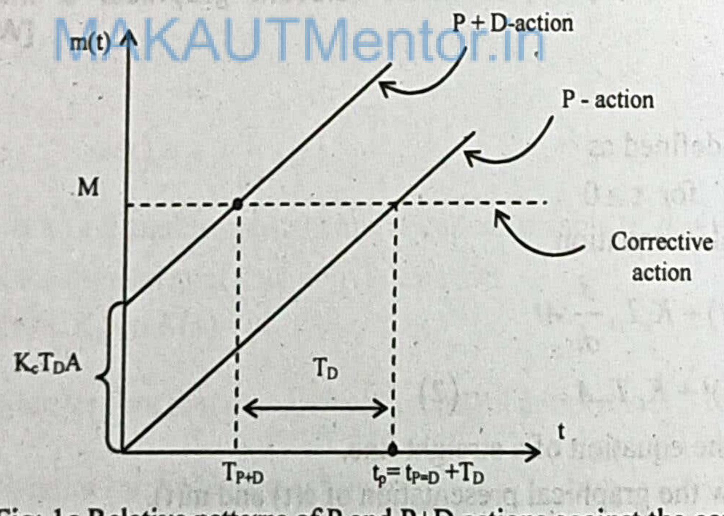


Fig: 1c Relative patterns of P and P+D actions against the control action

3) Fig: 1c says that a $P+D$ control action provides a corrective action T_d time earlier than the P-action, intended for the same response. Thus Derivative control anticipates the actuating errors and initiates an early corrective action thus providing faster response and with increased stability of the system.

- ***Nature of error signal $e(t)$ for a PD-control***

  For D-action in PD controller to be effective the error signal should very with time. If the error signal is constant with respect to time, then its time derivative

$\left(\frac{de(t)}{dt}\right)$ will be zero and the derivative section of the controller provides no corrective output.

- ***Effects of Step signal***

If the error signal $e(t)$ is step in nature (Fig: 1d) then its time rate of change $\left(\frac{de(t)}{dt}\right)$ will be very high (as the denominator $dt \rightarrow 0$) for a very short time $(dt \rightarrow 0)$. The output of PD-controller will involve an impulse function as shown in Fig: 1e

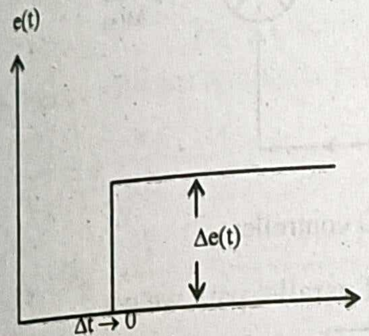


Fig: 1d Sudden change of e(t)

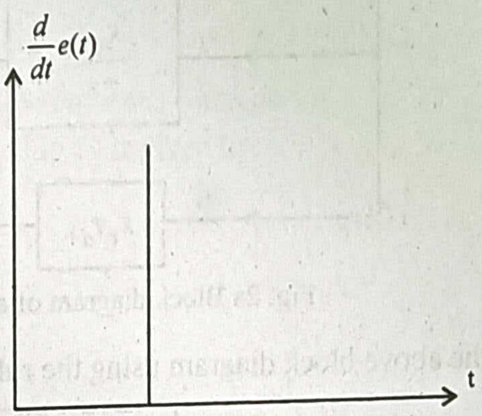


Fig: 1e Effect of D - action

2[nd] Part:

The combination of proportional, integral and derivative actions is terms as PID control action and has the advantages of each of the three individual control actions.

Mathematical Structure

$$m(t) = K_c e(t) + \frac{K_c}{T_i}\int e(t)dt + K_c T_D \frac{d}{dt}e(t) \quad \ldots (1)$$

where $m(t) \Rightarrow$ PID Controller's output

$K_C \Rightarrow$ proportional sensitivity

$T_i \Rightarrow$ integral time; sec.

$e(t) \Rightarrow$ error Signal

$T_D \Rightarrow$ derivative time; sec

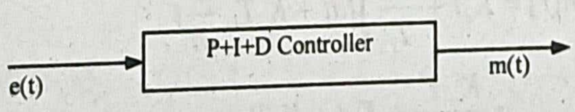


Fig: 1A PID block

Laplace Transformed Form

Laplace transforming equation (1), we get

$$M(s) = K_c E(s) + \frac{K_c}{T_i}\frac{E(s)}{s} + K_c T_D s E(s) = K_c\left(1 + \frac{1}{T_i s} + T_D s\right)E(s)$$

⇒ **Transfer Function**

$$\frac{M(s)}{E(s)} = K_c\left(1+\frac{1}{T_i s}+T_D s\right) \quad \ldots (3)$$

Block Diagram Representation

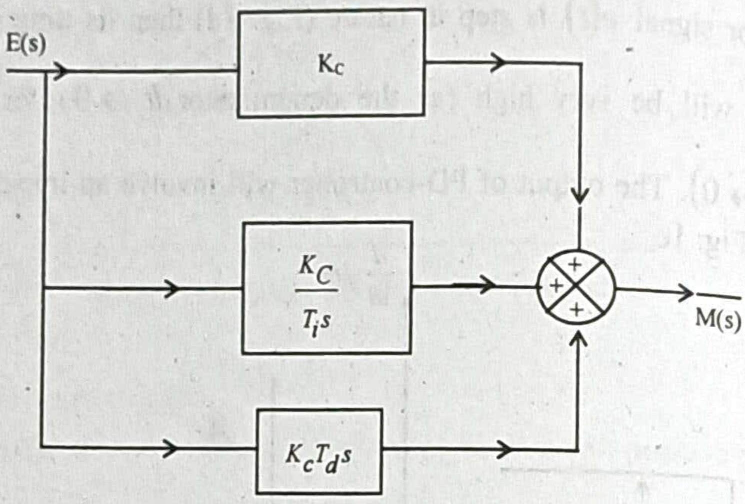


Fig: 2a Block diagram of a PID controller

Reducing the above block diagram using the rule of parallel path we get

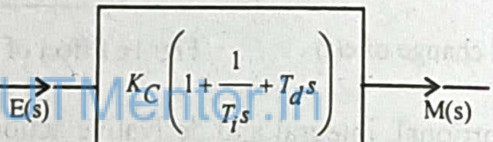


Fig: 2b Reduced block diagram

Characteristic Curve

Let the error signal be defined as

$e(t) = t \quad \text{for } t \geq 0 \quad \ldots (3)$

Putting this error signal in equation (15.37),

$$m(t) = K_c t + \frac{K_c}{T_i}\int t\,dt + K_c T_D \frac{d}{dt}t$$

$$= \frac{K_c}{T_i}\frac{t^2}{2} + K_c t + K_c T_D \quad \ldots (4)$$

which represents the equation of a parabola.

Realisation of P+I+D Control Action

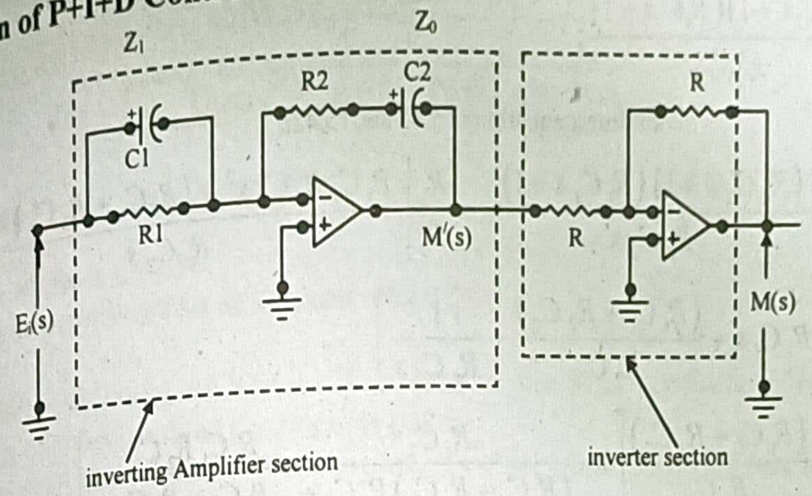


Fig: 3 Electronic circuit of a PID controller

Transfer function for the amplifier section(Fig.3) is given by

$$\frac{M'(s)}{E_i(s)} = -\frac{z_0}{z_i} \qquad \ldots (5)$$

where, $Z_0 = R_2 + \frac{1}{sC_2} = \frac{(R_2 C_2 s + 1)}{sC_2}$

and $Z_i = \frac{R_1 \cdot \frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}} = \frac{R_1}{(R_1 C_1 s + 1)}$

$$\therefore \frac{M'(s)}{E_i(s)} = -\frac{(R_2 C_2 s + 1)/sC_2}{\frac{R_1}{(R_1 C_1 s + 1)}}$$

$$\Rightarrow \frac{M'(s)}{E_i(s)} = -\frac{(R_2 C_2 s + 1)(R_1 C_1 s + 1)}{s R_1 C_2} \qquad \ldots (6)$$

T.F. for inverter section (Fig. 3) is given by

$$\frac{M(s)}{M'(s)} = -1 \qquad \ldots (6a)$$

$$\therefore \text{T.F.}\Big|_{PID} = \frac{M(s)}{E_i(s)}$$

$$\text{T.F.}\Big|_{PID} = \frac{M(s)}{E_i(s)} = \frac{M(s)}{M'(s)} \times \frac{M'(s)}{E_i(s)}$$

$$=\frac{(R_2C_2s+1)(R_1C_1s+1)}{sR_1C_2};$$

combining equations 15.42 and15.42a

$$=\frac{R_2}{R_1}\times\frac{(R_2C_2s+1)(R_1C_1s+1)}{R_2C_2s}=\frac{R_2}{R_1}\left[\frac{R_1C_1R_2C_2s^2+(R_1C_1+R_2C_2)s+1}{R_2C_2s}\right]$$

$$=\frac{R_2}{R_1}\left[R_1C_1s+\frac{(R_1C_1+R_2C_2)}{R_2C_2}+\frac{1}{R_2C_2s}\right]$$

$$=\frac{R_2}{R_1}\times\frac{(R_1C_1+R_2C_2)}{R_2C_2}\left[1+\frac{R_2C_2}{(R_1C_1+R_2C_2)R_2C_2s}+\frac{R_1C_1R_2C_2}{R_1C_1+R_2C_2}\cdot s\right]$$

$$\Rightarrow\frac{M(s)}{E_i(s)}=\frac{R_1C_1+R_2C_2}{R_1C_2}\left[1+\frac{1}{(R_1C_1+R_2C_2)}\cdot\frac{1}{s}+\left(\frac{R_1C_1R_2C_2}{R_1C_1+R_2C_2}\right)s\right]s$$

The equation 15.42 may be expressed as

$$\frac{M(s)}{E_i(s)}=K_P\left[1+\frac{1}{T_is}+T_Ds\right]$$

which satisfies the basic mathematical structure of a PID control action (refer to Eqn. 6)

where, $K_P=\frac{(R_1C_1+R_2C_2)}{R_1C_2}$, $T_i=(R_1C_1+R_2C_2)$ and $T_D=\frac{R_1C_1R_2C_2}{R_1C_1+R_2C_2}$.

3. Write short notes on the following:
a) PID controller **[WBUT 2008, 2009, 2013, 2015, 2019]**
b) PI & PD controllers **[WBUT 2012]**

Answer:

a) PID controller:

Refer to Question No. 1 of Long Answer Type Questions.

b) PI & PD controllers:

Refer to Question No. 2(a) & (b) of Long Answer Type Questions.

NICHOLS CHART

Long Answer Type Questions

1. Write short note on Nichols chart. **[WBUT 2006, 2008, 2019]**

OR,

a) What is Nichols chart? **[WBUT 2009]**

b) What is the application of Nichols chart? **[WBUT 2009]**

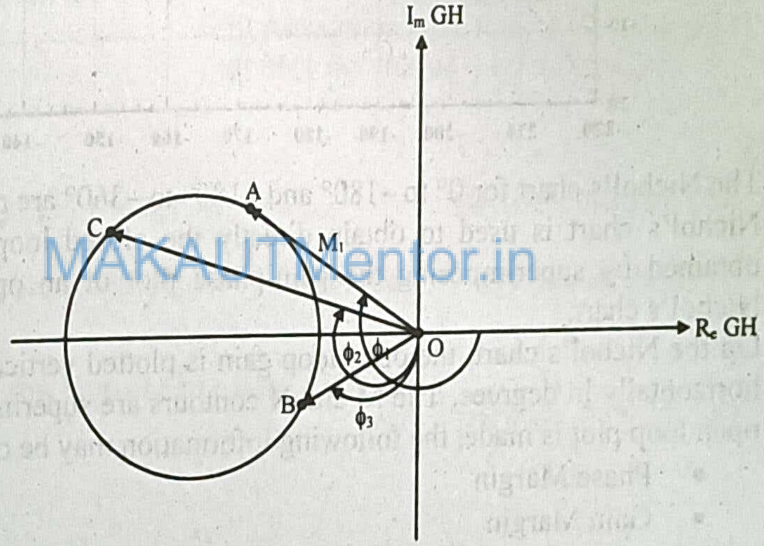
Answer:

a) The chart consisting of M and N circles in the log-magnitude (dB) versus phase diagram (in degree) is called the Nichols chart.

Though the constant M and N circles (plotted on the polar co-ordinates) are useful in the design of control system, it is more meaningful to plot them on the gain $|G(j\omega)H(j\omega)|$ – phase $\underline{|G(j\omega)H(j\omega)}$ plane where gain is expressed in dB and phase is expressed in degrees.

Approach

Consider a M-circle, as shown in the figure, on the GH-plane.



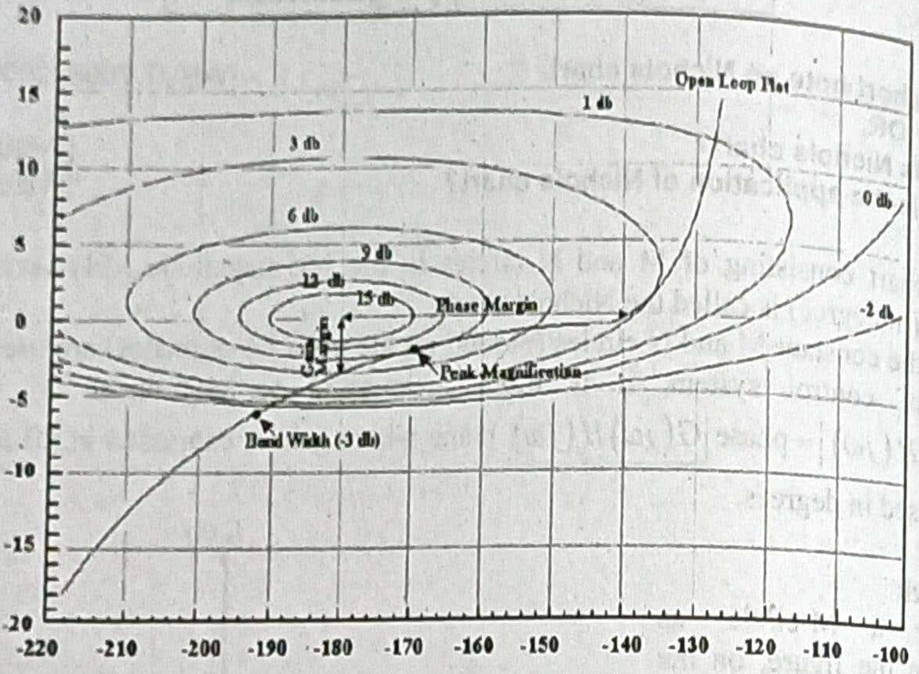
A point A on this circle, joined to the origin O, is considered and designated by the vector OA $(\overrightarrow{OA})$. The magnitude of this vector $(\overrightarrow{OA})$ is expressed in dB and the phase angle is measured negatively with respect to the positive R_e-GH-axis.

In the similar ways, points B and C are considered on the M-circle shown in figure above.

Points on the M-circle	M in dB	ϕ in degrees
A	M_1	ϕ_1
B	M_2	ϕ_2
C	M_3	ϕ_3

When the gain-phase plot of an open-loop is obtained from the constant M-circles and N-circles, the resultant plot, called as chart, is known as the Nichol's chart. In other words, when the constant M-circles (magnitude loci) and constant N-circles (phase angle loci) are transferred to the gain-phase plot, the resultant chart is known as the Nichol's chart.

Figure below shows the Nichol's chart



The Nichol's chart for 0° to –180° and –180° to –360° are mirror images of each other. Nichol's chart is used to obtain directly the closed loop frequency response. This is obtained by superimposing the gain-phase plot of an open loop transfer function on Nichol's chart.

On the Nichol's chart, the open loop gain is plotted vertically in dB and the phase angle horizontally in degrees. The M and N contours are superimposed on the chart. When the open loop plot is made, the following information may be obtained.

- Phase Margin
- Gain Margin
- Bandwidth (The frequency at which $M = -3\,\text{dB}$)

Peak Magnification and the frequency at which it occurs.

b) Applications of Nichol's Chart

The various applications of Nichol's chart are listed below:

To determine the closed loop frequency response against the given $G(j\omega)$. The value of resonant peak (M_r) of the closed loop and hence the corresponding resonant frequency (ω_r) can be evaluated. The 3-dB bandwidth of the closed-loop system can be evaluated.

For the specified M_r, it is possible to find the value of *K*. If M_r and ω_r are known, then other frequency domain specification can be evaluated. From the relationship between time domain and frequency domain The time-domain specifications can also be evaluated.

STATE VARIABLE ANALYSIS

Multiple Choice Type Questions

1. The second order system $\dot{X} = AX$ has $A = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$ [MODEL QUESTION]

The values of the damping ratio and natural frequency are

a) 1 & 1 b) 0.5 & 1 c) 0.707 & 2 d) 1 & 2

Answer: (b)

2. If both Eigen values of a second order system are real and –ve, then it is termed as [MODEL QUESTION]

a) Saddle point b) nodal point c) focus points d) none of these

Answer: (b)

3. A set of variables for a system is [MODEL QUESTION]

a) Not unique in general b) Always unique
c) Never unique d) May be unique

Answer: (b)

4. If $A = \begin{bmatrix} -0.5 & 0 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, then the system is [MODEL QUESTION]

a) controllable b) uncontrollable c) undefined d) none of these

Answer: (b)

5. In a series R – L – C circuit, the number of state variables is [MODEL QUESTION]

a) 3 b) 2 c) 1 d) 0

Answer: (b)

6. The state equation of a linear system is given by [MODEL QUESTION]

$\dot{X} = AX + BV$ where $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

The state transition matrix of the system is

a) $\begin{bmatrix} e^{2t} & 0 \\ 0 & e^{2t} \end{bmatrix}$ b) $\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-2t} \end{bmatrix}$ c) $\begin{bmatrix} \sin 2t & \cos 2t \\ -\cos 2t & \sin 2t \end{bmatrix}$ d) $\begin{bmatrix} \cos 2t & \sin 2t \\ -\sin 2t & \cos 2t \end{bmatrix}$

Answer: (d)

7. A 5×7 matrix has all entries as -1. Rank of the system is [MODEL QUESTION]

a) 1 b) 7 c) 5 d) 0

Answer: (a)

8. For SISO: $Y(s)=G(s)U(s)$ [MODEL QUESTION]

a) G (s) is a scalar
b) G (s) is a transfer function
c) G (s) is m × r dimensional matrix
d) both (a) and (b)

Answer: (d)

9. The transfer function for the state variable representation $\dot{X}=AX+BU$, $Y=CX+DU$, is given by [MODEL QUESTION]

a) $D+C(SI-A)^{-1}B$
b) $B+C(SI-A)^{-1}D$
c) $C+B(SI-A)^{-1}D$
d) $A+C(SI-B)^{-1}D$

Answer: (a)

10. A system is described by [MODEL QUESTION]

$$\frac{dx}{dt}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$y=\begin{bmatrix}1 & 0\end{bmatrix}x$$

The system is

a) controllable & observable
b) uncontrollable & observable
c) controllable & unobservable
d) uncontrollable & unobservable

Answer: (a)

11. The state variable description of a linear autonomous system is $\dot{X}=AX$ when X is a state vector & $A=\begin{bmatrix}0 & 2\\ 2 & 0\end{bmatrix}$

The poles of the system are located at [MODEL QUESTION]

a) – 2 and +2
b) –2j and +2j
c) –2 and –2
d) +2 and +2

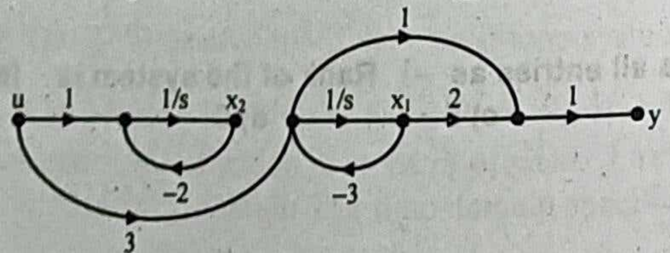
Answer: (a)

12. The value of a matrix in $\frac{dx}{dt}=AX$ for the system described by the differential equation $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+3y=0$ [MODEL QUESTION]

a) $\begin{bmatrix}1 & 0\\ -2 & -1\end{bmatrix}$
b) $\begin{bmatrix}1 & 0\\ -1 & -2\end{bmatrix}$
c) $\begin{bmatrix}0 & 1\\ -2 & -1\end{bmatrix}$
d) $\begin{bmatrix}1 & 0\\ -3 & -2\end{bmatrix}$

Answer: (d)

13. The state diagram of a system is shown in the given figure: [MODEL QUESTION]



The system is

a) controllable and observable
b) controllable but not observable
c) observable but not controllable
d) neither controllable nor observable

Answer: (b)

14. If the Eigen values of a second order system are complex conjugate with negative real parts, then the singularity point is termed as **[MODEL QUESTION]**
a) the stable nodal point
b) the unstable nodal point
c) the stable focus point
d) the vortex point

Answer: (c)

15. For the given LTI system $x' = \begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} x$, diagonalization matrix is **[MODEL QUESTION]**

a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
b) $\begin{bmatrix} 0 & 4 \\ 1 & 0 \end{bmatrix}$
c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
d) $\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

Answer: (d)

16. The property satisfied by a state transition matrix is **[MODEL QUESTION]**
a) $\phi(0) = 1$
b) $\phi^{-1}(t) = \phi(t)$
c) $[\phi(t)]^k = \phi(-kt)$
d) $\phi(t) \cdot \phi^T(t) = I$

Answer: (a)

17. Parallel decomposition gives: **[MODEL QUESTION]**
a) Diagonalisation
b) Eigen values of the given system
c) It gives the Jordan canonical form
d) all of these

Answer: (d)

18. A system is represented by the state equation given below: **[MODEL QUESTION]**

$$\dot{x} = \begin{bmatrix} -3 & -2 \\ -1 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

The poles of the system are:
a) 2 & 3
b) 1 & 4
c) –1 & –4
d) –6 & 3

Answer: (c)

19. A system describe by the state equation is **[MODEL QUESTION]**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

The state transition matrix of the system is
a) $\begin{bmatrix} e^{2t} & 0 \\ 0 & e^{2t} \end{bmatrix}$
b) $\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix}$
c) $\begin{bmatrix} e^{2t} & 1 \\ 1 & e^{2t} \end{bmatrix}$
d) $\begin{bmatrix} e^{-2t} & 1 \\ 1 & e^{-t} \end{bmatrix}$

Answer: (a)

20. Which one of the following statements regarding the state transition matrix is correct? [MODEL QUESTION]

a) $\phi(0)=0$
b) $\phi^{-1}(t)=\phi(1/t)$
c) $\phi(t_1+t_2)=\phi(t_1)+\phi(t_2)$
d) $\phi(t_2-t_1)\phi(t_1-t_0)=\phi(t_2-t_0)$

Answer: (d)

21. Consider the system $\dot{x}=\begin{bmatrix}0 & 1\\0 & -4\end{bmatrix}x+\begin{bmatrix}0\\2\end{bmatrix}u$ [MODEL QUESTION]

$$y=\begin{bmatrix}0 & 2\end{bmatrix}x$$

The system is
a) Not controllable, observable
b) Controllable, observable
c) Controllable, not observable
d) Not controllable, not observable

Answer: (c)

22. Consider the following state equations for a discrete system [MODEL QUESTION]

$$\begin{bmatrix}X_1(k+1)\\X_2(k+1)\end{bmatrix}=\begin{bmatrix}-\frac{1}{2} & 0\\-\frac{1}{4} & -\frac{1}{4}\end{bmatrix}\begin{bmatrix}X_1(k)\\X_2(k)\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}U(k)$$

$$Y(k)=\begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}X_1(k)\\X_2(k)\end{bmatrix}-4U(k)$$

The system given above is
a) controllable and observable
b) uncontrollable and observable
c) uncontrollable and unobservable
d) controllable and unobservable

Answer: (a)

23. The transfer function of a multi-input multi-output system, with the state space representation of

$$\dot{X}=AX+BU$$
$$Y=CX+DU$$

where X represents the state, Y the output and U the input vector, will be given by [MODEL QUESTION]

a) $C(sI-A)^{-1}B$
b) $C(sI-A)^{-1}B+D$
c) $(sI-A)^{-1}B+D$
d) $(sI-A)^{-1}B$

Answer: (b)

24. If the eigen values are on the imaginary axis, the phase portrait has [MODEL QUESTION]
a) closed path trajectories
b) spiral trajectories focusing at the origin
c) trajectories converging to the origin
d) unstable focus

Answer: (a)

25. If the state equation of a dynamic system is given by $\dot{X}(t) = AX(t)$

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 4 \\ 0 & 0 & 0 & -4 & -3 \end{bmatrix}.$$

the eigen values of the system would be **[MODEL QUESTION]**

a) real non-repeated
b) real non-repeated and complex
c) real repeated
d) real repeated and complex

Answer: (d)

26. The necessary and sufficient condition for pole placement approach is **[MODEL QUESTION]**

a) completely controllable
b) completely observable
c) completely controllable and observable
d) neither controllable and non-observable

Answer: (a)

27. The state and output equations of a system are as under **[MODEL QUESTION]**

$$\begin{bmatrix} x_1 & (t) \\ x_2 & (t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 & (t) \\ x_2 & (t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

output equation $c(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 & (t) \\ x_2 & (t) \end{bmatrix}.$

The system is

a) neither state controllable nor output controllable
b) state controllable but not output controllable
c) output controllable but not state controllable
d) both state & output controllable

Answer: (b)

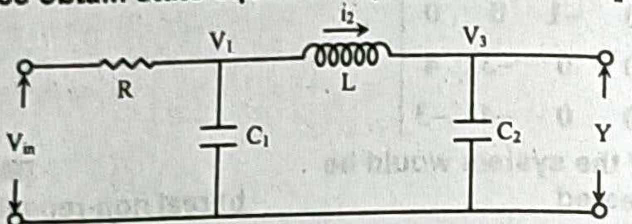
28. A system is describe by the state equation $\dot{X} = AX + BV$. The output being $Y = CX$ where $A = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$. The transfer function $G(S)$ of the system is **[MODEL QUESTION]**

a) $\frac{s}{s^2+5s+7}$
b) $\frac{1}{s^2+5s+7}$
c) $\frac{s}{s^2+3s+2}$
d) $\frac{1}{s^2+3s+2}$

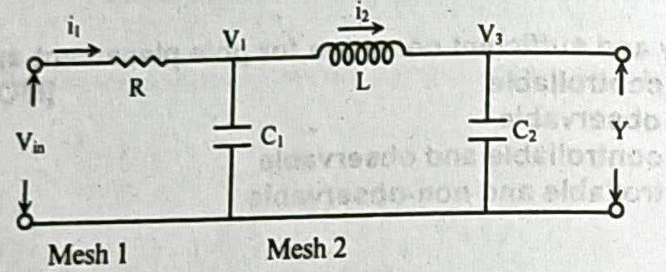
Answer: (a)

Short Answer Type Questions

1. For the circuit shown, choose $V_1(t)$, $i_2(t)$ and $V_3(t)$ on state variables, the output $Y(t)=V_3(t)$ and hence obtain state equation representation. **[MODEL QUESTION]**



Answer:
Consider the network with various currents and voltages as represented in the figure below:



Applying KVL in Mesh 1 of the network we've,

$$V_{in} = i_1 R + v_1 \quad \text{.... (1)}$$

i_1 being the current through resistance R as shown in figure.

Applying KVL in Mesh 2 of the network we've,

$$V_1 = L\frac{di_2}{dt} + v_3 \quad \text{.... (2)} \quad \Rightarrow \quad \frac{di_2}{dt} = \frac{1}{L}v_1 - \frac{1}{L}v_3 \quad \text{.... (2a)}$$

$$C_2\frac{dv_3}{dt} = i_2 \quad \text{.... (3)} \quad \Rightarrow \quad \frac{dv_3}{dt} = \frac{1}{C_2}i_2 \quad \text{.... (3a)}$$

$$C_1\frac{dv_1}{dt} = i_1 - i_2$$

i.e. $$i_1 = C_1\frac{dv_1}{dt} + i_2 \quad \text{.... (4)}$$

and output $y = V_3(t)$ .... (5)

From equations (1) and (4) we've,

$$V_{in} = R\left[C_1\frac{dv_1}{dt} + i_2\right] + v_1;$$

or, $$V_{in} = RC_1\frac{dv_1}{dt} + Ri_2 + v_1$$

or, $$\frac{dv_1}{dt} = -\frac{1}{RC_1}v_1 - \frac{R}{RC_1}i_2 + \frac{1}{RC_1}V_{in}$$

or, $$\frac{dv_1}{dt} = -\frac{1}{RC_1}v_1 - \frac{1}{C_1}i_2 + \frac{1}{RC_1}V_{in} \quad \text{.... (6)}$$

Now, equations (6), (2a) and (3a) can be written in the state space form using $V_1(t)$, $i_2(t)$ and $V_3(t)$ as state variables and $Y(t) = V_3(t)$ as the output of the system:

$$\begin{bmatrix} \frac{dv_1}{dt} \\ \frac{di_2}{dt} \\ \frac{dv_3}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC_1} & -\frac{1}{C_1} & 0 \\ \frac{1}{L} & 0 & -\frac{1}{L} \\ 0 & \frac{1}{C_2} & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ i_2 \\ V_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{RC_1} \\ 0 \\ 0 \end{bmatrix} V_{in}$$

and $$\begin{bmatrix} y(t) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1(t) \\ i_2(t) \\ V_3(t) \end{bmatrix}$$

2. Given $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$, determine $\phi(k) = A^k$ using Cayley – Hamilton method.

[MODEL QUESTION]

Answer:

We've the system matrix, $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

So the characteristic equation for the eigen value m is

$$|mI - A| = 0$$

or, $$\left| \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \right| = 0$$

or, $$\begin{vmatrix} m & -1 \\ 2 & m+3 \end{vmatrix} = 0$$

or, $m(m+3) + 2 = 0$

or, $m^2 + 3m + 2 = 0$

or, $m^2 + 2m + m + 2 = 0$

or, $m(m+2) + 1(m+2) = 0$

or, $(m+2)(m+1) = 0$

so the eigen values are $m = -2, -1$

Now, $e^{At} = p(t)I + q(t)A$

and $e^{mt} = p + qm$, for $m = -1, -2$

$\therefore \quad e^{-t} = p - q$ .... (1)

and $\quad e^{-2t} = p - 2q$ .... (2)

By subtracting Eqn. (2) from Eqn. (1),

$e^{-t} - e^{-2t} = q;$ and $p = e^{-t} + q$

i.e. $\quad p = e^{-t} + e^{-t} - e^{-2t}$

or, $\quad p = 2e^{-t} - e^{-2t}$

$\therefore \quad p = 2e^{-t} - e^{-2t}; \; q = e^{-t} - e^{-2t};$

now $\quad \phi(k) = A^k = e^{At}$

$$\therefore \quad e^{At} = pI + qA = \left(2e^{-t} - e^{-2t}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(e^{-t} - e^{-2t}\right)\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{-t} - e^{-2t} & 0 \\ 0 & 2e^{-t} - e^{-2t} \end{bmatrix} + \begin{bmatrix} 0 & e^{-t} - e^{-2t} \\ 2\left(-e^{-t} + e^{-2t}\right) & 3\left(-e^{-t} + e^{-2t}\right) \end{bmatrix}$$

$$\text{or,} \quad \phi(k) = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ 2\left(e^{-2t} + e^{-t}\right) & 3\left(e^{-2t} - e^{-t}\right) \end{bmatrix}$$

The above expression gives the state transition matrix as due to Cayley – Hamilton method.

3. The overall transfer function of a SISO system is given by $\dfrac{Y(s)}{U(s)} = \dfrac{s^2 + 4s + 4}{s^3 + 5s^2 + 4s}$. Obtain state model of the system. **[MODEL QUESTION]**

Answer:

Obtain the state model of system transfer function

$$T(s) = \frac{Y(s)}{V(s)} = \frac{\left(s^2 + 4s + 4\right)X(s)}{\left(s^3 + 5s^2 + 4s\right)X(s)}, \; X(s) \text{ being state variable}$$

$$\text{or,} \quad \frac{Y(s)}{V(s)} = \frac{\left(4s^{-3} + 4s^{-2} + s^{-1}\right)X(s)}{\left(4s^{-2} + 5s^{-1} + 1\right)X(s)} \; \text{[Dividing by } s^3 \text{ in numerator and denominator]}$$

Then, $\quad Y(s) = \left(4s^{-3} + 4s^{-2} + s^{-1}\right)X(s)$

and

$V(s) = \left(4s^{-2} + 5s^{-1} + 1\right)X(s);$ i.e.,

$X(s) = V(s) - \left(4s^{-2} + 5s^{-1}\right)X(s)$

The corresponding SFG is shown in Fig. 1.

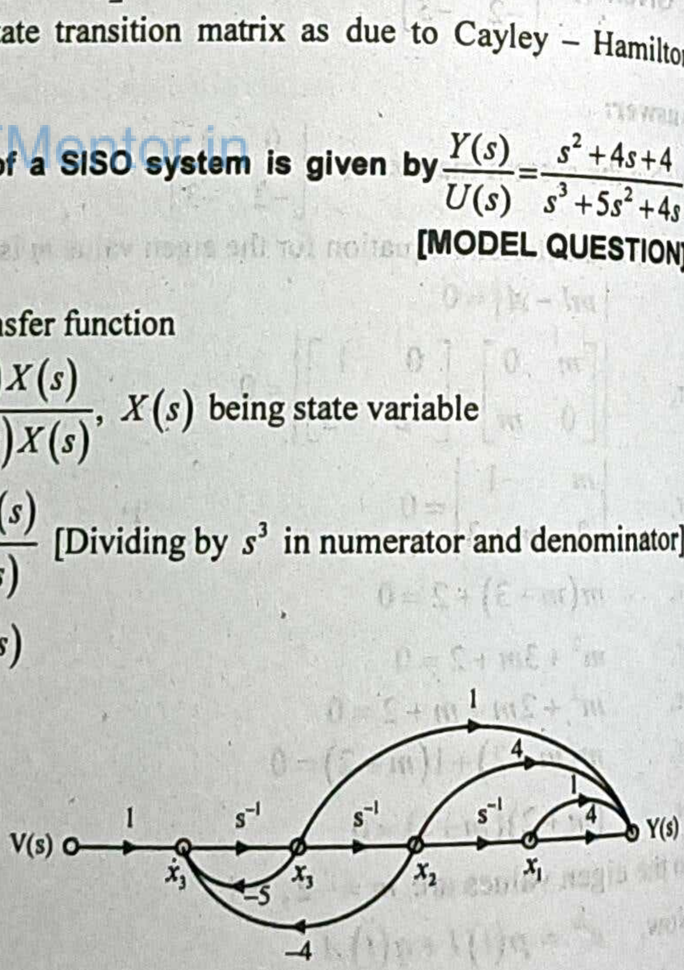


Fig: SFG

Consider $\dot{x}_1 = x_2$, $\dot{x}_2 = x_3$. Thus from SFG

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -4 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}V \quad \text{.... (1)}$$

$$y=\begin{bmatrix}4 & 4 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+[0]V \quad \text{.... (2)}$$

Equations (1) and (2) give the required state model.

4. The state space representation of a system is **[MODEL QUESTION]**

$$\dot{x}_1 = -x_1 + v$$
$$\dot{x}_2 = x_1 - 2x_2 + v$$

Comment on controllability and observability of the system.

Answer:

State space representation of the system is

$\dot{x}_1 = -x_1 + v$, $\dot{x}_2 = x_1 - 2x_2 + v$, which may be represented in matrix form as:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 1 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 1\end{bmatrix}u.$$

Comparing with standard state space form

$$\dot{X} = AX + Bu;\; A=\begin{bmatrix}-1 & 0\\ 1 & -2\end{bmatrix};\; B=\begin{bmatrix}1\\ 1\end{bmatrix}.$$

Thus $AB=\begin{bmatrix}-1 & 0\\ 1 & -2\end{bmatrix}_{2\times 2}\begin{bmatrix}1\\ 1\end{bmatrix}_{2\times 1}=\begin{bmatrix}-1\\ -1\end{bmatrix}_{1\times 1}$.

Controllability matrix is $s=\begin{bmatrix}B & AB\end{bmatrix}=\begin{bmatrix}1 & -1\\ 1 & -1\end{bmatrix}$; Now, $|s|=0$.

So the system is not controllable. Observability of the system cannot be commented, as output equation is not given.

5. Check the controllability and observability of the system: **[MODEL QUESTION]**

$$X'(t)=\begin{bmatrix}0 & 1\\ -6 & -5\end{bmatrix}X(t)+\begin{bmatrix}0\\ 1\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}1 & 0\end{bmatrix}X(t)$$

Answer:

Considering the state Eqn. as

$$\dot{x} = Ax + Bu,\quad y = cx;$$

$$A=\begin{bmatrix}0 & 1\\-6 & -5\end{bmatrix},\ B=\begin{bmatrix}0\\1\end{bmatrix};\ C=[1\ \ 0]$$

Now, $AB=\begin{bmatrix}0 & 1\\-6 & -5\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1\\-5\end{bmatrix}$

Controllability matrix $S=[B\ \ AB]=\begin{bmatrix}0 & 1\\1 & -5\end{bmatrix};|S|\neq 0$

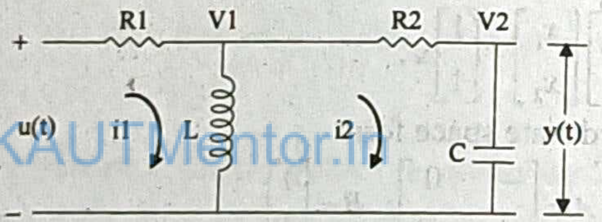
Thus the system is controllable.

Again, $CA=[1\ \ 0]_{1\times 2}\begin{bmatrix}0 & 1\\-6 & -5\end{bmatrix}_{2\times 2}=\begin{bmatrix}0 & 1\end{bmatrix};$

Observability matrix $V=\begin{bmatrix}C\\CA\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix};|V|\neq 0.$

Thus the system is observable
Hence, the system is controllable and observable.

6. Derive the state space representation of the network: **[MODEL QUESTION]**



Answer:

Now, the mesh Eqn.s are:

$$L\frac{d}{dt}(i_1-i_2)+R_1i_1=u(t)\text{.........(1)}$$

$$\&\quad \frac{1}{C}\int i_2dt+R_2i_2+L\frac{d}{dt}(i_2-i_1)=0\text{........(2)}$$

Define, $x_1=i_1-i_2,\text{–}(a)\ \&\ x_2=\frac{1}{c}\int i_2dt\text{.......}(b)$

Now, from Eqn. (2), $x_2+R_2i_2-L\frac{d}{dt}x_1=0$

$$or,\quad i_2=\frac{L}{R_2}\frac{d}{dt}x_1-\frac{x_2}{R_2}\quad\text{........(3)}$$

From Eqn. (1), $L\frac{d}{dt}x_1+R_1(x_1+i_2)=u(t)\cdot\begin{bmatrix}\text{From Eqn. }(a)\\ x_1=i_1-i_2, or, i_1=x_1+i_2\end{bmatrix}$

$$or,\ L\dot{x}_1+R_1x_1+R_1\left(\frac{L}{R_2}\dot{x}_1-\frac{x_2}{R_2}\right)=u(t)\cdot\text{[using Eqn. (3)]}$$

$$or, \dot{x}_1\left[L+\frac{LR_1}{R_2}\right]+R_1x_1-\frac{R_1x_2}{R_2}=u(t)$$

$$or,\ \dot{x}_1L\left(\frac{R_1+R_2}{R_2}\right)=-R_1x_1+\frac{R_1}{R_2}x_2+u(t)$$

$$or,\ \dot{x}_1=(-)\frac{R_1R_2}{L(R_1+R_2)}x_1+\frac{R_1}{L(R_1+R_2)}x_2+\frac{R_2}{L(R_1+R_2)}u(t) \text{......(4)}$$

From Eqn. (b), $x_2=\frac{1}{c}\int i_2dt\ ;\ \dot{x}_2=\frac{1}{c}i_2$...........(5)

From Eqns. (5) & (3)

$$\dot{x}_2=\frac{1}{c}\left[\frac{L}{R_2}\dot{x}_1-\frac{x_2}{R_2}\right]$$

$$or,\ \dot{x}_2=\frac{L}{CR_2}\dot{x}_1-\frac{1}{CR_2}x_2 \text{........(6)}$$

Putting value of $\dot{x}_1$ from Eqn. (4) in Eqn. (6)

$$\dot{x}_2=\frac{L}{CR_2}\left[-\frac{R_1R_2}{L(R_1+R_2)}x_1+\frac{R_1}{L(R_1+R_2)}x_2+\frac{R_2}{L(R_1+R_2)}u(t)\right]-\frac{1}{CR_2}x_2$$

$$or, \dot{x}_2=-\frac{R_1}{C(R_1+R_2)}x_1+\frac{R_1}{CR_2(R_1+R_2)}x_2-\frac{1}{CR_2}x_2+\frac{u(t)}{C(R_1+R_2)}$$

$$or,\ \dot{x}_2=-\frac{R_1}{C(R_1+R_2)}x_1+x_2\frac{1}{CR_2}\left[\frac{R_1}{R_1+R_2}-1\right]+\frac{1}{C(R_1+R_2)}\cdot u(t)$$

$$or,\ \dot{x}_2=(-)\frac{R_1}{C(R_1+R_2)}x_1-\frac{1}{C(R_1+R_2)}x_2+\frac{1}{C(R_1+R_2)}\cdot u(t) \text{......(7)}$$

Now, Eqns. (4) & (7) may be put in the form,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-\frac{R_1R_2}{L(R_1+R_2)} & \frac{R_1}{L(R_1+R_2)}\\ -\frac{R_1}{C(R_1+R_2)} & -\frac{1}{C(R_1+R_2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}\frac{R_2}{L(R_1+R_2)}\\ \frac{1}{C(R_1+R_2)}\end{bmatrix}u(t) \quad \text{Ans.}$$

& output, $y=\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$ Ans.

7. For the following system, obtain the state space equation. [MODEL QUESTION]

$$\left(\frac{d^3y}{dt^3}\right)+6\left(\frac{d^2y}{dt^2}\right)+11\left(\frac{dy}{dt}\right)+6y=u$$

where y = output and u = input.

Answer:
We have

$$\frac{d^3y}{dt^3} = -6\frac{d^2y}{dt^2} - 11\frac{dy}{dt} - 6y + u \quad \text{.... (1)}$$

for output y and input u.
Consider $y = x_1 =$ one state variable and

$$\left.\begin{aligned} \frac{dy}{dt} &= \frac{d}{dt}x_1 = \dot{x}_1 = x_2 = \text{another state variable} \\ \frac{d^2y}{dt^2} &= \frac{d^2x_1}{dt^2} = \frac{d}{dt}x_2 = x_3 = \dot{x}_2 = \text{another state variable} \\ \frac{d^3y}{dt^3} &= \frac{dx_3}{dt} = \dot{x}_3 \end{aligned}\right\} \text{.... (2)}$$

Thus from Eqns. (1) and (2), we can form the matrix equation as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad \text{.... (3a)}$$

$$\begin{bmatrix} y \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u \quad \text{.... (3b)}$$

From Eqns. (3a) and (3b) we have the state space equation as:

$$\dot{X} = AX + Bu;\; Y = CX + Du;$$

where, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$; $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$; $D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

8. Consider a system given by **[MODEL QUESTION]**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$

Check for the state controllability.

Answer:
There is misprint in the question. It may as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}_{3\times 2} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{2\times 1};$$

Check for controllability.

Here system matrix $A=\begin{bmatrix}2&0&0\\0&2&0\\0&3&1\end{bmatrix}$; Input matrix $B=\begin{bmatrix}0&1\\1&0\\0&1\end{bmatrix}$

Controllability matrix is $S=\begin{bmatrix}B & AB & A^2B\end{bmatrix}$

Here $A^2=AA=\begin{bmatrix}2&0&0\\0&2&0\\0&3&1\end{bmatrix}\begin{bmatrix}2&0&0\\0&2&0\\0&3&1\end{bmatrix}=\begin{bmatrix}4&0&0\\0&4&0\\0&9&1\end{bmatrix}$

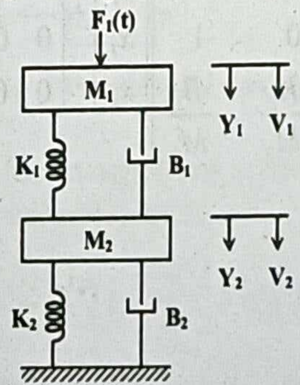
$$AB=\begin{bmatrix}2&0&0\\0&2&0\\0&3&1\end{bmatrix}_{3\times3}\begin{bmatrix}0&1\\1&0\\0&1\end{bmatrix}_{3\times2}=\begin{bmatrix}0&2\\2&0\\3&1\end{bmatrix}_{3\times2};$$

$$A^2B=\begin{bmatrix}4&0&0\\0&4&0\\0&9&1\end{bmatrix}_{3\times3}\begin{bmatrix}0&1\\1&0\\0&1\end{bmatrix}_{3\times2}=\begin{bmatrix}0&4\\4&0\\9&1\end{bmatrix}_{3\times2}$$

So the controllability matrix is $S=\begin{bmatrix}B & AB & A^2B\end{bmatrix}=\begin{bmatrix}01&02&04\\10&20&40\\01&31&91\end{bmatrix}$

Now, the controllability matrix to be of full rank $(=3)$ should have three linearly independent columns or rows, as it happens here. So the system is controllable.

9. For the mechanical system shown in the figure below, obtain the state model in standard form, with the velocity of M_2 as the output: **[MODEL QUESTION]**



Answer:

The dynamics of the spring mass system may be represented as

$$M_1\frac{d^2y_1}{dt^2}+B_1\frac{dy_1}{dt}+K_1y_1=F_1(t) \quad \text{.... (1)}$$

$$M_2\frac{d^2y_2}{dt^2}+B_2\frac{d}{dt}(y_2-y_1)+K_2(y_2-y_1)=0 \quad \text{.... (2)}$$

From Eqn. (1),

$$\frac{d^2y_1}{dt^2} = -\frac{B_1}{M_1}\frac{dy_1}{dt} - \frac{K_1y_1}{M_1} + \frac{F_1}{m} \quad \text{.... (3)}$$

Consider $y_1 = x_1, \dot{y}_1 = \dot{x}_1 = x_2$; stable vector $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

From Eqn. (3),

$$\dot{x}_2 = -\frac{B_1}{M_1}x_2 - \frac{K_1}{M_1}x_1 + \frac{F_1}{M_1} \quad \text{.... (3a)}$$

From Eqn. (2),

$$\frac{d^2y_2}{dt^2} = -\frac{B_2}{M_2}\frac{dy_2}{dt} + \frac{B_2}{M_2}\frac{dy_1}{dt} - \frac{K_2}{M_2}y_2 + \frac{K_2}{M_2}y_1 \quad \text{.... (4)}$$

Consider, $y_2 = x_3$; $\dot{y}_2 = \dot{x}_3 = x_4$; i.e., $\dot{x}_4 = \ddot{x}_3 = \ddot{y}_2$

From Eqn. (4),

$$\dot{x}_4 = -\frac{B_2}{M_2}x_4 + \frac{B_2}{M_2}x_2 - \frac{K_2}{M_2}x_3 + \frac{K_2}{M_2}x_1 \quad \text{.... (4a)}$$

Consider, $x_4 = y =$ output = velocity of M_2 .... (5)

Now, from Eqns. (3a), (4a) and (5), we can from the state equation as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}_{4\times1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_1}{M_1} & -\frac{B_1}{M_1} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_2}{M_2} & \frac{B_2}{M_2} & -\frac{K_2}{M_2} & -\frac{B_2}{M_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}_{4\times2} \begin{bmatrix} 0 \\ F/M_1 \end{bmatrix}_{2\times1}$$

and $$y = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

10. The transfer function of a dynamic system is given by **[MODEL QUESTION]**

$$\frac{Y(s)}{U(s)} = \frac{2(s+5)}{(s+2)(s+3)(s+4)}$$

Obtain the diagonal canonical state model of the system. Also determine the output $y(t)$.

Answer:

$$\frac{Y(s)}{U(s)}=\frac{2(s+5)}{(s+2)(s+3)(s+4)}=\frac{A}{s+2}+\frac{B}{s+3}+\frac{C}{s+4}=\frac{3}{s+2}-\frac{4}{s+3}+\frac{1}{s+4}$$

Thus, $\dot{x}_1=-2x_1+u(t);\ \dot{x}_2=-3x_2+u(t);\ \dot{x}_3=-4x_3+u(t)$

and output $\quad y(t)=3x_1-4x_2+x_3$

State model is:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-2 & 0 & 0\\ 0 & -3 & 0\\ 0 & 0 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$$

or, $\quad y(t)=\begin{bmatrix}3 & -4 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$

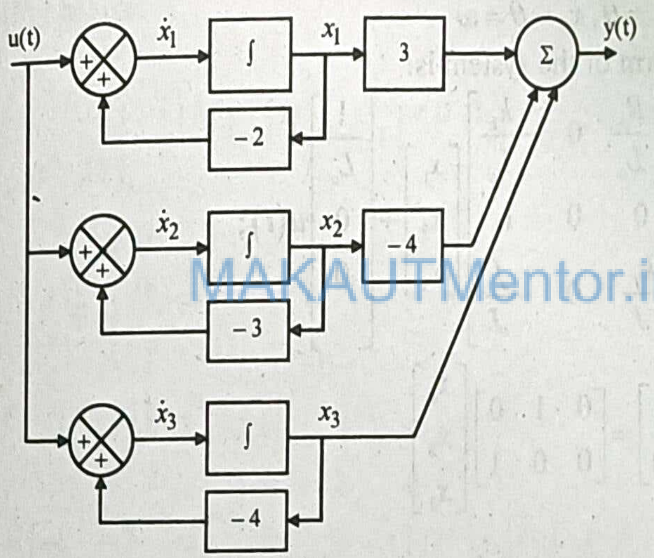
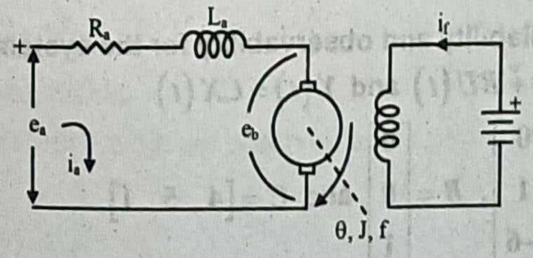


Fig: 1 For diagonal canonical state model

11. Develop the state model for an armature controlled separately excited dc shunt motor, considering the state variables as x_1

$x_1(t)=i_a(t),\ x_2(t)=\theta(t),\ x_3(t)=\frac{d\theta(t)}{dt}=\omega(t).$ **[MODEL QUESTION]**

Answer:



The dynamics of the motor can be expressed as:

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a ;$$

and $$J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} = T_m = k_T i_a ;$$

Again, $e_b = k_b \frac{d\theta}{dt} = k_b \omega$; as $\omega = \frac{d\theta}{dt}$

$$\therefore \quad \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}$$

So the system dynamics can be expressed as:

$$\frac{di_a}{dt} = -\frac{R_a}{L_a} i_a - \frac{k_b}{L_a}\omega + \frac{1}{L_a} e_a ; \quad \text{and} \quad \frac{d\omega}{dt} = \frac{k_T}{J} i_a - \frac{f}{J}\omega$$

Now considering the state variables as:

$$x_1 = i_a, x_2 = \theta, x_3 = \dot{\theta} = \omega$$

The state space form of the system is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & 0 & -\frac{k_b}{L_a} \\ 0 & 0 & 1 \\ \frac{k_T}{J} & 0 & -\frac{f}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} u(t);$$

and output $$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

12. Write down the advantages of state space techniques. [MODEL QUESTION]

Answer:

Advantages of state space technique:

(i) This technique can be used for linear as well as non-linear system.

(ii) This technique can be very easily used for multiple input multiple output (MIMO) systems in matrix form.

(iii) In this analysis initial condition of the system need not be ignored.

13. Predict the controllability and observability for the system [MODEL QUESTION]

$$\dot{X}(t) = AX(t) + BU(t) \text{ and } Y(t) = CX(t)$$

where, $A \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 5 & 1 \end{bmatrix}$

Answer:

Controllability Matrix $M=\begin{bmatrix}B & AB & A^2B\end{bmatrix}$;

Observability Matrix $N=\begin{bmatrix}C\\CA\\CA^2\end{bmatrix}$;

Now, $AB=\begin{bmatrix}0&1&0\\0&0&1\\-6&-11&-6\end{bmatrix}_{3\times3}\begin{bmatrix}0\\0\\1\end{bmatrix}_{3\times1}=\begin{bmatrix}0\\1\\-6\end{bmatrix}_{3\times1}$;

So, $A^2B=\begin{bmatrix}0&1&0\\0&0&1\\-6&-11&-6\end{bmatrix}_{3\times3}\begin{bmatrix}0\\1\\-6\end{bmatrix}_{3\times1}=\begin{bmatrix}1\\-6\\-11+36\end{bmatrix}_{3\times1}=\begin{bmatrix}1\\-6\\25\end{bmatrix}_{3\times1}$

So the controllability Matrix is

$$M=\begin{bmatrix}0&0&1\\0&1&-6\\1&-6&25\end{bmatrix}_{3\times3};\ |M|=(1)\begin{vmatrix}0&1\\1&-6\end{vmatrix}\neq0$$ thus the system is controllable.

Again $CA=\begin{bmatrix}4&5&1\end{bmatrix}_{1\times3}\begin{bmatrix}0&1&0\\0&0&1\\-6&-11&-6\end{bmatrix}_{3\times3}$

$=\begin{bmatrix}(-6)&(4-11)&(5-6)\end{bmatrix}_{1\times3}=\begin{bmatrix}-6&-7&-1\end{bmatrix}$

So, $CA^2=(CA)A=\begin{bmatrix}-6&-7&-1\end{bmatrix}_{1\times3}\begin{bmatrix}0&1&0\\0&0&1\\-6&-11&-6\end{bmatrix}_{3\times3}=\begin{bmatrix}(6)&(-6+11)&(-7+6)\end{bmatrix}_{1\times3}$

or, $CA^2=\begin{bmatrix}6&5&-1\end{bmatrix}$;

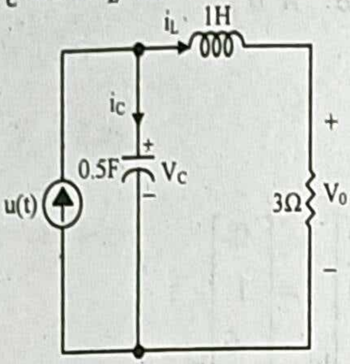
So the observability matrix is $N=\begin{bmatrix}C\\CA\\CA^2\end{bmatrix}=\begin{bmatrix}4&5&1\\-6&-7&-1\\6&5&-1\end{bmatrix}$;

So, $|N|=4[7+5]-5[6+6]+1[-30+42]=48-60+12=0$

So the system is not observable.

Thus the system is controllable but not observable.

14. Find the state space model of the electrical systems shown below in terms of physical variable. Take V_C and i_L as state variables and V_0 as output variable.
[MODEL QUESTION]



Answer:

We have by KCL in the network,

$$u = i_C + i_L = i_L + C\frac{dV_C}{dt}$$

$$\text{or, } \dot{V}_C = 0.V_C - \frac{1}{C}i_L + \frac{1}{C}u \text{.........(1)}$$

Again by KVL in the network,

$$V_C = L\frac{d}{dt}i_L + R\,i_L\,;$$

$$\text{or, } \frac{d}{dt}i_L = \frac{1}{L}V_C - \left(\frac{R}{L}\right)i_L + (0).u \text{........(2)}$$

From equations (1) & (2) we have,

$$\frac{d}{dt}\begin{bmatrix} V_C \\ i_L \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R}{L} \end{bmatrix}\begin{bmatrix} V_C \\ i_L \end{bmatrix} + \begin{bmatrix} 1/C \\ 0 \end{bmatrix} u \text{.........(3a)}$$

$$\text{\& output } V_O = R\,i_L = \begin{bmatrix} 0 & R \end{bmatrix}\begin{bmatrix} V_C \\ i_L \end{bmatrix} \text{.........(3b)}$$

Equations (3a) and (3b) represent the state variable formulation of the given network. Putting the values of C = (0.5F), L = (1H) & R = (3Ω) we've state equation as:

$$\frac{d}{dt}\begin{bmatrix} V_C \\ i_L \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} V_C \\ i_L \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(t) \text{.........(4a)}$$

$$\text{\& output } V_0 = R\,i_L = \begin{bmatrix} 0 & 3 \end{bmatrix}\begin{bmatrix} V_C \\ i_L \end{bmatrix} \text{.........(4b)}$$

Eqns. 4(a) & 4(b) represent the required state variable model.

15. The state equation of a linear time invariant system is given by
[MODEL QUESTION]

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u(t)$$

Find the characteristic equation and the state transition matrix using the Caley-Hamilton theorem.

Answer:

For state equation $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u(t)$

Using Caley-Hamilton theorem, the characteristic equation is:

$q(\lambda)=|\lambda I-A|=\left|\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}-\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}\right|=\begin{vmatrix}\lambda & -1\\ 1 & \lambda+2\end{vmatrix}=0; \Rightarrow \lambda^2+1+2\lambda=0;$ which is the characteristic equation.

i.e., $(\lambda+1)^2=0$; Thus the system matrix A has Eigen values, $\lambda_1,\lambda_2=-1$

Since the system matrix A is of second order, the reminder polynomial $R(\lambda)$ is of the form: $R(\lambda)=\alpha_0+\alpha_1\lambda$

Since A is of second order, the polynomial $R(\lambda)$ will be of the following form

$R(\lambda)=\alpha_0+\alpha_1\lambda$

$f(\lambda)=f(-1)=e^{\lambda t}=e^{-t}=\alpha_0+\lambda\alpha_1=(\alpha_0-\alpha_1);\ [\because \lambda=\lambda_1=\lambda_2=-1]$

$\frac{d}{d\lambda}f(\lambda)\Big|_{\lambda=-1}=te^{-t}=\frac{d}{d\lambda}R(\lambda)\Big|_{\lambda=-1}=\alpha_1$

$[R(\lambda)$ being the reminder polynomial defined by equation:

$f(\lambda)=Q(\lambda)q(\lambda)+R(\lambda)$

$q(\lambda)$ being zero, $\frac{d}{d\lambda}q(\lambda)=0]$

The result is $\alpha_0=\alpha_1+e^{-t}=te^{-t}+e^{-t}=e^{-t}(1+t)$

Thus, $\alpha_0=e^{-t}(1+t);\alpha_1=te^{-t}$

$$f(A)=e^{At}=\alpha_0 I+\alpha_1 A=e^{-t}(1+t)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+te^{-t}\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}$$

$$=\begin{bmatrix}e^{-t}(1+t) & 0\\ 0 & e^{-t}(1+t)\end{bmatrix}+\begin{bmatrix}0 & te^{-t}\\ -te^{-t} & -2te^{-t}\end{bmatrix}=\begin{bmatrix}e^{-t}(1+t) & te^{-t}\\ -te^{-t} & e^{-t}+te^{-t}-2te^{-t}\end{bmatrix}$$

or, $f(A)=\begin{bmatrix}e^{-t}(1+t) & te^{-t}\\ -te^{-t} & e^{-t}(1-t)\end{bmatrix}$.

16. The overall transfer function of a SISO system is given by [MODEL QUESTION]

$$\frac{Y(s)}{U(s)}=\frac{s^2+3s+2}{s^3+9s^2+26s+24}$$

Obtain the state equation.

Answer:

From the transfer function, $\frac{Y(s)}{U(s)}=\frac{s^2+3s+2}{s^3+9s^2+26s+24}$: to find the state equation.

Now, $\frac{Y(s)}{U(s)}=\frac{s^2+3s+2}{s^3+9s^2+26s+24}=\frac{(s+1)(s+2)}{(s+2)(s+3)(s+4)}=\frac{s+1}{(s+3)(s+4)}$;

Consider,

$$\frac{Y(s)}{U(s)}=\frac{s+1}{(s+3)(s+4)}\left[\frac{s+1}{(s+3)(s+4)}=\frac{C_1}{s+3}+\frac{C_2}{s+4}:\text{ i.e., }s+1=C_1(s+4)+C_2(s+3)\right]$$

Putting $s=-3,-2=C_1$; i.e., $C_1=-2$;;

Putting $s=-4,+3=+C_2$; i.e., $C_2=3$

or, $\frac{Y(s)}{U(s)}=b_0u+\frac{C_1}{s+\lambda_1}+\frac{C_2}{s+\lambda_2}$; where $C_1=-2, C_2=3, \lambda_1=-3, \lambda_2=-4, b_0=0$;

The diagonal canonical form of the state model is:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 1\end{bmatrix}u$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-3 & 0\\ 0 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 1\end{bmatrix}u \quad \cdots(1)$$

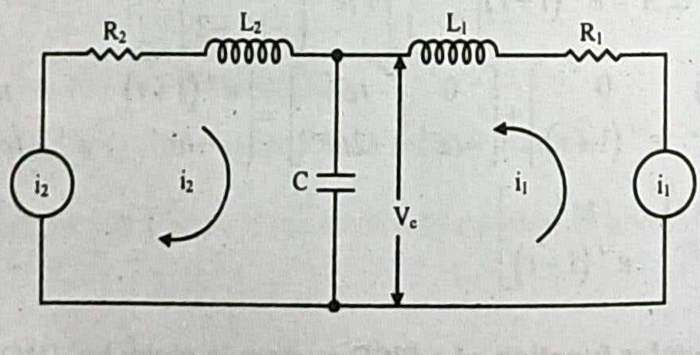
and $y=\begin{bmatrix}c_1 & c_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+b_0u$

$$y=\begin{bmatrix}-2 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \quad \cdots(2)$$

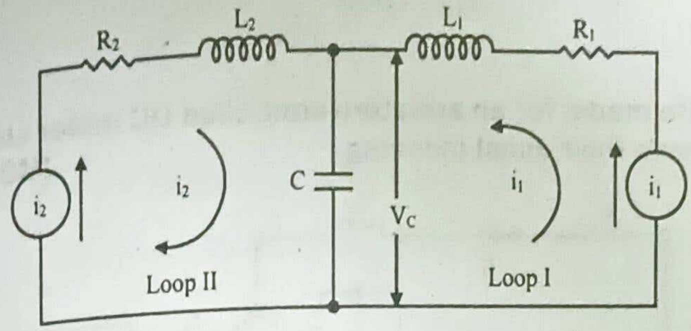
Equations (1) and (2) represent the state equation. Ans.

17. Obtain the state space representation of the network shown.

[MODEL QUESTION]



Answer:



In loop I,

$$V_C + R_1 i_1 + L_1 \frac{di_1}{dt} = 0 \quad \text{.... (1)}$$

In loop II,

$$V_C + R_2 i_2 + L_2 \frac{di_2}{dt} = 0 \quad \text{.... (2)}$$

From Eqn. (1),

$$\frac{di_1}{dt} = -\frac{R_1}{L_1} i_1 - \frac{1}{L_1} V_C \quad \text{.... (3)}$$

and Eqn. (2),

$$\frac{di_2}{dt} = -\frac{R_2}{L_2} i_2 - \frac{1}{L_2} V_C \quad \text{.... (4)}$$

Consider the voltage drop across R_1 if $V_{R_1} = i_1 R_2$ and voltage drop across R_2 if $V_{R_2} = i_2 R_2$ are outputs.

Thus, we have

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 \\ 0 & -\frac{R_2}{L_2} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} -\frac{1}{L_1} \\ -\frac{1}{L_2} \end{bmatrix} V_C \quad \text{.... (5)}$$

$$\begin{bmatrix} V_{R_1} \\ V_{R_2} \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad \text{.... (6)}$$

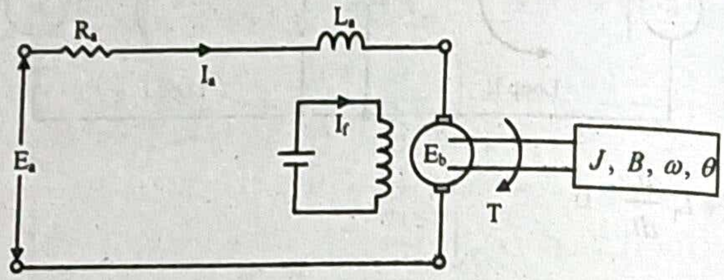
Eqns. (5) and (6) give the state space form as

$$\dot{X} = AX + BU$$

$$Y = CX, \text{ with } A = \begin{bmatrix} -\frac{R_1}{L_1} & 0 \\ 0 & -\frac{R_2}{L_2} \end{bmatrix}, \ B = \begin{bmatrix} -\frac{1}{L_1} \\ -\frac{1}{L_2} \end{bmatrix}, \ C = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix},$$

state vector $X = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$, input V_C, output vector $\begin{bmatrix} V_{R_1} \\ V_{R_2} \end{bmatrix}$.

18. Obtain the state model for an armature controlled DC motor shown in the figure below. Symbols have their usual meaning. **[MODEL QUESTION]**



Answer:

The dynamics of the motor can be expressed as:

$$L_a \frac{dI_a}{dt} + R_a I_a + E_b = E_a; \ \&$$

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T_m = K_T I_a;$$

$$E_b = K_b \frac{d\theta}{dt} = K_b \omega; \text{ as}$$

$$\omega = \frac{d\theta}{dt}; \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}$$

So the system dynamics can be expressed as:

$$\frac{dI_a}{dt} = -\frac{R_a}{L_a} I_a - \frac{K_b}{L_a}\omega - \frac{1}{L_a} E_a; \ \&$$

$$\frac{d\omega}{dt} = \frac{K_T}{J} I_a - \frac{B}{J}\omega;$$

Now considering the state variables as:

$$x_1 = I_a, x_2 = \theta, x_3 = \dot{\theta} = \omega.$$

The state space form of the system is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & 0 & -\frac{K_b}{L_a} \\ 0 & 0 & 1 \\ \frac{K_T}{J} & 0 & -\frac{B}{J} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} E_a; \ \&$$

Output, $\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

19. Draw the phase plane portrait of the system described by $\ddot{x}+0.5\dot{x}+2x+x^2=0$.
[MODEL QUESTION]

Answer:

$\ddot{x}+0.5\dot{x}+2x+x^2=0$ .... (1)

Now, $\frac{d^2x}{dt^2}=\frac{d\dot{x}}{dt}=\frac{d\dot{x}}{dx}\cdot\frac{dx}{dt}=a\dot{x};a=\frac{d\dot{x}}{dx}$ considering (a) the slope of the phase plane trajectory. Thus the given system equation becomes:

$a\dot{x}+0.5\dot{x}+2x+x^2=0$ .... (2)

$a=-(0.5\dot{x}+2x+x^2)/\dot{x}$, For, $(x=0,\dot{x}=0)$, & $(x=-2,\dot{x}=0)$

$a=$ undefined; Hence $(0,0)$ & $(-2,0)$ are singular points of the system. The nature of these points can be determined as follows.

In a neighbourhood of the origin (x being infinitesimally positive quantity), the system Eqn. (1) is given as:

$\ddot{x}+0.5\dot{x}+2x=0$;

or, $s^2X(s)+0.5sX(s)+2X(s)=0$;

$(s^2+0.5s+2)=0$

$s_1=-0.25+j1.39$; $s_2=-0.25-j1.39$

The characteristic equation is considering Laplace transform and ignoring initial condition:

Thus this singular point (0,0) is a stable focus.

In a neighbourhood of the singular point (–2,0), Eqn.(1) becomes by letting $y=x+2$, i.e $x=y-2$ ....(1a).

Thus from Eqns. (1) & (1a), we have,

$y=x+2;x=y-2;\ddot{x}=\ddot{y}$;

$\dot{x}=\dot{y}$;

$\ddot{y}+0.5\dot{y}+2(y-2)+(y-2)^2=0$;

$\ddot{y}+0.5\dot{y}+2y-4+y^2+4-4y=0$;

$\ddot{y}+0.5\dot{y}-2y=0$;

Considering y to be infinitesimally small, so that y^2 tends to zero.

The characteristic equation is considering Laplace transform and ignoring initial condition:

$s^2Y(s)+0.5sY(s)-2Y(s)=0$;

$Y(s)\left[s^2+0.5s-2\right]=0$;

or, $s = \frac{-0.5 \pm \sqrt{(0.5)^2 + 8}}{2}$;

or, $s = -0.25 \pm 1.44$;

$s_1 = 1.19, s_2 = -1.69$.

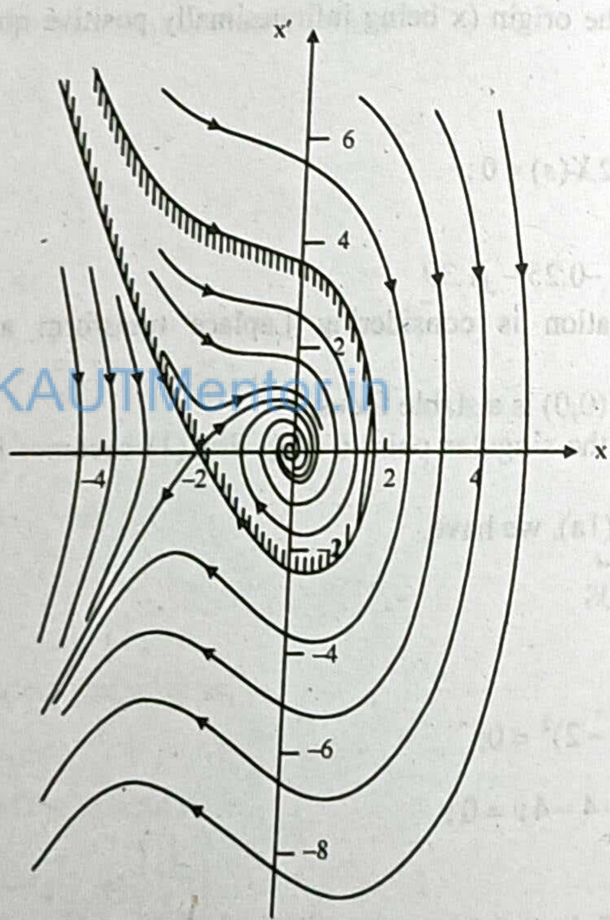
Thus the singular point (–2,0) is a saddle point.

From Eqn. (2),

$$\dot{x} = \frac{-x(x+2)}{a+0.5} \quad \text{.... (3)}$$

or, $x^2 + 2x + \dot{x}(a+0.5) = 0$ .... (4)

Now, using various values starting point $x(0)$, with different values of (a), $\dot{x}$ may be evaluated from Eqn.(4), and accordingly the phase plane portrait may be drawn as shown in Fig.1.



Long Answer Type Questions

1. Determine the state feedback gain matrix so that closed loop poles of the following linear system are located at $-2, -5, -6$. **[MODEL QUESTION]**

$$\dot{X}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -30 & -11\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u(t)$$

$$Y=\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Answer:

Now, with specific pole placement at –2, –5, –6, the characteristic equation of the system becomes:

$$(s+2)(s+5)(s+6)=0$$

i.e., $(s+2)(s^2+11s+30)=0$

or, $s^3+11s^2+30s+2s^2+22s+60=0$

or, $s^3+13s^2+52s+60=0$ .... (1)

Now, suppose the desired state feedback matrix be $K=\begin{bmatrix}a & b & c\end{bmatrix}$. With the desired state feedback matrix K, the characteristics equation is

$$|(sI-A+BK)|=0$$

where $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -30 & -11\end{bmatrix}$; $B=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}$; $I=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

i.e., $sI=\begin{bmatrix}s & 0 & 0\\ 0 & s & 0\\ 0 & 0 & s\end{bmatrix}$

Now, $BK=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}_{3\times1}\begin{bmatrix}a & b & c\end{bmatrix}_{1\times3}$

or, $BK=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ a & b & c\end{bmatrix}$

Again, $sI-A=\begin{bmatrix}s & 0 & 0\\ 0 & s & 0\\ 0 & 0 & s\end{bmatrix}-\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -30 & -11\end{bmatrix}$

or, $$(sI-A)=\begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 30 & s+11+c \end{bmatrix}$$

Thus we have the characteristic equation condition

$$|sI-A+Bk|=0$$

or, $$\begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 30 & s+11 \end{bmatrix}+\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a & b & c \end{bmatrix}=0$$

or, $$\begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ a & (30+b) & (s+11) \end{vmatrix}=0$$

or, $s[s(s+c+11)+30+b]+a[1]=0$

or, $s[s^2+sc+11s+30+b]+a=0$

or, $s^3+s^2(c+11)+s(30+b)+a=0$ .... (2)

Now, in the Eqns. (1) and (2), equating the coefficients of s^3, s^2 & s^1, we have

$c+11=13$

or, $c=13-11$

or, $c=2$

and $b+30=52$ .... (3)

i.e., $b=52-30=22$

or, $b=22$ .... (4)

and $a=60$ .... (5)

Thus the required state feedback gain matrix is $K=[a \quad b \quad c]$

i.e., $K=[60 \quad 22 \quad 2]$ Ans.

2. a) Describe the advantages of state space analysis over the classical analysis.
b) In what condition all the closed loop poles of a system can be arbitrarily positioned?
c) Consider the following differential equation of a system:

$$\frac{d^3y}{dt^3}+\frac{9d^2y}{dt^2}+\frac{11dy}{dt}+6y(t)=3x(t)$$

Convert it into state space form, and find state feedback gain k, so that the closed loop poles will be located at -3, -4, and -5 respectively. Obtain the closed loop system matrix. **[MODEL QUESTION]**

Answer:

a) Advantages of state space analysis over classical control system are: state space analysis can be very easily used

i) to analyse multi input multi output (MIMO) systems;
ii) to determine controllability and observability of a system from controllability and observability matrix determining their ranks;
iii) to analyse homogenious as well as non-homogenious systems;
iv) to find out state observer;
to find out state transition matrix which yields a good number of information.

b) If any of the poles lie in the right half s-plane, the system with the poles becomes unstable. Hence, the condition for one system to be stable, all the poles need to be in the left half s-plane. So for unstable system, the poles can be arbitrarily placed.

c) Change the given differential equation as $y(t) \to x(t) =$ state variable and $x(t) \to u(t) =$ input variable.

Consider $x(t) = x_1(t)$, $\dot{x}(t) = \dot{x}_1(t) = x_2(t)$; $\dot{x}_2(t) = \ddot{x}_1(t) = x_3(t)$;
$\dot{x}_3(t) = \ddot{x}_2(t) = \dddot{x}_1(t) = \dddot{x}(t)$.

Now the given differential equation becomes.
$\dddot{x} = \dot{x}_3 = -9x_3 - 11x_2 - 6x_1(t) + 3u(t)$; which may be represented as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} u(t),$$

which is comparable to the standard state space form $\dot{X} = AX + Bu$; where

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}; \ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -9 \end{bmatrix}; \ B = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

Consider the feedback matrix $K = [K_1 \quad K_2 \quad K_3]$ for closed loop poles at $-3, -4, -5$.
Now the characteristics equation is:

$$(s+3)(s+4)(s+5) = 0;$$

or, $$(s+3)(s^2+9s+20) = 0;$$

or, $$s^3 + 9s^2 + 20s + 3s^2 + 27s + 60 = 0$$

or, $$s^3 + 12s^2 + 47s + 60 = 0 \qquad \text{.... (1)}$$

Again the characteristic equation with A and K is given as:

$$|sI - A + BK| = 0$$

Now, $sI - A + BK$

$$= \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -9 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}_{3\times1} [K_1 \quad K_2 \quad K_3]_{1\times3}$$

$$=\begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 6 & 11 & (s+9) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3K_1 & 3K_2 & 3K_3 \end{bmatrix} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ (3K_1+6) & (3K_2+11) & (3K_3+s+9) \end{bmatrix}$$

Thus $|sI - A + BK| = 0$ yields

$$s\{s(3K_3+s+9)+1(3K_2+11)\}+(3K_1+6)\cdot 1=0$$

or, $s\{s^2+3K_3s+9s+3K_2+11\}+3K_1+6=0$;

or, $s^3 + s^2(3K_3+9) + s(3K_2+11) + (3K_1+6) = 0$ ------------- (2).

Now comparing coefficients of s^3, s^2, s^1, s^0 in Eqns. (1) and (2), we have

$3K_3 + 9 = 12$ .... (3),

$3K_2 + 11 = 47$ .... (4)

$3K_1 + 6 = 60$ .... (5)

From Eqn. (5), $K_1 = 18$; From Eqn. (4), $K_2 = 12$; From Eqn. (3), $K_3 = 1$.

Thus the state feedback gain $K = [K_1 \quad K_2 \quad K_3] = [18 \quad 12 \quad 1]$ **(Ans.)**

Closed system matrix $= A - BK = s$;

$$=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -9 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}_{3\times1} [18 \quad 12 \quad 1]_{1\times3} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -9 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 54 & 36 & 3 \end{bmatrix}$$

or, $$s = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -60 & -47 & -12 \end{bmatrix}$$ **(Ans.)**

3. a) Determine the controllability and observability of the system **[MODEL QUESTION]**

$$\dot{X} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} X + \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 2 & 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \end{bmatrix} X$$

b) Obtain the solution of the state equation for u(t) = 1 for ≥ 0

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} X + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

$$X(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; Y(t) = [1 \quad 1] X(1)$$

Answer:

a) For the state equation $x = Ax + Bu; \; Y = cx + Du$,

Controllability matrix is

$$S = \left[B : AB : A^2B\right]$$

And the observability matrix is

$$V = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$$

For controllability of the system, which is of order 3, the rank of the controllable matrix (S) should be 3, and for observability of the system, the rank of the observable matrix V should be 3.

Now, in the problem,

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}_{3\times3} ; B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 2 & 1 \end{bmatrix}_{3\times2} ; C = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \end{bmatrix}_{2\times3}$$

$$AB = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}_{3\times3} \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 2 & 1 \end{bmatrix}_{3\times2} = \begin{bmatrix} -1 & 0 \\ 0 & -4 \\ -6 & -3 \end{bmatrix}_{3\times2}$$

$$A^2B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}_{3\times3} \begin{bmatrix} -1 & 0 \\ 0 & -4 \\ -6 & -3 \end{bmatrix}_{3\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 8 \\ 18 & 9 \end{bmatrix}_{3\times2}$$

So the controllability matrix is

$$S = \begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & -4 & 0 & 8 \\ 2 & 1 & -6 & -3 & 18 & 9 \end{bmatrix}_{3\times6}$$

Now since $|S| \neq 0$, the system is controllable.

Again,

$$CA = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \end{bmatrix}_{2\times3} \times \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}_{3\times3}$$

$$\text{or, } CA = \begin{bmatrix} -1 & -2 & -6 \\ -3 & -2 & 0 \end{bmatrix}_{2\times3}$$

$$CA^2 = \begin{bmatrix} -1 & -2 & -6 \\ -3 & -2 & 0 \end{bmatrix}_{2\times3} \times \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}_{3\times3} = \begin{bmatrix} 1 & 4 & 18 \\ 3 & 4 & 0 \end{bmatrix}_{2\times3}$$

Then the observability matrix is

$$V = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ -1 & -2 & -6 \\ -3 & -2 & 0 \\ 1 & 4 & 18 \\ 3 & 4 & 0 \end{bmatrix}_{6\times3}$$

for which $|V| \neq 0$

So the system is observable.

b) In the problem, system matrix $A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$;

So $[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 3 & s+4 \end{bmatrix}$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+4 & 1 \\ -3 & s \end{bmatrix}}{\begin{vmatrix} s & -1 \\ 3 & s+4 \end{vmatrix}} = \frac{1}{s^2+4s+3}\begin{bmatrix} s+4 & 1 \\ -3 & s \end{bmatrix} = \frac{\begin{bmatrix} s+4 & 1 \\ -3 & s \end{bmatrix}}{(s+1)(s+3)}$$

$$X(s) = (sI - A)^{-1} \times (0) + (sI - A)^{-1} Bu(s).$$

$$\text{or, } X(s) = \frac{1}{(s+1)(s+3)}\begin{bmatrix} s+4 & 1 \\ -3 & s \end{bmatrix}_{2\times2}\begin{bmatrix} 0 \\ 1 \end{bmatrix}_{2\times1} + \frac{\begin{bmatrix} s+4 & 1 \\ -3 & s \end{bmatrix}}{(s+1)(s+3)}\begin{bmatrix} 0 \\ 2 \end{bmatrix}\frac{1}{s}$$

$$\text{or, } X(s) = \frac{1}{(s+1)(s+3)}\begin{bmatrix} 1 \\ s \end{bmatrix} + \frac{1.\begin{bmatrix} 2 \\ 2s \end{bmatrix}}{(s+1)(s+3)}.\frac{1}{s}.$$

$$\text{or, } X(s) = \frac{1}{(s+1)(s+3)}\left\{\begin{bmatrix} 1 \\ s \end{bmatrix} + 2.\begin{bmatrix} 1/s \\ 1 \end{bmatrix}\right\}.$$

or, $X(s)=\frac{1}{(s+1)(s+3)}\left\{\begin{bmatrix}1+\frac{2}{s}\\ s+2\end{bmatrix}\right\}.$

or, $X(s)=\begin{bmatrix}\frac{(s+2)}{s(s+1)(s+3)}\\ \frac{(s+2)}{(s+1)(s+3)}\end{bmatrix}$ ....(1)

Again,

$Y(t)=C\times(t)$

i.e.

$Y(s)=C\times(s).=[1\quad 1]_{1\times 2}\begin{bmatrix}\frac{(s+2)}{s(s+1)(s+3)}\\ \frac{(s+2)}{(s+1)(s+3)}\end{bmatrix}_{2\times 1}$

or, $Y(s)=\frac{s+2}{s(s+1)(s+3)}+\frac{s+2}{(s+1)(s+3)}=\frac{s+2+s(s+2)}{s(s+1)(s+3)}$

or, $Y(s)=\frac{s^2+3s+2}{s(s+1)(s+3)}=\frac{\cancel{(s+1)}(s+2)}{s\cancel{(s+1)}(s+3)}=\frac{(s+2)}{s(s+3)}.$

or, $Y(s)=\frac{(s+2)}{s(s+3)};$

Consider,

$\frac{(s+2)}{(s+1)(s+3)}=\frac{A_1}{s+1}+\frac{A_2}{s+3}$

i.e. $s+2=A_1(s+3)+A_2(s+1).$

or, $-1+2=A_1(-1+3)+A_2\times 0.$

or, $1=2A_1; A_1=\frac{1}{2}.$

Again,

$-3+2=A_2; A_2(-3+1)$

or, $+1=+2A_2;\Rightarrow A_2=\frac{1}{2}.$

$$\therefore \frac{s+2}{(s+1)(s+3)} = \frac{1}{2}\left[\frac{1}{s+1} + \frac{1}{s+3}\right]$$

Again consider,

$$\therefore \frac{s+2}{s(s+1)(s+3)} = \frac{B_1}{s} + \frac{B_2}{s+1} + \frac{B_3}{s+3}$$

i.e. $s+2 = B_1(s+1)(s+3) + B_2 s(s+3) + B_3 s(s+1)$

Put s = 0 i.e.

$2 = B_1(1)(3);\ i.e. B_1 = \frac{2}{3}$

Put s = −1,

i.e. $-1+2 = B_2(-1)(-1+3)$

or, $1 = B_2(-1)(2) = -2B_2$

$\therefore B_2 = -\frac{1}{2};$

Put, s = −3;

$-3+2 = B_3(-3)(-3+1) = B_3(-3)(-2) = 6B_3$

or, $-1 = 6B_3;\ i.e. B_3 = -\frac{1}{6}$

$$\therefore \frac{s+2}{s(s+1)(s+3)} = \frac{2/3}{s} - \frac{1/2}{(s+1)} - \frac{1/6}{(s+3)}$$

Consider,

$$\therefore \frac{s+2}{s(s+3)} = \frac{A_1}{s} + \frac{A_2}{s+3}$$

i.e. $s+2 = A_1(s+3) + A_2 s$

$2 = 3A;$ Put s = 0

i.e. $A_1 = 2/3;$

$-1 = -3A_2$ Put s = −3

$A_2 = 1/3;$

$$\therefore Y(s) = \frac{2/3}{s} + \frac{1/3}{s+3};\ \therefore Y(t) = \frac{2}{3} + \frac{1}{3}e^{-3t}$$

So the solution is

$y(t) = \frac{2}{3} + \frac{1}{3}e^{-3t}$ (Ans.)

From equation (1) by inverse Laplace

$x_1(t)$ and $x_2(t)$ can be evaluated.

State space vector in s domain is thus give by the relation:

$$\begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = \begin{bmatrix} \left(\frac{2}{3}\right)s - \frac{\frac{1}{2}}{(s+1)} - \frac{\frac{1}{6}}{s+3} \\ \frac{1}{2}\left(\frac{1}{s+1} + \frac{1}{s+3}\right) \end{bmatrix}.$$

Taking Laplace inverse of the above matrix equation, the state vector in time domain is given by the following relation:

$$\text{or, } \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \frac{2}{3} - \frac{1}{2}e^{-t} - \frac{1}{6}e^{-3t} \\ \frac{1}{2}\left(e^{-t} + e^{-3t}\right) \end{bmatrix}. \qquad \text{(Ans.)}$$

4. a) Show that the arbitrary pole placement of a liner state feedback system is possible if the system is completed controllable.

b) Determine the state feedback gain matrix so that the closed loop poles of the following system are located at $(-2+j4)$, $(-2-j4)$, -10

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ -1 & -5 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

[MODELQUESTION]

Answer:

a) Consider the state space representation of an nth order system is

$$\dot{x} = Ax + Bu \qquad y = Cx + Du \quad \ldots\ldots\ldots\ldots(1)$$

With appropriate gain matrix K maintaining the controllability of the system, for having the system poles at desired locations, consider desired closed loop poles be at $s = \sigma_1, \sigma_2, \ldots, \sigma_n$. We choose the control signal $u = -Kx$........(2)

Now from Eqns. (1) & (2) we've

$\dot{x} = Ax - BKx = (A - BK)x$

or $\dot{x} = (A - BK)x$..........(3)

The solution of the above Eqns. is $x = e^{(A-BK)t}x(0)$...........(4), $x(0)$ being the initial state caused by an external disturbance.

Suppose that the system representing Eqn. (1) is not completely state controllable, then the rank of the controllability matrix $\left[B \;\; AB \ldots\ldots A^{n-1}B\right] = q < n$

This means that these are 'q' linearity independent column vectors in the controllability matrix. Let us define such 'q' linearity independent column vectors as $f_1, f_2, \ldots f_q$.

Also let us choose $(n-q)$ additional vectors $V_{q+1}, V_{q+2}, \ldots V_n$ such that $P = \left[f_1 f_2 \ldots f_q, V_{q+1}, V_{q+2} \ldots V_n \right]$ is of rank n.

Now it can be shown that $\hat{A} = P^{-1}AP = \begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix}$, $\hat{B} = P^{-1}B = \begin{bmatrix} B_{11} \\ 0 \end{bmatrix}$

define $\hat{K} = KP = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$

Then we've the characteristic polynomial is

$$|sI - A - BK| = |sI - A + BK| = \left|P^{-1}(sI - A + BK)P\right|$$

$$= \left|sI - \hat{A} + \hat{B}\hat{K}\right| = \left|sI - \hat{A} + \hat{B}\hat{K}\right|$$

$$= \left| sI - \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} + \begin{bmatrix} B_{11} \\ --- \\ 0 \end{bmatrix} \begin{bmatrix} K_1 & K_2 \end{bmatrix} \right| = \begin{vmatrix} sI_q - A_{11} + B_{11}K_1 & -A_{12} + B_{12}K_2 \\ 0 & sI_{n-q}A_{22} \end{vmatrix}$$

$$= \left|sI_q - A_{11} + B_{11}K_1\right| . \left|sI_{n-q}A_{22}\right| = 0$$

where I_q is a q-dimensional identity matrix and I_{n-q} is $(n-q)$ dimensional identity matrix. Notice that the eigen values of A_{22} do not depend on K. Thus if the system is not completely state controllable, then there are eigen values of matrix A that cannot be arbitrary placed. Therefore, to place the eigen values of matrix (A–BK) arbitrarily, the system must be completely state controllable.

b) Define the desired state feedback gain matrix K as $K = \left[K_1\ K_2\ K_3\right]$.

Now for the specific pole placement at pole $(-2 + j4), (-2 - j4), -10$, the characteristic Eqn. becomes

$$(s+2-j4)(s+2+j4)(s+10) = 0$$

or, $\left[(s+2)^2 + 16\right](s+10) = 0$

or, $(s^2+4s+20)(s+10) = 0$

or, $s^3 + 4s^2 + 20s + 10s^2 + 40s + 200 = 0$

or, $s^3 + 14s^2 + 60s + 200 = 0$ ....(1)

Again for the system with system matrix A, and the B matrix as,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The characteristic Eqn. should be $|sI - A + BK| = 0$

or, $$\left|\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times 1} [K_1 K_2 K_3]_{1\times 3}\right| = 0$$

i.e., $$\left|\begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 5 & s+6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ K_1 & K_2 & K_3 \end{bmatrix}\right| = 0$$

i.e., $$\begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1+K_1 & s+K_2 & s+6+K_3 \end{vmatrix} = 0$$

or, $s^3 + (6+K_3)s^2 + (5+K_2)s + (1+K_1) = 0$ ..... (2).

Comparing the coefficients of equal powers of s in Eqns. (1) and (2) we have,

$6+K_3 = 14$; $5+K_2 = 60$; $1+K_1 = 200$.

i.e. $K_3 = 8$, $K_2 = 55$, $K_1 = 199$.

Hence the feedback gain matrix is $K - [K_1 \quad K_2 \quad K_3] = [199 \quad 55 \quad 8]$ Ans.

5. a) For the system represented by **[MODEL QUESTION]**

$$\dot{X} = \begin{bmatrix} -7 & 1 \\ -12 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ -1 \end{bmatrix} u$$

$$Y = [3 \quad -4] x + [2] u$$

Compute output response when $u(t) = 3e^{-t}$ **and** $X[0] = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$.

Answer:

We have the system,

$\dot{X} = AX + Bu$; $Y = CX + Du$;

where, $A = \begin{bmatrix} -7 & 1 \\ -12 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$; $C = [3 \quad -4]$, $D = [2]$

Now taking Laplace transformation in the state equation:

$$L[\dot{X}] = L[AX + Bu]$$

or, $$sX(s) - x(0) = AX(s) + Bu(s)$$

or, $$X(s)[(sI - A)] = x(0) + Bu(s)$$

i.e., $$X(s) = (sI - A)^{-1} x(0) + (sI - A)^{-1} Bu(s) \quad \cdots(1)$$

Now, $sI-A=\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}-\begin{bmatrix} -7 & 1 \\ -12 & 0 \end{bmatrix}=\begin{bmatrix} s+7 & -1 \\ -12 & s \end{bmatrix}$

$\therefore (sI-A)^{-1}=\dfrac{1}{s(s+7)+12}\begin{bmatrix} s & 1 \\ -12 & s+7 \end{bmatrix}$

or, $[sI-A]^{-1}=\dfrac{1}{(s+3)(s+4)}\begin{bmatrix} s & 1 \\ -12 & s+7 \end{bmatrix}$

then using equation (1)

$$X(s)=\frac{1}{(s+3)(s+4)}\begin{bmatrix} s & 1 \\ -12 & s+7 \end{bmatrix}_{2\times2}\begin{bmatrix} -6 \\ 1 \end{bmatrix}_{2\times1}+\frac{\begin{bmatrix} s & 1 \\ -12 & s+7 \end{bmatrix}}{(s+3)(s+4)}\begin{bmatrix} 2 \\ -1 \end{bmatrix}\left(\frac{3}{s+1}\right)$$

$$=\frac{1}{(s+3)(s+4)}\begin{bmatrix} -6s+1 \\ 72+s+7 \end{bmatrix}+\frac{1}{(s+3)(s+4)}\begin{bmatrix} 2s-1 \\ -24-s-7 \end{bmatrix}\left(\frac{3}{s+1}\right)$$

$$=\frac{1}{(s+3)(s+4)}\begin{bmatrix} -6s+1+\dfrac{3(2s-1)}{s+1} \\ 79+s-\dfrac{3(31+s)}{s+1} \end{bmatrix}$$

or, $$X(s)=\frac{1}{(s+3)(s+4)}\begin{bmatrix} \dfrac{(1-6s)(s+1)+(2s-1).3}{(s+1)} \\ \dfrac{(79+s)(s+1)-(31+s).3}{(s+1)} \end{bmatrix}$$

$$=\frac{1}{(s+3)(s+4)}\begin{bmatrix} \dfrac{s+1-6s^2-6s+6s-3}{s+1} \\ \dfrac{79s+79+s^2+s-93-35}{s+1} \end{bmatrix}=\begin{bmatrix} \dfrac{s-2-6s^2}{(s+1)(s+3)(s+4)} \\ \dfrac{77s+s^2-14}{(s+1)(s+3)(s+4)} \end{bmatrix}$$

or, $$X(s)=\begin{bmatrix} \dfrac{-(6s^2-s+2)}{(s+1)(s+3)(s+4)} \\ \dfrac{s^2+77s-14}{(s+1)(s+3)(s+4)} \end{bmatrix} \quad \cdots(2)$$

Again for output state space equation,

$Y=CX+Du$

i.e., $X(s)=CX(s)+Du(s)$

$$\text{or, } Y(s)=\begin{bmatrix}3 & -4\end{bmatrix}_{1\times 2}\begin{bmatrix}\dfrac{-(6s^2-s+2)}{(s+1)(s+3)(s+4)}\\ \dfrac{s^2+77s-14}{(s+1)(s+3)(s+4)}\end{bmatrix}+\frac{2.3}{s+1}$$

$$\text{or, } Y(s)=\frac{-3(6s^2-s+2)}{(s+1)(s+3)(s+4)}-\frac{4(s^2+77s-14)}{(s+1)(s+3)(s+4)}+\frac{6}{s+1}$$

$$\text{or, } Y(s)=\frac{-18s^2+3s-6-4s^2-308s+56+6(s^2+7s+12)}{(s+1)(s+3)(s+4)}=\frac{-16s^2-263s+122}{(s+1)(s+3)(s+4)}$$

$$Y(s)=\frac{-16s^2-263s+122}{(s+1)(s+3)(s+4)}$$

$$\text{Now, } \frac{-16s^2-263s+122}{(s+1)(s+3)(s+4)}=\frac{A_1}{(s+1)}+\frac{A_2}{(s+2)}+\frac{A_3}{(s+4)}$$

$$\text{i.e., } -16s^2-263s+122=A_1(s+3)(s+4)+A_2(s+1)(s+4)+A_3(s+1)(s+3)$$

Put $s=1$, i.e., $-16+263+122=A_1(-1+3)(-1+4)$

or, $369=A_1(2)(3)=6A_1$

$$\therefore A_1=\frac{369}{6}$$

or, $A_1=61.5$

Again, put $s=-3$

i.e., $-16\times 9+263\times 3+122=A_2(-3+1)(-3+4)$

or, $-144+789+122=A_2(-2)(1)=-2A_2$

or, $-\dfrac{767}{2}=A_2$

$\therefore A_2=-383.5$

Again, put $s=-4$

i.e., $-16\times 16+263\times 4+122=A_3(-4+1)(-4+3)=A_3(-3)(-1)$

or, $A_3=\dfrac{1}{3}[-256+122+1052]$

or, $A_3=306$

$$\therefore Y(s)=\frac{61.5}{s+1}-\frac{383.5}{s+3}+\frac{306}{s+4}$$

So the output response is taking Laplace inverse transform

$$y(t)=61.5e^{-t}-383.5e^{-3t}+306e^{-4t}$$

b) Determine the state feedback gain matrix so that the closed loop poles of the following system are located at $-2\pm j3.464, -5$. Give a block diagram of the control configuration.

$$\dot{X}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}x+\begin{bmatrix}0\\ 0\\ -1\end{bmatrix}u$$

[MODEL QUESTION]

$$Y=\begin{bmatrix}1 & 0 & 0\end{bmatrix}x.$$

Answer:

With the closed loop poles at $-2+j3.464, -2-j3.464, -5$, the characteristic equation of the system becomes:

$$(s+2-j3.464)(s+2+j3.464)(s+5)=0$$

or, $\left[(s+2)^2+3.464^2\right](s+5)=0$

or, $(s^2+4s+4+12)(s+5)=0$

or, $(s^2+4s+4+16)(s+5)=0$

or, $s^3+4s^2+16s+5s^2+20s+80=0$

or, $s^3+9s^2+36s+80=0$ $\quad\cdots(1)$

Now suppose the desired state feedback matrix be $k=\begin{bmatrix}a & b & c\end{bmatrix}$. With the desired state feedback matrix the characteristic equation is:

$$|sI-A+Bk|=0$$

where $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}$; $B=\begin{bmatrix}0\\ 0\\ -1\end{bmatrix}$; $I=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$;

so that $SI=\begin{bmatrix}s & 0 & 0\\ 0 & s & 0\\ 0 & 0 & s\end{bmatrix}$

Now, $SI-A=\begin{bmatrix}s & 0 & 0\\ 0 & s & 0\\ 0 & 0 & s\end{bmatrix}-\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}=\begin{bmatrix}s & -1 & 0\\ 0 & s & -1\\ 6 & 11 & s+6\end{bmatrix}$

$$Bk=\begin{bmatrix}0\\ 0\\ -1\end{bmatrix}_{3\times1}\begin{bmatrix}a & b & c\end{bmatrix}_{1\times3}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ -a & -b & -c\end{bmatrix}$$

$$\therefore |sI - A + Bk| = \left\{\begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 6 & 11 & s+6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -a & -b & -c \end{bmatrix}\right\}$$

$$= \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 6-a & 11-b & s+6-c \end{bmatrix} = s\left[s(s+6-c)+11-b\right]+1\left[(6-a)\right]$$

$$= s\left[s^2+6s-cs+11-b\right]+6-a = s^3+s^2(6-c)-s(11-b)+(6-a)$$

So with the state feedback the characteristic equation is

$s^3+s^2(6-c)+s(b-11)+6-a=0$ .... (2)

Now equating coefficients of s^3, s^2, s^1, s^0 in equations (1) and (2) we have

$6-c=9$; i.e. $c=6-9$

$\therefore c=-3.$

$b-11=36$; i.e. $b=47$ & $6-a=80$

or, $a=6-80=-74$

$\therefore a=-74, b=47, c=-3$

So the state feedback gain matrix is $k=[-74 \quad 47 \quad -3]$ Ans.

Control Configuration:

Consider the system's control signals, $k=-kx$;

Now, $\dot{x}=Ax+Bk=Ax-Bkx$

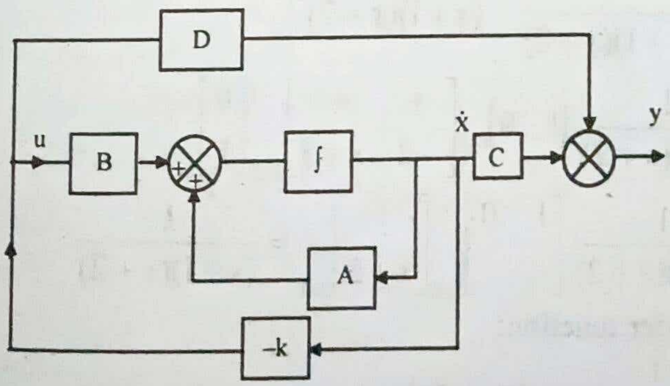
or, $\dot{x}=(A-Bk)x$

Now the state equations are:

$\dot{x}=Ax+Bk$

$y=cx+Dk$

So the configuration is as shown below:



6. A system is characterized by the following state equation **[MODELQUESTION]**

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-3 & 1\\ -2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u \qquad y=[1\ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

a) Find the transfer function of the system.

Answer:

For the state space system, $\dot{x} = Ax + Bu, y = cx$

$$y(s) = c\,x(s)$$

$$\&\quad sx(s) = Ax(s) + Bu(s).$$

$$or, (s\ I\ A)x(s) = Bu(s)$$

$$or,\ x(s) = (sI - A)^{-1}Bu(s).$$

Since, $y(s) = c\ x(s) = c(sI - A)^{-1}Bu(s)$

So transfer function $G(s) = y(s)u^{-1}(s) = c(sI - A)^{-1}B$

$$or,\ G(s) = [1\quad 0]\begin{bmatrix}s+3 & -1\\ 2 & s\end{bmatrix}^{-1}\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$\left[sI - A = \begin{pmatrix}s & 0\\ 0 & s\end{pmatrix} - \begin{pmatrix}-3 & 1\\ -2 & 0\end{pmatrix} = \begin{pmatrix}s+3 & -1\\ 2 & s\end{pmatrix}\right]$$

Now, $|(sI - A)| = \begin{vmatrix}s+3 & -1\\ 2 & s\end{vmatrix} = s(s+3) + 2 = s^2 + 3s + 2 = (s+1)(s+2)$

Now, $Adj\begin{bmatrix}sI - A\end{bmatrix} = \begin{bmatrix}s & -2\\ 1 & s+3\end{bmatrix}^T = \begin{bmatrix}s & 1\\ -2 & s+3\end{bmatrix}$

$$\therefore (sI - A)^{-1} = \begin{bmatrix}\dfrac{s}{(s+1)(s+2)} & \dfrac{1}{(s+1)(s+2)}\\ -\dfrac{2}{(s+1)(s+2)} & \dfrac{s+3}{(s+1)(s+2)}\end{bmatrix}$$

Thus, $G(s) = \dfrac{1}{(s+1)(s+2)}[1\quad 0]_{1\times 2}\begin{bmatrix}s & +1\\ -2 & s+3\end{bmatrix}_{2\times 2}\begin{bmatrix}0\\ 1\end{bmatrix}_{2\times 1}$

$$G(s) = \frac{1}{(s+1)(s+2)}\begin{bmatrix}1 & 0\end{bmatrix}_{1\times 2}\begin{bmatrix}1\\ s+3\end{bmatrix}_{2\times 1} = \frac{1}{(s+1)(s+2)}$$

So the required transfer function:

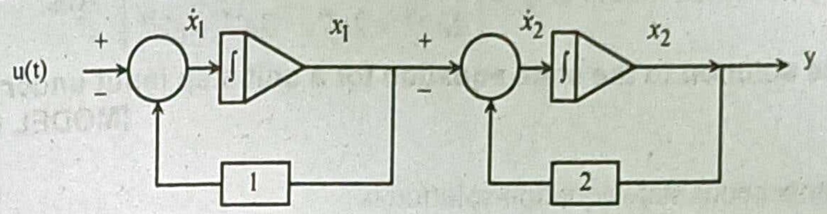
$$G(s) = \frac{1}{(s+1)(s+2)}\quad \text{Ans. (a)}$$

b) Draw the block Diagram of the above transfer function. **[MODEL QUESTION]**

Answer:

$$G(s)=\frac{1}{(s+1)(s+2)}=\frac{1}{s+1}\cdot\frac{1}{s+2}$$

So the required block diagram is



c) Compute the state transition matrix. **[MODEL QUESTION]**

Answer:

State transition matrix is

$$L^{-1}\left[(sI-A)^{-1}\right]=\begin{bmatrix} L^{-1}\dfrac{s}{(s+1)(s+2)} & L^{-1}\dfrac{1}{(s+1)(s+2)} \\ L^{-1}\dfrac{s}{(s+1)(s+2)} & L^{-1}\dfrac{s+3}{(s+1)(s+2)} \end{bmatrix}=\begin{bmatrix}\varphi_{11} & \varphi_{12}\\ \varphi_{21} & \phi_{22}\end{bmatrix}$$

Now, $\dfrac{s}{(s+1)(s+2)}=\dfrac{B_1}{S+1}+\dfrac{B_2}{s+2}$; $s=B_1(s+2)+B_2(s+1)$

Putting $s=-2,\ -2=B_2(-1)$; $B_2=2$,

Putting $s=-1,\ -1=B_1(1)$; $B_1=-1$;

Thus $\dfrac{s}{(s+1)(s+2)}=-\dfrac{1}{s+1}+\dfrac{2}{s+2}$

$$\therefore L^{-1}\left[\frac{s}{(s+1)(s+2)}\right]=\left(-e^{-t}+2e^{-2t}.\right)=\varphi_{11}$$

Again, $\dfrac{s+3}{(s+1)(s+2)}=\dfrac{A_1}{s+2}+\dfrac{A_2}{s+2}$; $s+3=A_1(s+2)+A_2(s+1)$

Putting, $s=-2,\ 1=(-1)A_2$; *if* $A_2=(-)1$,

Putting $s=-1,2=A_1$; $A_1=2$

i.e. $\dfrac{s+3}{(s+1)(s+2)}=\dfrac{2}{s+1}-\dfrac{1}{s+2}$

$$\therefore L^{-1}\left[\frac{s+3}{(s+1)(s+2)}\right]=\left[2e^{-t}-e^{-2t}\right]=\varphi_{22}$$

Again, $L^{-1}\left[\dfrac{1}{(s+1)(s+2)}\right]=L^{-1}\left[\dfrac{1}{s+1}-\dfrac{1}{s+2}\right]=e^{-t}-e^{-2t}=\varphi_{12}$

& $$L^{-1}\left[-\frac{2}{(s+1)(s+2)}\right]=(-)2e^{-t}+2e^{-2t}=\varphi_{21};$$

Thus the state transition matrix is $\varphi=\begin{bmatrix}(2e^{-2t}-e^{-t}) & (e^{-t}-e^{-2t})\\ 2e^{-2t}-2e^{-t} & 2e^{-t}-e^{-2t}\end{bmatrix}$ Ans.

d) Obtain the solution to the state equation for a unit step input under zero initial conditions. **[MODEL QUESTION]**

Answer:

For non-homogeneous state equation solution is

$$x(t)=\varphi(t)\,x(0)+\int_0^t\varphi(t-\tau)Bu(\tau)d\tau=\begin{bmatrix}\varphi_{11} & \varphi_{12}\\ \varphi_{21} & \varphi_{22}\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}+\int_0^t\begin{bmatrix}\varphi_{11}(t-\tau) & \varphi_{12}(t-\tau)\\ \varphi_{21}(t-\tau) & \varphi_{22}(t-\tau)\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\cdot 1d\tau$$

$$=\int_0^t\begin{bmatrix}\varphi_{12}(t-\tau)\\ \varphi_{22}(t-\tau)\end{bmatrix}d\tau=\int_0^t\begin{Bmatrix}e^{-(t-\tau)}-e^{-2(t-\tau)}\\ 2e^{-(t-\tau)}-e^{-2(t-\tau)}\end{Bmatrix}d\tau$$

$$=\begin{bmatrix}e^{-t\int_0^t\tau d\tau} & -e^{-2t\int_0^t e^{2\tau}d\tau}\\ 2e^{-t\int_0^t e^{\tau}d\tau} & -e^{-2t\int_0^t e^{2\tau}d\tau}\end{bmatrix}=\begin{bmatrix}e^{-t}.e^{\tau}\Big|_0^t-\frac{e^{-2t}}{2}\cdot e^{2\tau}\Big|_0^t\\ 2e^{-t}\cdot e^{\tau}\Big|_0^t-\frac{e^{-2t}}{2}\cdot e^{2\tau}\Big|_0^t\end{bmatrix}$$

$$=\begin{bmatrix}e^{-t}(e^{t}-1) & -\frac{e^{-2t}}{2}(e^{2t}-1)\\ 2e^{-t}.(e^{t}-1) & -\frac{e^{-2t}}{2}(e^{2t}-1)\end{bmatrix}=\begin{bmatrix}(1-e^{-t})+\frac{1}{2}(-1+e^{-2t})\\ 2(1-e^{-t})+\frac{1}{2}(-1+e^{-2t})\end{bmatrix}$$

$$=\begin{bmatrix}1-\frac{1}{2}-e^{-t}+\frac{1}{2}e^{-2t}\\ 2-2e^{-t}-\frac{1}{2}+\frac{1}{2}e^{-2t}\end{bmatrix}$$

or, $$\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}\frac{1}{2}+\frac{1}{2}e^{-2t}-\frac{1}{2}e^{-t}\\ \frac{3}{2}+\frac{1}{2}e^{-2t}-2e^{-t}\end{bmatrix}$$ Ans.

7. Consider the system defined by $\dot{X}=AX+BU$, where **[MODELQUESTION]**

$$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -30 & -11\end{bmatrix},\ B=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

By using feedback control $U=-Kx$, it is desired to have closed loop poles at $S=-2,-5$ and -6. Determine the state feedback gain matrix *K*.

Answer:

Consider the feedback gain matrix $k = [k_1 \quad k_2 \quad k_3]$. For the specific poles at $-2, -5, -6$ the characteristic equation becomes

$(s+2)(s+5)(s+6) = 0$

$or, (s^2+7s+10)(s+6) = 0; or (s+6)(s^2+7s+10) = 0$

$or, s^3+7s^2+10s+6s^2+42s+60 = 0$

$or, s^3+13s^2+52s+60 = 0$ .... (1)

For the system with given A and B, the characteristic equation is

$|s - A + Bk| = 0$;

$$or, \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -30 & -11 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times1} [k_1 \quad k_2 \quad k_3]_{1\times3} = 0$$

$$or, \left| \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 30 & s+11 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k_1 & k_2 & k_3 \end{bmatrix} \right| = 0$$

$$or, \left| \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ k_1 & (30+k_2) & (s+11+k_3) \end{bmatrix} \right|$$

$$= 0; or, s \begin{vmatrix} s & -1 \\ (30+k_2) & (s+11+k_3) \end{vmatrix} + \begin{vmatrix} 0 & -1 \\ k_1 & (s+11+k_2) \end{vmatrix} = 0$$

$or, s[s^2+11s+k_3 s+30+k_2]+k_1 = 0$

$or, s^3+s^2(11+k_3)+s(30+k_2)+k_1 = 0$ .... (2)

Now, comparing coefficients of s^3, s^2, s^0 of Eqns. (1) & (2)

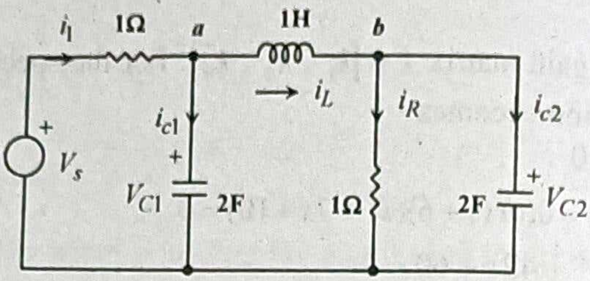
We have, $11+k_3 = 13; \Rightarrow k_3 = 2$

$30+k_2 = 52; \Rightarrow k_2 = 22.$

$k_1 = 60$

So the feedback gain matrix is $[k_1 \quad k_2 \quad k_3] = [60 \quad 22 \quad 2]$ Ans.

8. a) Write the state equation for the circuit below: **[MODEL QUESTION]**



Answer:

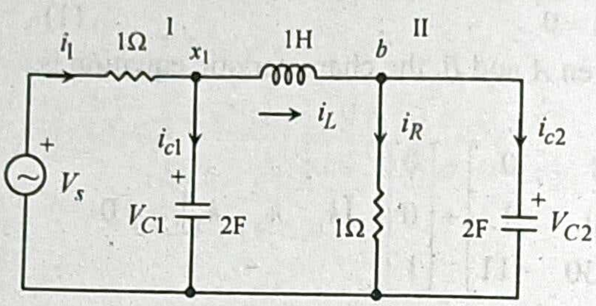


Fig: 1 $\quad C_1 = C_2 = 2F$

Consider KCL at nodes I and II where the voltages are considered as $x_1(=V_{C1})$ and $x_2(=V_{C2})$ respectively, x_1, x_2 being states.

Thus at node I by KCL

$$\frac{V_s - x_1}{1} = i_L - C_1\frac{dx_1}{dt}$$

or, $$\frac{dx_1}{dt} = \frac{1}{C_1}[x_1 + i_L - V_s] = \frac{1}{2}[x_1 + i_L - V_s]$$

i.e., $$\frac{dx_1}{dt} = \frac{1}{2}x_1 + \frac{1}{2}i_L - \frac{1}{2}V_s \quad \text{.... (1)}$$

Again by KCL at node II, we have,

$$i_L = \frac{x_2}{1} - C_2\frac{dx_2}{dt}$$

or, $$\frac{dx_2}{dt} = \frac{1}{C_2}[x_2 - i_L] = \frac{1}{2}[x_2 - i_L]$$

i.e., $$\frac{dx_2}{dt} = \frac{1}{2}x_2 - \frac{1}{2}i_L \quad \text{.... (2)}$$

Eqns. (1) and (2) can be represented in state space form as:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\frac{1}{2} & -\frac{1}{2}\\ -\frac{1}{2} & 0\end{bmatrix}\begin{bmatrix}i_L\\ V_s\end{bmatrix};$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}.$$

b) Determine the state feedback gain matrix, so that closed loop poles of the following linear system are located at –2, –5 and –6.

$$\dot{x}=\begin{bmatrix}0&1&0\\0&0&1\\0&-30&-11\end{bmatrix}x+\begin{bmatrix}0\\0\\1\end{bmatrix}u(t),\ y=\begin{bmatrix}1&0&0\end{bmatrix}x$$ **[MODEL QUESTION]**

Answer:

Let us define the state feedback gain matrix as $K=\begin{bmatrix}K_1 & K_2 & K_3\end{bmatrix}$

Now for the specific pole placement at poles $-2,-5,-6$ the characteristic equation becomes $(s+2)(s+5)(s+6)=0$

or, $(s+6)(s^2+7s+10)=0$;

or, $s^3+13s^2+52s+60=0$ .... (1)

Now for the given system with system matrix A and input matrix B as

$$A=\begin{bmatrix}0&1&0\\0&0&1\\0&-30&-11\end{bmatrix};\ B=\begin{bmatrix}0\\0\\1\end{bmatrix};$$

The characteristic equation becomes $|sI-A+BK|=0$

i.e., $$\left|\begin{bmatrix}s&0&0\\0&s&0\\0&0&s\end{bmatrix}-\begin{bmatrix}0&1&0\\0&0&1\\0&-30&-11\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}K_1&K_2&K_3\end{bmatrix}\right|=0$$

or, $$\left|\begin{bmatrix}s&-1&0\\0&s&-1\\0&30&s+11\end{bmatrix}+\begin{bmatrix}0&0&0\\0&0&0\\K_1&K_2&K_3\end{bmatrix}\right|=0$$

or, $$\begin{vmatrix}s&-1&0\\0&s&-1\\K_1&30+K_2&(s+11+K_3)\end{vmatrix}=0$$

or, $s\left[s(s+11+K_3)+(30+K_2)\right]+K_1=0$

or, $s\left[s^2+(11+K_3)s+(30+K_2)\right]+K_1=0$

or, $s^3+(11+K_3)s^2+(30+K_2)s+K_1=0$ .... (2)

Now comparing coefficients of s^2, s^1, s^0 in Eqns. (1) & (2), we have,

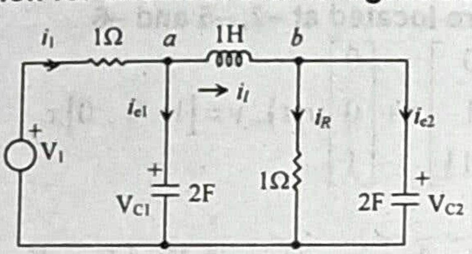
$11+K_3=13;\quad\Rightarrow\quad K_3=13-11=2$; i.e., $K_3=2$;

$30+K_2=52;\quad\Rightarrow\quad K_2=52-30=22$

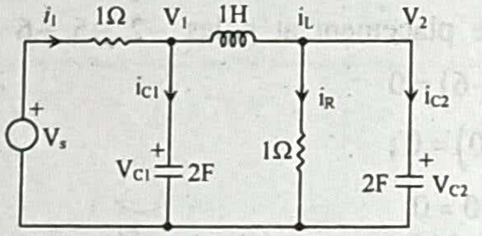
$K_1=60$

Hence, the required state feedback gain matrix is $K=\begin{bmatrix}60 & 22 & 2\end{bmatrix}$ (Ans.)

9. a) Write the state equation for circuit shown in figure below: **[MODEL QUESTION]**



Answer:



We have, $V_s = i_1 \times 1 + V_{C1} = i + V_1$; $V_1 = \frac{Ldi_L}{dt} + V_2 = \frac{di_L}{dt} + V_2$

$$\frac{dV_{C1}}{dt} = \frac{1}{C}\cdot i_{C1} = \frac{1}{2}\cdot i_{C1};\ \frac{dV_{C2}}{dt} = \frac{1}{2} i_{C2};\ V_{C2} = V_2 = i_R \times 1 = i_R;$$

$\therefore$ $V_2 = i_R$

$\therefore$ $\frac{di_L}{dt} = V_1 - V_2;$

Thus: $\frac{dV_1}{dt} = \frac{1}{2} i_{C1} = \frac{1}{2}[i_1 - i_L] = \frac{1}{2}[V_s - V_1 - i_L] = -\frac{1}{2}V_1 - \frac{1}{2}i_L + \frac{1}{2}V_s$

$\frac{dV_2}{dt} = \frac{1}{2} i_{C2} = \frac{1}{2}[i_L - i_R] = \frac{1}{2}[i_L - V_2] = \frac{1}{2}i_L - \frac{1}{2}V_2$ and $\frac{di_L}{dt} = V_1 - V_2$.

Hence the state equation is

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & -\frac{1}{2} \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ 0 \\ 0 \end{bmatrix} u \quad \textbf{Ans.}$$

b) Determine the state feedback gain matrix so that close loop poles of the following linear system are located at –2, –5 & –6.

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -30 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t),\ y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

[MODEL QUESTION]

Answer:

Consider the feedback matrix $K=[k_1 \quad k_2 \quad k_3]$

For poles at –2, –5, –6, the characteristic equation is $(s+2)(s+5)(s+6)=0$ i.e.,

$s^3+13s^2+52s+60=0$ .... (1)

For the system with $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -30 & -11\end{bmatrix}, B=\begin{bmatrix}0\\0\\1\end{bmatrix}$

The characteristic equation is $|sI+A+Bk|=0$

$$\Rightarrow \left|\begin{bmatrix}s & 0 & 0\\ 0 & s & 0\\ 0 & 0 & s\end{bmatrix}-\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -30 & -11\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}[k_1 \quad k_2 \quad k_3]\right|=0;$$

$$\Rightarrow \left|\begin{bmatrix}s & -1 & 0\\ 0 & s & -1\\ 0 & 30 & s+11\end{bmatrix}+\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ k_1 & k_2 & k_3\end{bmatrix}\right|=0$$

$$\Rightarrow \quad s^3+s^2(11+k_3)+s(30+k_2)+k_1=0 \qquad \text{.... (2)}$$

Now, comparing the coefficients of s in Eqns. (1) and (2), we have,

$k_1=60;\ 3+k_2=52;\ k_2=22;\ 11+k_3=13;$ i.e., $k_3=2$

Thus the state feedback gain matrix is $[60, 22, 2]$ **Ans.**

10. a) Consider a system defined by **[MODEL QUESTION]**

$$\dot{X}=AX+BU, \text{ where } A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1 & -5 & -6\end{bmatrix} \ \& \ B=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

Using state feedback control $U=-KX$, it is desired to have closed loop poles at $s=2\pm j4$, $s=-10$. Determine the state feedback gain matrix K.

Answer:

Consider the feedback matrix $K=(K_1 \quad K_2 \quad K_3)$.

Now the closed loop poles are at $s=2+4j$, $s=2-4j$, $s=-10$. So the characteristics equation is $(s-2-4j)(s-2+4j)(s+10)=0$.

or, $(s+10)\left[s^2-2s+4js-2s+4-8j-4js+8j+16\right]=0$

or, $(s+10)\left[s^2-4s+20\right]=0$

or, $s^3-4s^2+20s+10s^2-40s+200=0$

or, $s^3+6s^2-20s+200=0$ .... (1)

Again for the system with $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1 & -5 & -6\end{bmatrix}$ and $B=\begin{bmatrix}0\\0\\1\end{bmatrix}$

The characteristics equation is $|sI-A+BK|=0$

$$sI=\begin{bmatrix}s & 0 & 0\\ 0 & s & 0\\ 0 & 0 & s\end{bmatrix};$$

So, $$sI-A=\begin{bmatrix}s & 0 & 0\\ 0 & s & 0\\ 0 & 0 & s\end{bmatrix}-\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1 & -5 & -6\end{bmatrix}$$

or, $$sI-A=\begin{bmatrix}s & -1 & 0\\ 0 & s & -1\\ 1 & 5 & s+6\end{bmatrix};$$

and $$BK=\begin{bmatrix}0\\0\\1\end{bmatrix}_{3\times1}\begin{bmatrix}K_1 & K_2 & K_3\end{bmatrix}_{1\times3}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ K_1 & K_2 & K_3\end{bmatrix}_{3\times3}$$

Thus, $$sI-A-BK=\begin{bmatrix}s & -1 & 0\\ 0 & s & -1\\ 1 & 5 & s+6\end{bmatrix}-\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ K_1 & K_2 & K_3\end{bmatrix}$$

$$=\begin{bmatrix}s & -1 & 0\\ 0 & s & -1\\ 1-K_1 & 5-K_2 & (s+6-K_3)\end{bmatrix}$$

So, $|sI-A-BK|=0$;

$$\Rightarrow \begin{vmatrix}s & -1 & 0\\ 0 & s & -1\\ (1-K_1) & (5-K_2) & (s+6-K_3)\end{vmatrix}=0$$ is the characteristic equation.

or, $s\left[s(s+6-K_3)+5-K_2\right]+(1-K_1)[1]=0$

or, $s(s^2+6s-sK_3+5-K_2)+(1-K_1)=0$

or, $s^3+6s^2-s^2K_3+5s-sK_2+1-K_1=0$

or, $s^3+s^2(6-K_3)+s(5-K_2)+1-K_1=0$ .... (2)

Now comparing the coefficients of s^2, s^1, s^0 of Eqns. (1) and (2), as both represent the characteristic equations of the system, we have,

$6-K_3=6$ .... (a)

$5-K_2=-20$ .... (b)

$1-K_1=200$ .... (c)

Solving Eqns. (a), (b) and (c), we have, $K_1=-199,\ K_2=25,\ K_3=0$.

So the state feedback matrix is $K=\begin{bmatrix}-199 & 25 & 0\end{bmatrix}$ **(Ans.)**

b) A system is described by the state equation $\dot{X}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}X+\begin{bmatrix}0\\1\end{bmatrix}U$ & $Y=\begin{bmatrix}1 & 0\end{bmatrix}X$, where $X(0)=\begin{bmatrix}0 & 0\end{bmatrix}^T$, the input to the system is unit step. Find out the state solution of the above system. **[MODEL QUESTION]**

Answer:

For the system in the present problem, the system matrix is $A=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}$;

So the state transition matrix (STM) is $=L^{-1}\left[(sI-A)^{-1}\right]=\begin{bmatrix}\phi_{11}(t) & \phi_{12}(t)\\ \phi_{21}(t) & \phi_{22}(t)\end{bmatrix}$

Now, $sI-A=\begin{pmatrix}s & 0\\ 0 & s\end{pmatrix}-\begin{pmatrix}0 & 1\\ -2 & -3\end{pmatrix}=\begin{pmatrix}s & -1\\ 2 & s+3\end{pmatrix}$

$$\therefore STM=L^{-1}\left\{\frac{\begin{bmatrix}s+3 & 1\\ -2 & s\end{bmatrix}}{\begin{vmatrix}s & -1\\ 2 & s+3\end{vmatrix}}\right\}=L^{-1}\begin{Bmatrix}\dfrac{s+3}{s^2+3s+2} & \dfrac{1}{s^2+3s+2}\\ -\dfrac{2}{s^2+3s+2} & \dfrac{s}{s^2+3s+2}\end{Bmatrix}$$

$$=L^{-1}\begin{Bmatrix}\dfrac{s+3}{(s+1)(s+2)} & \dfrac{1}{(s+1)(s+2)}\\ -\dfrac{2}{(s+1)(s+2)} & \dfrac{s+3}{(s+1)(s+2)}\end{Bmatrix}$$

Now, $\phi_{11}(t)=L^{-1}\left[\dfrac{2}{s+1}-\dfrac{1}{s+2}\right]=2e^{-t}-e^{-2t}$

$$\phi_{12}(t)=L^{-1}\left\{\frac{1}{(s+1)(s+2)}\right\}=L^{-1}\left[\frac{1}{s+1}-\frac{1}{s+2}\right]=e^{-t}-e^{-2t}$$

$$\phi_{21}(t)=L^{-1}\left[-\frac{2}{(s+1)(s+2)}\right]=L^{-1}(-)\left[\frac{2}{s+1}-\frac{2}{s+2}\right]$$

$$=(-)\left[2e^{-t}-2e^{-2t}\right]=2e^{-2t}-2e^{-t}$$

$$\phi_{22}(t)=L^{-1}\left[\frac{s}{(s+1)(s+2)}\right]=L^{-1}\left[\frac{2}{s+2}-\frac{1}{s+1}\right]$$

$$\phi_{22}(t)=2e^{-2t}-e^{-t}.$$

Thus, the state transition matrix is

$$\phi(t)=\begin{bmatrix}\left(2e^{-t}-e^{-2t}\right) & \left(e^{-t}-e^{-2t}\right)\\ \left(2e^{-2t}-2e^{-t}\right) & \left(2e^{-2t}-e^{-t}\right)\end{bmatrix}$$

Now the solution of the state equation is:

$$X(t)=\phi(t)x(0)+\int_0^t\phi(t-\tau)Bu(\tau)\,d\tau$$

$$=\int_0^t\phi(t-\tau)Bu(\tau)\,d\tau \qquad \left[\because x(0)=\begin{bmatrix}0 & 0\end{bmatrix}^T\right]$$

$$=\int_0^t\begin{bmatrix}\phi_{11}(t-\tau) & \phi_{12}(t-\tau)\\ \phi_{21}(t-\tau) & \phi_{22}(t-\tau)\end{bmatrix}_{2\times2}\begin{bmatrix}0\\1\end{bmatrix}_{2\times1}1\cdot d\tau$$

$$\begin{bmatrix}x_1(t)\\x_2(t)\end{bmatrix}=\int_0^t\begin{bmatrix}\phi_{12}(t-\tau)\,d\tau\\ \phi_{22}(t-\tau)\,d\tau\end{bmatrix}=\begin{bmatrix}\int_0^t e^{-(t-\tau)}d\tau-\int_0^t e^{-2(t-\tau)}d\tau\\ 2\int_0^t e^{-2(t-\tau)}d\tau-\int_0^t e^{-(t-\tau)}d\tau\end{bmatrix}$$

$$=\begin{bmatrix}e^{-t}\int_0^t e^{\tau}d\tau-e^{-2t}\int_0^t e^{2\tau}d\tau\\ 2e^{-2t}\int_0^t e^{2\tau}d\tau-e^{-t}\int_0^t e^{\tau}d\tau\end{bmatrix}=\begin{bmatrix}e^{-t}\left\{e^{\tau}\Big|_0^t\right\}-e^{-2t}\cdot\frac{1}{2}\cdot e^{2\tau}\Big|_0^t\\ 2e^{-2t}\frac{1}{2}e^{2\tau}\Big|_0^t-e^{-t}e^{\tau}\Big|_0^t\end{bmatrix}$$

$$=\begin{bmatrix}e^{-t}\left[e^{t}-1\right]-\frac{1}{2}e^{-2t}\left[e^{2t}-1\right]\\ e^{-2t}\left[e^{2t}-1\right]-e^{-t}\left[e^{t}-1\right]\end{bmatrix}=\begin{bmatrix}\left(1-e^{-t}\right)-\frac{1}{2}\left(1-e^{-2t}\right)\\ \left(1-e^{-2t}\right)-\left(1-e^{-t}\right)\end{bmatrix}$$

So the state solution is

$$x(t)=\begin{bmatrix}\frac{1}{2}-e^{-t}+\frac{1}{2}e^{-2t}\\ e^{-t}-e^{-2t}\end{bmatrix} \qquad \textbf{(Ans.)}$$

11. A system is characterized is the following state **[MODEL QUESTION]**

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}u$$

$$x(0)=\begin{bmatrix}1\\1\end{bmatrix},\ y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

(i) Find the transfer function of the system;
(ii) Draw the block diagram of the above transfer function;
(iii) Compute the state transition matrix;
(iv) Determine the output response of the system to unit step input.

Answer:

(i) State equation is $\dot{X}=AX+Bu$, $x(0)=\begin{bmatrix}1\\1\end{bmatrix}$

Output is $y=\begin{bmatrix}1 & 0\end{bmatrix}X$, CX

i.e., $A=\begin{bmatrix}0 & 1\\-2 & -3\end{bmatrix}$; $B=\begin{bmatrix}0\\1\end{bmatrix}$; $C=\begin{bmatrix}1 & 0\end{bmatrix}$

Now, $[sI-A]=\begin{bmatrix}s & 0\\0 & s\end{bmatrix}-\begin{bmatrix}0 & 1\\-2 & -3\end{bmatrix}=\begin{bmatrix}s & -1\\2 & s+3\end{bmatrix}$

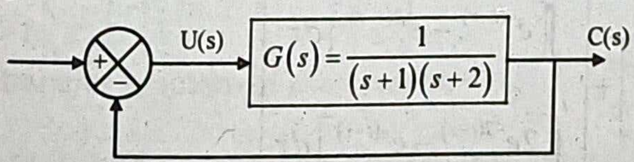
So, $$(sI-A)^{-1}=\frac{1}{s(s+3)+2}\begin{bmatrix}s+3 & 1\\-2 & s\end{bmatrix}=\begin{bmatrix}\frac{(s+3)}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)}\\ \frac{-2}{(s+1)(s+2)} & \frac{s}{s(s+1)(s+2)}\end{bmatrix}$$

So, $$G(s)=\frac{1}{(s+1)(s+2)}\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}(s+3) & 1\\-2 & s\end{bmatrix}_{2\times2}\begin{bmatrix}0\\1\end{bmatrix}_{2\times1}$$

$$=\frac{1}{(s+1)(s+2)}\begin{bmatrix}1 & 0\end{bmatrix}_{1\times2}\begin{bmatrix}1\\s\end{bmatrix}_{2\times1}$$

or, $$G(s)=\frac{1}{(s+1)(s+2)}$$ **(Ans.)**

(ii) Block diagram of the derived transfer function:



(iii) State transition matrix is $\phi(t)=L^{-1}\left[(sI-A)^{-1}\right]=\begin{bmatrix}\phi_{11}(t) & \phi_{12}(t)\\ \phi_{21}(t) & \phi_{22}(t)\end{bmatrix}$;

where $\phi_{11}(t)=L^{-1}\frac{s+3}{(s+1)(s+2)}$; $\phi_{12}(t)=L^{-1}\left[\frac{1}{(s+1)(s+2)}\right]$

$$\phi_{21}(t)=L^{-1}\left[\frac{-2}{(s+1)(s+2)}\right];\ \phi_{22}(t)=L^{-1}\left[\frac{s}{(s+1)(s+2)}\right]$$

Now, $$\phi_{11}(t)=L^{-1}\left[\frac{s+3}{(s+1)(s+2)}\right]=L^{-1}\left[\frac{2}{s+1}-\frac{1}{s+2}\right]=\left(2e^{-t}-e^{-2t}\right)$$

$$\phi_{12}(t)=L^{-1}\left[\frac{1}{(s+1)(s+2)}\right]=L^{-1}\left[\frac{1}{s+1}-\frac{1}{s+2}\right]=\left(e^{-t}-e^{-2t}\right)$$

$$\phi_{21}(t)=L^{-1}\left[\frac{-2}{(s+1)(s+2)}\right]=L^{-1}(-)\left[\frac{2}{s+1}-\frac{2}{s+2}\right]=2\left(e^{-2t}-e^{-t}\right)$$

$$\phi_{22}(t)=L^{-1}\left[\frac{s}{(s+1)(s+2)}\right]=L^{-1}\left[\frac{2}{s+2}-\frac{1}{s+1}\right]=\left(2e^{-2t}-e^{-t}\right)$$

Thus, the state transition matrix $\phi(t)$ is computed with the above values of $\phi_{ij}(t)$.

(iv) Now, $X(t)=\phi(t)x(0)+\int_0^t\phi(t-\tau)Bu(\tau)\,d\tau$

and response is $y(t)=\begin{bmatrix}1 & 0\end{bmatrix}X(\tau)$

$$B=\begin{bmatrix}0\\1\end{bmatrix};\text{ for step input } u(\tau)=1;\ x(0)=\begin{bmatrix}1\\1\end{bmatrix}\text{ given.}$$

Thus $$X(t)=\begin{bmatrix}\phi_{11} & \phi_{12}\\ \phi_{21} & \phi_{22}\end{bmatrix}_{2\times2}\begin{bmatrix}1\\1\end{bmatrix}_{2\times1}+\int_0^t\phi(t-\tau)\begin{bmatrix}0\\1\end{bmatrix}d\tau$$

$$=\begin{bmatrix}\phi_{11}+\phi_{12}\\ \phi_{21}+\phi_{22}\end{bmatrix}_{2\times1}+\int_0^t\begin{bmatrix}\phi_{11}(t-\tau) & \phi_{12}(t-\tau)\\ \phi_{21}(t-\tau) & \phi_{22}(t-\tau)\end{bmatrix}_{2\times2}\cdot\begin{bmatrix}0\\1\end{bmatrix}_{2\times1}d\tau$$

$$=\begin{bmatrix}2e^{-t}-e^{-2t}+e^{-t}-e^{-2t}\\ 2e^{-2t}-2e^{-t}+2e^{-2t}-e^{-t}\end{bmatrix}+\int_0^t\begin{bmatrix}\phi_{12}(t-\tau)\\ \phi_{22}(t-\tau)\end{bmatrix}_{2\times1}d\tau$$

$$=\begin{bmatrix}3e^{-t}-2e^{-2t}\\ -3e^{-t}+4e^{-2t}\end{bmatrix}+\begin{bmatrix}\int_0^t\left[e^{-(t-\tau)}-e^{-2(t-\tau)}\right]d\tau\\ \int_0^t\left[2e^{-2(t-\tau)}-e^{-(t-\tau)}\right]d\tau\end{bmatrix}$$

$$=\begin{bmatrix}3e^{-t}-2e^{-2t}\\ -3e^{-t}+4e^{-2t}\end{bmatrix}+\begin{bmatrix}e^{-t}\int_0^t e^{\tau}d\tau-e^{-2t}\int_0^t e^{2\tau}d\tau\\ 2e^{-2t}\int_0^t e^{2\tau}d\tau-e^{-t}\int_0^t e^{\tau}d\tau\end{bmatrix}$$

$$=\begin{bmatrix} 3e^{-t}-2e^{-2t} \\ -3e^{-t}+4e^{-2t} \end{bmatrix}+\begin{bmatrix} e^{-t}\left[e^{\tau}\Big|_0^t\right]-\frac{1}{2}e^{-2t}\left[e^{2\tau}\Big|_0^t\right] \\ 2e^{-2t}\left[e^{2\tau}\Big|_0^t\right]-e^{-t}\left[e^{\tau}\Big|_0^t\right] \end{bmatrix}$$

$$=\begin{bmatrix} 3e^{-t}-2e^{-2t} \\ -3e^{-t}+4e^{-2t} \end{bmatrix}+\begin{bmatrix} e^{-t}\left[e^{t}-1\right]-\frac{1}{2}e^{-2t}\left[e^{2t}-1\right] \\ 2e^{-2t}\left[e^{2t}-1\right]-e^{-t}\left[e^{t}-1\right] \end{bmatrix}$$

$$=\begin{bmatrix} 3e^{-t}-2e^{-2t} \\ -3e^{-t}+4e^{-2t} \end{bmatrix}+\begin{bmatrix} \left(1-e^{-t}\right)-\frac{1}{2}\left(1-e^{-2t}\right) \\ 2\left(1-e^{-2t}\right)-\left(1-e^{-t}\right) \end{bmatrix}$$

$$=\begin{bmatrix} 3e^{-t}-2e^{-2t} \\ -3e^{-t}+4e^{-2t} \end{bmatrix}+\begin{bmatrix} \frac{1}{2}-e^{-t}+\frac{1}{2}e^{-2t} \\ 1-2e^{-2t}+e^{-t} \end{bmatrix}$$

$$=\begin{bmatrix} 3e^{-t}-2e^{-2t}+\frac{1}{2}-e^{-t}+\frac{1}{2}e^{-2t} \\ -3e^{-t}+4e^{-2t}+1-2e^{-2t}+e^{-t} \end{bmatrix}=\begin{bmatrix} \frac{1}{2}+2e^{-t}-\frac{3}{2}e^{-2t} \\ 1-2e^{-t}+2e^{-2t} \end{bmatrix}$$

Thus, $X(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}=\begin{bmatrix} \frac{1}{2}+2e^{-t}-\frac{3}{2}e^{-2t} \\ 1-2e^{-t}+2e^{-2t} \end{bmatrix}$

Output $y(t)=\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}=x_1(t)$

Then $y(t)=\left(\frac{1}{2}+2e^{-t}-\frac{3}{2}e^{-2t}\right)$ **(Ans.)**

12. A system is characterized by the following state equation: [MODEL QUESTION]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}=\begin{bmatrix} 3 & -1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}+\begin{bmatrix} 1 \\ 1 \end{bmatrix}u,\ Y=\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a) Find the transfer function of the system.

Answer:

$\dot{X}=AX+BU;\ Y=CX;$ i.e., $SX(s)=AX(s)+BU(s); Y(s)=CX(s)$

or, $Y(s)=C[sI-A]^{-1}BU(s)$; Transfer function $G(s)=C[sI-A]^{-1}B$;

In the given problem, $A=\begin{bmatrix} 3 & -1 \\ 0 & -1 \end{bmatrix}; B=\begin{bmatrix} 1 \\ 1 \end{bmatrix}; C=\begin{bmatrix} 1 & 1 \end{bmatrix}$

Transfer function is $G(s)=\begin{bmatrix}1 & 1\end{bmatrix}[sI-A]^{-1}\begin{bmatrix}1\\1\end{bmatrix}$

$$sI-A=\begin{bmatrix}s & 0\\0 & s\end{bmatrix}\cdot\begin{bmatrix}3 & -1\\0 & -1\end{bmatrix}=\begin{bmatrix}s-3 & 1\\0 & s+1\end{bmatrix}$$

Now,

$$|sI-A|=(s-3)(s+1)$$

$$[sI-A]^{-1}=\frac{1}{(s+3)(s+1)}\begin{bmatrix}s+1 & -1\\0 & s-3\end{bmatrix}$$

$$G(s)=\begin{bmatrix}1 & 1\end{bmatrix}\frac{1}{(s+3)(s+1)}\begin{bmatrix}s+1 & -1\\0 & s-3\end{bmatrix}_{2\times2}\begin{bmatrix}1\\1\end{bmatrix}_{2\times1}$$

$$=\frac{1}{(s+3)(s+1)}\begin{bmatrix}1 & 1\end{bmatrix}_{1\times2}\begin{bmatrix}s\\s-3\end{bmatrix}_{2\times1}=\frac{s+s-3}{(s+3)(s+1)}=\frac{2s-3}{(s+3)(s+1)}=\frac{2s-3}{s^2+4s+3}$$

b) Draw the block diagram of the above transfer function. **[MODEL QUESTION]**

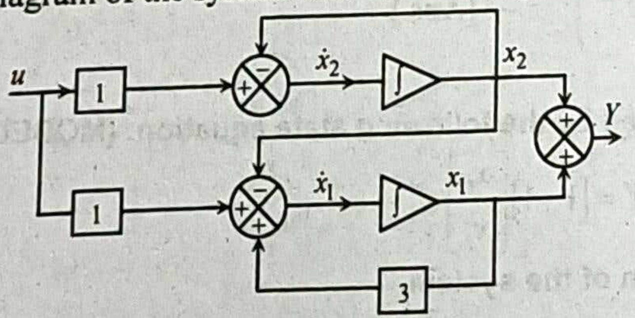
Answer:

State space model is represented as:

$$\begin{bmatrix}\dot{x}_1\\\dot{x}_2\end{bmatrix}=\begin{bmatrix}3 & -1\\0 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}u;\ Y=\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}.$$

i.e., $\begin{bmatrix}\dot{x}_1\\\dot{x}_2\end{bmatrix}=\begin{bmatrix}3x_1-x_2\\-x_2\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}u;\ Y=x_1+x_2;$

So the block diagram of the system is as shown in figure below.



c) Compute the state transition matrix. **[MODEL QUESTION]**

Answer:

$$\text{State transition matrix}=L^{-1}\begin{bmatrix}\dfrac{s+1}{(s-3)(s+1)} & -\dfrac{1}{(s-3)(s+1)}\\ 0 & \dfrac{s-3}{(s-3)(s+1)}\end{bmatrix}=\begin{bmatrix}\phi_{11}(t) & \phi_{12}(t)\\\phi_{21}(t) & \phi_{22}(t)\end{bmatrix}$$

$$\phi_{11}(t)=L^{-1}\left[\frac{s+1}{(s-3)(s+1)}\right]=L^{-1}\left[\frac{1}{s-3}\right]=e^{3t}$$

$$\phi_{12}(t)=L^{-1}\left[\frac{1}{(s-3)(s+1)}\right]=L^{-1}\left(\frac{1}{4}\right)\left[\frac{1}{s-3}-\frac{1}{s+1}\right]$$

or, $\phi_{12}(t)=L^{-1}\left\{\frac{1}{4}\left[\frac{1}{s-3}-\frac{1}{s+1}\right]\right\}=\frac{1}{4}\left[e^{3t}-e^{-t}\right]$

$\phi_{21}(t)=0;$

$\phi_{22}(t)=L^{-1}\left[\frac{1}{s+1}\right]=e^{-t}$

So the state transition matrix is

$$STM=\phi(t)=\begin{bmatrix} e^{3t} & \frac{1}{4}\left(e^{3t}-e^{t}\right) \\ 0 & e^{t} \end{bmatrix}$$

d) Obtain the solution to the state equation for a unit step input under zero initial conditions. [MODEL QUESTION]

Answer:

Now the solution of the state equation is

$$X(t)=\phi(t)x(0)+\int_0^t \phi(t-\tau)Bu(\tau)d\tau;$$

$$=\int_0^t \phi(t-\tau)Bu(\tau)d\tau;$$ $\because$ initial condition is zero, i.e., $x(0)=0$,

or, $X(t)=\int_0^t \phi(t-\tau)Bd\tau;$ Again for unit step input $u(\tau)=1$

Now, $$\phi(t-\tau)B=\begin{bmatrix} \left\{e^{3(t-\tau)}\right\} & \left\{\frac{1}{4}\left(e^{3(t-\tau)}-e^{-(t-\tau)}\right)\right\} \\ 0 & \left\{e^{-(t-\tau)}\right\} \end{bmatrix}_{2\times 2}\begin{bmatrix}1\\1\end{bmatrix}_{2\times 1}$$

$$\phi(t-\tau)B=\begin{bmatrix} \left\{e^{3(t-\tau)}+\frac{1}{4}\left(e^{3(t-\tau)}-e^{-(t-\tau)}\right)\right\} \\ \left\{e^{-(t-\tau)}\right\} \end{bmatrix}_{2\times 1}$$

Now, $\int_0^t e^{3(t-\tau)}d\tau+\frac{1}{4}\int_0^t\left[e^{3(t-\tau)}-e^{-(t-\tau)}\right]d\tau=e^{3t}.\int_0^t e^{-3\tau}d\tau+\frac{1}{4}\left[e^{3t}.\int_0^t e^{-3\tau}d\tau-e^{-t}.\int_0^t e^{\tau}d\tau\right]$

$$= e^{3t}.\frac{1}{3}.e^{-3\tau}\Big|_t^0 + \frac{1}{4}\left[e^{3t}.\frac{1}{3}.e^{-3\tau}\Big|_t^0 - e^{-t}e^{\tau}\Big|_0^t\right]$$

$$= \frac{1}{3}e^{3t}\left(1-e^{-3t}\right) + \frac{1}{4}\left[e^{3t}.\frac{1}{3}\left(1-e^{-3t}\right) - e^{-t}\left(e^t - 1\right)\right]$$

$$= \frac{1}{3}e^{3t}\left[1-e^{-3t}\right] + \frac{1}{4}\left[e^{3t}\left(\frac{1}{3}-\frac{1}{3}e^{-3t}\right) - 1 + e^{-t}\right]$$

$$= \left(\frac{1}{3}e^{3t} - \frac{1}{3}\right) + \frac{1}{4}\left[\frac{1}{3}e^{3t} - \frac{1}{3} - 1 + e^{-t}\right] = \frac{1}{3}e^{3t} - \frac{1}{3} + \frac{1}{12}e^{3t} - \frac{1}{12} - \frac{1}{4} + \frac{1}{4}e^{-t}$$

$$= e^{3t}\left(\frac{1}{3}+\frac{1}{12}\right) + \frac{1}{4}e^{-t} - \left(\frac{1}{3}+\frac{1}{4}+\frac{1}{12}\right) = e^{3t}\left(\frac{5}{12}\right) + \frac{1}{4}e^{-t} - \left(\frac{8}{12}\right)$$

$$= \frac{5}{12}e^{3t} + \frac{1}{4}e^{-t} - \frac{2}{3}$$

Again, $\int_0^t e^{-(t-\tau)}d\tau = e^{-t}\int_0^t e^{\tau}d\tau = e^{-t}.\left[e^t - 1\right] = 1 - e^t$

So the solution of the state matrix is $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \left(\frac{5}{12}e^{3t} + \frac{1}{4}e^{-t} - \frac{2}{3}\right) \\ \left(1-e^t\right) \end{bmatrix}$ Ans.

13. a) Find the state transition matrix using Cayley-Hamilton theorem of the system having $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$. **[MODEL QUESTION]**

Answer:

We've the system matrix, $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

So the characteristic equation for the eigen value m is

$$|mI - A| = 0$$

or, $\left|\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\right| = 0$

or, $\begin{vmatrix} m & -1 \\ 2 & m+3 \end{vmatrix} = 0$

or, $m(m+3) + 2 = 0$

or, $m^2 + 3m + 2 = 0$

or, $m^2 + 2m + m + 2 = 0$

or, $m(m+2)+1(m+2)=0$

or, $(m+2)(m+1)=0$

so the eigen values are $m=-2,\ -1$

Now, $e^{At}=p(t)I+q(t)A$

and $e^{mt}=p+qm$, for $m=-1,-2$

$\therefore\quad e^{-t}=p-q$ .... (1)

and $e^{-2t}=p-2q$ .... (2)

By subtracting Eqn. (2) from Eqn. (1),

$e^{-t}-e^{-2t}=q$; and $p=e^{-t}+q$

i.e. $p=e^{-t}+e^{-t}-e^{-2t}$

or, $p=2e^{-t}-e^{-2t}$

$\therefore\quad p=2e^{-t}-e^{-2t};\ q=e^{-t}-e^{-2t};$

now $\phi(k)=A^k=e^{At}$

$$\therefore\quad e^{At}=pI+qA=\left(2e^{-t}-e^{-2t}\right)\begin{bmatrix}1&0\\0&1\end{bmatrix}+\left(e^{-t}-e^{-2t}\right)\begin{bmatrix}0&1\\-2&-3\end{bmatrix}$$

$$=\begin{bmatrix}2e^{-t}-e^{-2t}&0\\0&2e^{-t}-e^{-2t}\end{bmatrix}+\begin{bmatrix}0&e^{-t}-e^{-2t}\\2\left(-e^{-t}+e^{-2t}\right)&3\left(-e^{-t}+e^{-2t}\right)\end{bmatrix}$$

$$\text{or,}\quad \phi(k)=\begin{bmatrix}2e^{-t}-e^{-2t}&e^{-t}-e^{-2t}\\2\left(e^{-2t}+e^{-t}\right)&3\left(e^{-2t}-e^{-t}\right)\end{bmatrix}$$

b) The dynamic equation of a non-homogeneous system is given as $\dot{X}(t)=\begin{bmatrix}0&1\\-6&-5\end{bmatrix}X(t)+\begin{bmatrix}0\\1\end{bmatrix}U(t)$ & output $Y(t)=[1\quad 0]X(t)$. The initial condition is $X(0)=[1\quad 0]^T$, where $U(t)$ is unit step input. Determine the output of $Y(t)$ at $t=1$ sec. **[MODEL QUESTION]**

Answer:

Comparing with the standard state space form of control system as

$\dot{X}=AX+BU;$

$Y=CX;$

In the problem:

$$A=\begin{bmatrix}0&1\\-6&-5\end{bmatrix};B=\begin{bmatrix}0\\1\end{bmatrix};C=[1\quad 0];$$

For the system in the present problem, the system matrix A is given as:

$$A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix};$$

So, the state transition matrix (STM) is given as:

$$STM = L^{-1}\left[(sI - A)^{-1}\right] = \begin{bmatrix} \phi_{11}(t) & \phi_{12}(t) \\ \phi_{21}(t) & \phi_{22}(t) \end{bmatrix};$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 6 & (s+5) \end{bmatrix};$$

$$(sI - A)^{-1} = \frac{\begin{bmatrix} (s+5) & -6 \\ 1 & s \end{bmatrix}^T}{\begin{vmatrix} s & -1 \\ 6 & (s+5) \end{vmatrix}} = \frac{\begin{bmatrix} (s+5) & 1 \\ -6 & s \end{bmatrix}}{s^2 + 5s + 6};$$

$$\phi_{11}(t) = L^{-1}\left[\frac{(s+5)}{s^2 + 5s + 6}\right] = L^{-1}\left[\frac{(s+5)}{(s+2)(s+3)}\right];$$

$$\phi_{12}(t) = L^{-1}\left[\frac{1}{(s+2)(s+3)}\right];$$

$$\phi_{21}(t) = L^{-1}\left[\frac{-6}{(s+2)(s+3)}\right];$$

$$\phi_{22}(t) = L^{-1}\left[\frac{s}{(s+2)(s+3)}\right]$$

Now, $\phi_{11}(t) = L^{-1}\left[\frac{(s+5)}{(s+2)(s+3)}\right] = L^{-1}\left[\frac{A_1}{(s+2)} + \frac{A_2}{(s+3)}\right];$

$$\phi_{11}(t) = L^{-1}\left[\frac{3}{(s+2)} - \frac{2}{(s+3)}\right];$$

$$\phi_{11}(t) = 3e^{-2t} - 2e^{-3t}$$

$$\phi_{12}(t) = L^{-1}\left[\frac{1}{(s+2)(s+3)}\right] = L^{-1}\left[\frac{1}{(s+2)} - \frac{1}{(s+3)}\right];$$

$$\phi_{12}(t) = e^{-2t} - e^{-3t};$$

$$\phi_{21}(t) = L^{-1}\left[\frac{-6}{(s+2)(s+3)}\right] = L^{-1}\left[\frac{-6}{(s+2)} + \frac{6}{(s+3)}\right]$$

$$\phi_{21}(t) = -6e^{-2t} + 6e^{-3t}$$

$$\phi_{22}(t) = L^{-1}\left[\frac{s}{(s+2)(s+3)}\right] = L^{-1}\left[\frac{-2}{(s+2)} + \frac{3}{(s+3)}\right]$$

$$\phi_{22}(t) = -2e^{-2t} + 3e^{-3t}$$

$$X(t) = \begin{bmatrix} \phi_{11}(t) & \phi_{12}(t) \\ \phi_{21}(t) & \phi_{22}(t) \end{bmatrix} x(0) + \int_0^t \phi(t-\tau)Bu(\tau)d\tau;$$

or, the state matrix X(t) is given as:

$$X(t) = \varphi(t)x(0) + \int_0^t \phi(t-\tau)Bu(\tau)d\tau$$

$$X(t) = \phi(t)\left[x(0) + \int_0^t \phi(-\tau)Bu(\tau)d\tau \right]$$

$$X(t) = \phi(t)\left[x(0) + \int_0^t \phi(-\tau)\begin{bmatrix} 0 \\ 1 \end{bmatrix} d\tau \right]$$

$$X(t) = \phi(t)\left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \int_0^t \begin{bmatrix} \phi_{11}(-\tau) & \phi_{12}(-\tau) \\ \phi_{21}(-\tau) & \phi_{22}(-\tau) \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} d\tau \right] = \phi(t)\left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \int_0^t \begin{bmatrix} \phi_{12}(-\tau) \\ \phi_{22}(-\tau) \end{bmatrix} d\tau \right]$$

$$= \phi(t)\left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \int_0^t \begin{bmatrix} e^{2\tau} - e^{3\tau} \\ -2e^{2\tau} + 3e^{3\tau} \end{bmatrix} d\tau \right] = \begin{bmatrix} \phi_{11}(t) & \phi_{12}(t) \\ \phi_{21}(t) & \phi_{22}(t) \end{bmatrix} \cdot \left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} \left(\frac{1}{2}e^{2t} - \frac{1}{3}e^{3t} - \frac{1}{6}\right) \\ (-e^{-2t} + e^{3t}) \end{bmatrix}\right]$$

$$= \begin{bmatrix} \phi_{11}(t) & \phi_{12}(t) \\ \phi_{21}(t) & \phi_{22}(t) \end{bmatrix} \cdot \begin{bmatrix} \left(\frac{1}{2}e^{2t} - \frac{1}{3}e^{3t} + \frac{5}{6}\right) \\ 0 \end{bmatrix} = \begin{bmatrix} \phi_{11}(t).(\frac{1}{2}e^{2t} - \frac{1}{3}e^{3t} + \frac{5}{6}) \\ \phi_{21}(t).(\frac{1}{2}e^{2t} - \frac{1}{3}e^{3t} + \frac{5}{6}) \end{bmatrix}$$

$$\text{or, } X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \phi_{11}(t).(\frac{1}{2}e^{2t} - \frac{1}{3}e^{3t} + \frac{5}{6}) \\ \phi_{21}(t).(\frac{1}{2}e^{2t} - \frac{1}{3}e^{3t} + \frac{5}{6}) \end{bmatrix}$$

$$y(t) = CX(t) = C\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = [1 \quad 0].\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$y(t) = x_1(t) = \phi_{11}(t).\left(\frac{1}{2}e^{2t} - \frac{1}{3}e^{3t} + \frac{5}{6}\right)$$

$$y(t) = (3e^{-2t} - 2e^{-3t})\left(\frac{1}{2}e^{2t} - \frac{1}{3}e^{3t} + \frac{5}{6}\right)$$

$$y(t)\big|_{t=1} = \left(3e^{-2} - 2e^{-3}\right)\left(\frac{1}{2}e^{2} - \frac{1}{3}e^{3} + \frac{5}{6}\right)$$

$$y(t)\big|_{t=1} = \left(\frac{3}{2} - e + \frac{5}{2}e^{-2}\right) + \left(-e^{-1} + \frac{2}{3} - \frac{5}{3}e^{-3}\right)$$

$$=\left(\frac{13}{6}-e-e^{-1}+\frac{5}{2}e^{-2}-\frac{5}{3}e^{-3}\right)$$
$$=2.16-2.72-0.37+0.34-0.083$$
$$=-0.673$$

The above value gives the output $Y(t)$ at $t=1$ sec with unit step input.

c) A system is characterised by $\dot{X}=\begin{bmatrix}0 & 1\\-2 & -3\end{bmatrix}X+\begin{bmatrix}0\\1\end{bmatrix}U$. $X(0)=\begin{bmatrix}0\\1\end{bmatrix}$, $Y=[1\ \ 0]X$. Find the transfer function of the system. **[MODEL QUESTION]**

Answer:

Comparing with the standard state space form of control system as

$$\dot{X}=AX+BU$$
$$Y=CX;$$

In the problem:

$$A=\begin{bmatrix}0 & 1\\-2 & -3\end{bmatrix};B=\begin{bmatrix}0\\1\end{bmatrix};C=[1\ \ 0];$$

$$G(s)=C(sI-A)^{-1}B;=[1\ \ 0].\left[\begin{bmatrix}s & 0\\0 & s\end{bmatrix}-\begin{bmatrix}0 & 1\\-2 & -3\end{bmatrix}\right]^{-1}\begin{bmatrix}0\\1\end{bmatrix}$$

$$=[1\ \ 0].\begin{bmatrix}s & -1\\2 & (s+3)\end{bmatrix}^{-1}\begin{bmatrix}0\\1\end{bmatrix}.=\frac{[1\ \ 0].\begin{bmatrix}(s+3) & -2\\1 & s\end{bmatrix}^{T}.\begin{bmatrix}0\\1\end{bmatrix}}{(s^2+3s+2)}$$

$$=\frac{[1\ \ 0].\begin{bmatrix}(s+3) & 1\\-2 & s\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}}{(s^2+3s+2)}.=\frac{[(s+3)\ \ 1].\begin{bmatrix}0\\1\end{bmatrix}}{(s^2+3s+2)}=\frac{1}{(s^2+3s+2)}$$

The transfer function G(s) is given by the above expression.

14. Comment on the controllability & observability of the given discrete time system shown.

$$X(k+1)=\begin{bmatrix}0 & 1\\-6 & -5\end{bmatrix}X(k)+\begin{bmatrix}0\\1\end{bmatrix}U(k)$$
$$Y(k)=[2\ \ 2]X(k).$$

[MODEL QUESTION]

Answer:

Comparing with the standard state space form of control system as

$$\dot{X}=AX+BU$$
$$Y=CX;$$

In the problem:

$$A=\begin{bmatrix}0 & 1\\ -6 & -5\end{bmatrix}; B=\begin{bmatrix}0\\1\end{bmatrix}; C=\begin{bmatrix}2 & 2\end{bmatrix}$$

$$AB=\begin{bmatrix}0 & 1\\ -6 & -5\end{bmatrix}\cdot\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1\\-5\end{bmatrix}$$

$$S_c=\begin{bmatrix}B & (AB)\end{bmatrix}=\begin{bmatrix}0 & 1\\ 1 & -5\end{bmatrix}; |S_c|\neq 0;$$

$$A^T=\begin{bmatrix}0 & -6\\ 1 & -5\end{bmatrix}; C^T=\begin{bmatrix}2\\2\end{bmatrix}; A^T.C^T=\begin{bmatrix}0 & -6\\ 1 & -5\end{bmatrix}\cdot\begin{bmatrix}2\\2\end{bmatrix}=\begin{bmatrix}-12\\-8\end{bmatrix}$$

$$S_o=\begin{bmatrix}C^T & (A^TC^T)\end{bmatrix}=\begin{bmatrix}2 & -12\\ 2 & -8\end{bmatrix}; |S_o|\neq 0;$$

S_c & S_o being controllability & observability matrices.
Thus the system is controllable and observable.

15. Write short notes on the following:
a) Properties of state transition matrix **[MODEL QUESTION]**
b) Controllability and observability **[MODEL QUESTION]**
c) State space model of field controlled d.c. motor **[MODEL QUESTION]**
d) Pole placement method **[MODEL QUESTION]**

Answer:

a) Properties of State Transition Matrix:

Consider free running (no input i.e., u(t)=0) operation of one control system.

The state equation may be written as: $\dot{x}(t)=Ax(t)$ ... (1)

where $x(t)$ is the state vector in the system with system matrix A.
Above equation is the homogeneous state equation.

Taking Laplace transformation on both sides of that equation we've,

$$sX(s)-x(0)=AX(s);\text{ where } L[x(t)]=X(s)$$

or, $$(sI-A)X(s)=x(0)$$

or, $$X(s)=(sI-A)^{-1}x(0) \quad \text{... (2)}$$

Taking Laplace inverse of Eqn. (2), we've

$$x(t)=L^{-1}\left[(sI-A)^{-1}x(0)\right] \quad \text{...(3)}$$

Now,

$$(sI-A)^{-1}=s^{-1}\left(I-\frac{A}{s}\right)^{-1}$$

$$=s^{-1}\left[I+\frac{A}{s}+\frac{A^2}{s^2}+\frac{A^3}{s^3}+........\right]$$

$$=\frac{I}{s}+\frac{A}{s^2}+\frac{A^2}{s^3}+\frac{A^3}{s^4}+....$$

So, $L^{-1}\left[(sI-A)^{-1}\right]=I+At+\frac{A^2t^2}{2!}+\frac{A^3t^3}{31}+....=e^{At}$ ...(4)

Now, from Eqn. (3) and (4) we've

$x(t)=e^{At}x(0)$ ...(5)

Now the solution to the homogenous state Eqn. (1) may be written as:

$x(t)=\phi(t)x(0)$ ...(6)

where $\phi(t)$ is an $n\times n$ matrix, which is the unique solution of the Eqn. (1). $\phi(t)=e^{At}$ is called state transition matrix, as it helps for the transition of state of the system.

$\dot{\phi}(t)=A\phi(t), \phi(0)=I$

$\phi^{-1}(t)=e^{-At}=\phi(-t)$ ...(7)

Hence, the properties of state transition matrix may be written as:

(i) $\phi(0)=e^{A.0}=I$

(ii) $\phi(t)=e^{At}=(e^{-At})^{-1}=[\phi\{-t\}]^{-1}$ i.e. $\phi^{-1}(t)=\phi(-t)$

(iii) $\phi(t_1+t_2)=e^{A(t_1+t_2)}=e^{At_1}\cdot e^{At_2}=\phi(t_1)\phi(t_2)=\phi(t_2)\phi(t_1)$

(iv) $[\phi(t)]^n=\phi(nt)$

(v) $\phi(t_2-t_1)\phi(t_1-t_0)=\phi(t_2-t_0)=\phi(t_1-t_0).\phi(t_2-t_1)$

b) Controllability and observability:

State Controllability:

Consider the state space equation of one time invariant control system as:

$\dot{x}(t)=Ax(t)+Bu(t)$.............(1)

$y(t)=Cx(t)+Du(t)$.............(2)

where $x(t)$ is $n\times1$ state vector, $u(t)$ is $r\times1$ input vector, $y(t)$ is $p\times1$ output vector, A,B,C,D the co-efficient matrixes of appropriate dimensions.

The state $x(t)$ is said to be controllable at $t=t_0$, if there exists a piecewise continuous input that will drive the state to any final state $x(t_f)$ for a finite time interval

$(t_f - t_0) \geq 0$. If every state $x(t_0)$ of the system is controllable at a finite time interval, the system is said to be completely state controllable or simply controllable.

The following theorem shows that the condition of controllability depends on the co-efficient matrices A and B of the system. The theorem also gives the method of testing state controllability.

Theorem: For the system described by the state Eqn. (1) to be completely state controllable, it is necessary and sufficient that the following n × nr controllability matrix has a rank n:

$$S = \begin{bmatrix} B & AB & A^2B & A^{n-1}B \end{bmatrix} \quad .....(3)$$

It may be noted that if the controllable matrix S is non square, we can form the matrix SS' which is n × n square matrix and if SS' is non singular then S has rank n.

Output controllability:

In particular design of a control system, we may desire to control the output rather than the stability of the system. Complete state controllable condition is neither necessary nor sufficient condition for complete output controllability. For one system as represented by Eqns. (1) and (2), the system is completely output controllable, if it is possible to construct one unconstrained control vector u(t) which will transfer any initial output $y(t_0)$ to any final output $y(t_f)$ in a finite time interval $t_f - t_0 \geq 0$. It can be proved that the condition for complete output controllability is that the output controllable matrix $M = \begin{bmatrix} CB : CAB : CA^2B .... CA^{n-1}B : D \end{bmatrix}$ has rank n.

Observability

A system is completely observable if every state variable of the system affects some of the outputs. That is, as it is often desirable to obtain information on the state variables, from the measurements of the outputs and the inputs, if any of the states cannot be observed from the measurement of the outputs, the state is said to be unobservable, and the system is not completely observable, or simply the system is unobservable.

Following figure shows the state diagram of a linear system, in which the state x_2 is not connected to the output in any way, while the state x_1 is connected to the output $y(t)$. Hence once we measure the output $y(t)$, we can observe the state $x(t)$, since

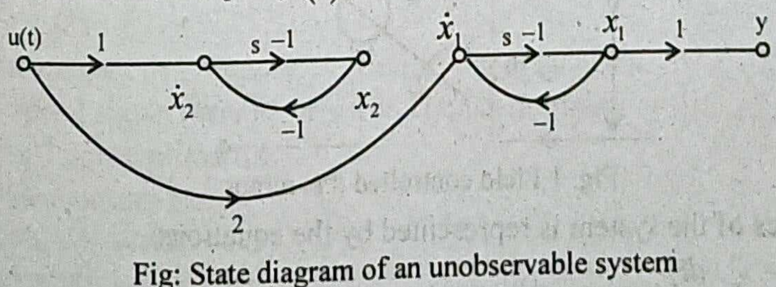


Fig: State diagram of an unobservable system

$x_1(t) = y(t)$. But the state $x_2(t)$ cannot be observed from the information on $y(t)$. Thus the system is not observable.

Condition of Observability

Given a linear time invariant system that is described by the state Eqns. (1) and (2). The state $x(t_0)$ is said to be observable, if given any input u(t), there exists a finite time $t_f \geq t_0$, such that the knowledge of u(t) for $t_0 \leq t < t_f$, the matrices A,B,C,D and the output y(t), for $t_0 < t < t_f$ are sufficient to determine $x(t_0)$. If every state of the system is observable for a finite t_f the system is said to be completely observable or simply observable.

The following theorem shows that the condition of observability depends on the matrices A and C of the system. The theorem provides the method of testing observability of the system.

Theorem: For the system described by the state Eqns. (1) and (2) to be completely observable, it is necessary and sufficient that the following $n \times np$ observability matrix has a rank n:

$$V = \begin{bmatrix} C \\ CA \\ CA^2 \\ \cdot \\ \cdot \\ \cdot \\ CA^{n-1} \end{bmatrix}$$

c) State space model of field controlled d.c. motor:

Fig.1 represents the field controlled d.c. motor with field current i_f through the field winding having resistance R_f and inductance L_f producing torque T in the motor which rotates with an angular velocity w, so that θ is considered an angular displacement at an instant t.

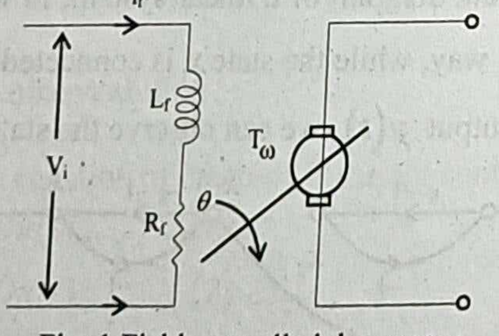


Fig: 1 Field controlled d.c. motor

Now, the dynamics of the system is represented by the equations.

$$v_i = R_f i_f + L_f \frac{di_f}{dt}$$

i.e. $\frac{di_f}{dt} = -\left(\frac{R_f}{L_f}\right)i_f + \frac{v_i}{L_f}$ ....(1)

$J\frac{d\omega}{dt} + B\omega = T = Ki_f$ [where J is the moment of initia of the motor, B and K are constants of proportionality]

i.e., $\frac{d\omega}{dt} = \left(\frac{K}{J}\right)i_f - \left(\frac{B}{J}\right)\omega$ ....(2)

$\frac{d\theta}{dt} = \omega$ ....(3)

Equations (1), (2) and (3) may be represented combiningly in the matrix form as:

$$\begin{bmatrix} \frac{di_f}{dt} \\ \frac{d\omega}{dt} \\ \frac{d\theta}{dt} \end{bmatrix} = \begin{bmatrix} -R_f/L_f & 0 & 0 \\ K/J & -\frac{B}{J} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_f \\ \omega \\ \theta \end{bmatrix} + \begin{bmatrix} 1/L_f \\ 0 \\ 0 \end{bmatrix} v_i \quad ....(4)$$

Eqn. (4) gives the state spice model of the field controlled d.c. motor.

d) Pole placement method:

For the purpose of pole placement it is assumed that all the states are measurable and available for feedback. It can be proved that if the system is completely state controllable, then the pole of the closed loop system may be placed at any desired locations by state feedback trough an appropriate gain matrix.

Let us suppose that the desired closed loop poles be at s = $\sigma_1, \sigma_2, \ldots\ldots\sigma_n$. By choosing an appropriate gain matrix for state feedback, it is possible to force the system to have closed loop poles at desired locations, provided that the original system is completely state controllable. For the purpose of simplicity, we consider the system under consideration to be of single input single output (SISO) type with control signal u(t) and output signal y(t) as scalars. We also assume the reference input r(t) to be zero.

Consider a control system as

$\dot{x} = Ax + Bu$ ......(1)

$y = Cx + Du.$

where, x = state vector (n – vector), y = output signal (scalar),

u = Control signal (scalar), $A = n \times n$ constant matrix

$B = n \times 1$ constant matrix

$C = 1 \times n$ constant matrix

D = constant scalar.

We choose the control signal, $u = -Kx$ .....(2)

This means that the control signal u is determined by an instantaneous state. Such a scheme is known as state feedback. The $(1\times n)$ matrix K is called the state feedback gain matrix. We assume here that all the state variables are available for feedback.
Now, from Eqns. (1) and (2) we have,

$$\dot{x} = Ax + Bu = Ax - BKx$$

$$\text{or, } \dot{x} = (A - BK)x$$

$$\text{i.e. } \dot{x}(t) = (A - BK)x(t) \quad \text{..................(3)}$$

The solution of the above Eqn. is given as

$$x(t) = e^{(A-BK)t}x(o) \quad \text{....................(4)}$$

$x(o)$ being the initial state caused by an external disturbance. The system represented by Eqn. (4) is shown in Fig. 1.

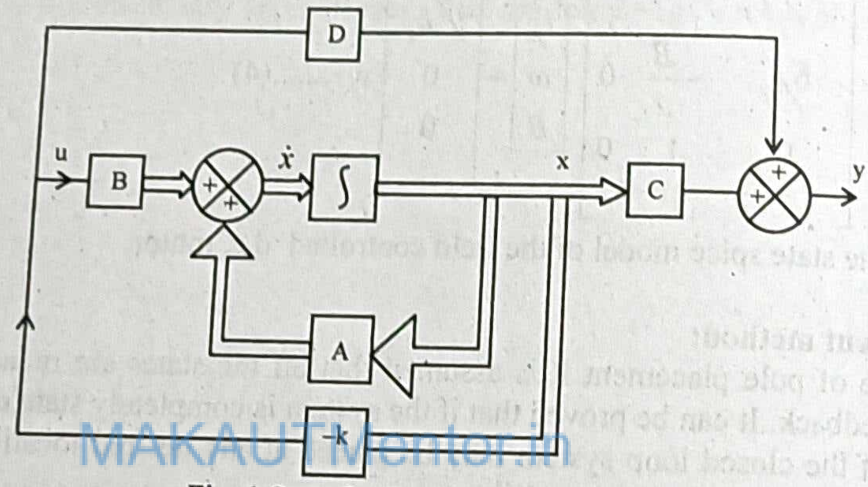


Fig: 1 State feedback system

The stability and the transient response characteristics are determined by the eigen values of the matrix (A-BK).
Now if the matrix K be chosen properly, the matrix (A-BK) can be made asymptotically stable, and for all $x(o) \neq o$, it is possible to make $x(t)$ approaches to 0, as t approaches infinity. The eigen values of matrix (A-BK) are called the regulator poles. If these regulator poles be placed in the left half s plane, then x(t) approaches zero as t approaches infinity. The problem of placing the regulator poles (closed loop poles) at the desired locations is the pole placement problem. It can be proved that arbitrary pole placement for a given system is possible, if and only if the system is completely state controllable.

COMPENSATORS

Multiple Choice Type Questions

1. The phase lead compensation is used to [WBUT 2006, 2010, 2017, 2019]
   a) Increase rise time & decrease overshoot
   b) decrease both rise time & overshoot
   c) increase both rise time & overshoot
   d) decrease rise time & increase overshoot

Answer: (b)

2. A lag network for compensation normally consists of [WBUT 2012]
   a) *R* only
   b) *R* & *C* elements
   c) *R* & *L* elements
   d) *R*, *L* & *C* elements

Answer: (b)

3. The transfer function of a phase-lead compensator is $G_c(s) = \dfrac{1+Ts}{1+0.333Ts}$
The maximum phase contribution from the compensator is [WBUT 2015]
   a) 20°
   b) 30°
   c) 45°
   d) 60°

Answer: (d)

4. The transfer function of a network is $\dfrac{1+0.3s}{2+s}$. It represents a [WBUT 2016, 2018]
   a) lag network
   b) lead network
   c) lag-lead network
   d) proportional controller

Answer: (a)

Short Answer Type Questions

1. What is compensation? What is compensated system? What is compensator? [WBUT 2010]

Answer:

1st Part:

Compensation is the process of altering the response of a control system in order to meet set design criteria. If a system is poor in terms of stability or if the designed system does not provide the desired performance then compensation is needed.

2nd Part:

An uncompensated system is said to be compensated which provides the desired performance when a compensator is added in the system.

3rd Part:
Compensators are the elements added to the control system either in forward or feedback or both path(s) to alter the response of a control system in order to accommodate desired performance from the system. By introducing additional poles and/or zeros to a system the response of the system will change significantly.
Three types of compensators are there such as lead, lag, and lead-lag. Lead compensation alters the transient response of systems. This includes overshoot, rise time (TR), settling time (TS), and peak time (TP). Lag compensation alters steady-state error of systems. Lead –lag compensators are used to alter transient response and steady-state error simultaneously.

2. Explain with the help of an example how improvement of system performance is achieved through compensation. **[WBUT 2010]**

Answer:
To obtain the desired behaviour / performance as per performance specifications the transient response specifications are to be translated into desired locations of closed loop poles.

Approach I
In some control systems the gain adjustment (K_c for proportional, τ_i Integral Time, τ_d derivative time) moves the closed loop poles to desired locations.

Approach II
It is used for most of the cases. Here,
- We add a compensator in the control system.
- Then, its parameters are to be obtained in such a way so that the desired performance specifications may be met.

Long Answer Type Questions

1. Derive the transfer function for lead & lead-lag compensators with necessary passive equivalent circuits. **[WBUT 2016]**

Answer:
Realization of Lead Network
The LEAD network is shown in the Fig. 1

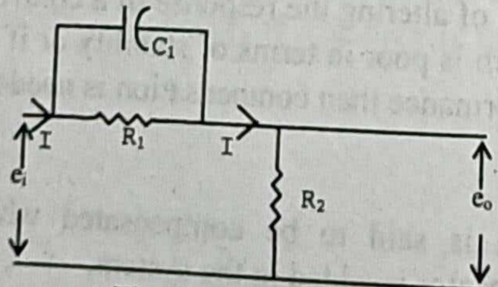


Fig: 1 Lead network

The "Transformed network of the above network (Fig.1) is shown in Fig. 2

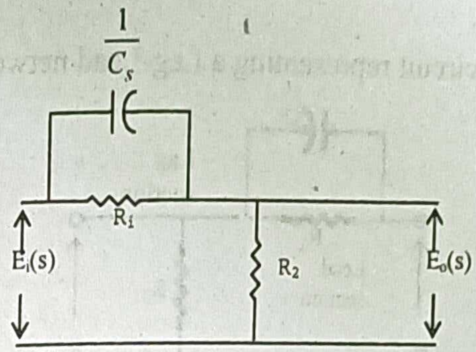


Fig: 2 Transformed lead network

Applying potential divider rule we get

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2}{\dfrac{R_1 \cdot \dfrac{1}{Cs}}{R_1 + \dfrac{1}{Cs}} + R_2} = \frac{R_2}{\dfrac{\dfrac{R_1}{Cs}}{\dfrac{R_1Cs+1}{Cs}} + R_2}$$

$$= \frac{R_2}{R_1+R_2} \cdot \frac{(R_1Cs+1)}{\left(\dfrac{R_1R_2}{R_1+R_2}\right)Cs+1} \quad \text{.... (1)}$$

Substituting

$$R_1C = \tau, \text{ and } \frac{R_2}{R_1+R_2} = \alpha(<1),$$

$G_C(s)$ may be written in two forms

$$G_C(s) = \frac{\alpha(s\tau+1)}{(\alpha\tau s+1)} \quad \text{.... (2). Form I}$$

$$= \frac{\alpha\tau\left(s+\dfrac{1}{\tau}\right)}{\alpha\tau\left(s+\dfrac{1}{\alpha\tau}\right)} = \frac{\left(s+\dfrac{1}{\tau}\right)}{s+\dfrac{1}{\alpha\tau}} \quad \text{.... (2a) Form II}$$

Form I is suitable for frequency domain design where factor α gets cancelled by an amplifier having gain $A = \dfrac{1}{\alpha}$.

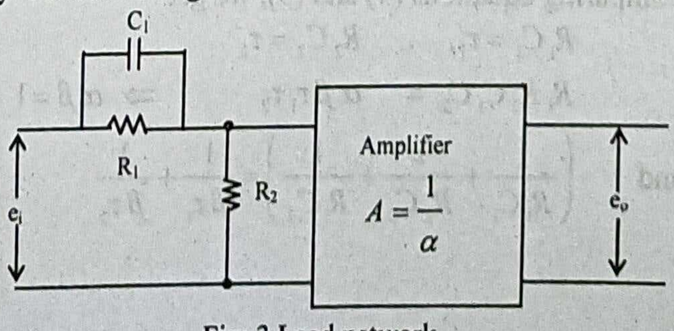


Fig: 3 Lead network

Form II is suitable for Root Locus design.

Lag-Lead Network

Fig. 4 shows an electrical circuit representing a Lag-Lead network

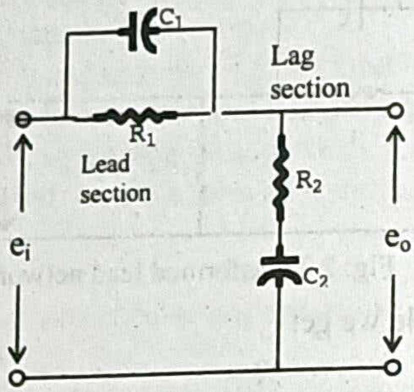


Fig: 4 Lag -Lead network

- ***Transfer Function:*** Applying potential divider rule

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2 + \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2} + \frac{R_1/sC_1}{R_1 + \frac{1}{sC_1}}} \quad \text{.... (3)}$$

After Simplification,

$$= \frac{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}{(R_1 C_1 s + 1)(R_2 C_2 s + 1) + R_1 C_2 s}$$

$$= \frac{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}{R_2 C_2 R_1 C_1 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2)s + 1}$$

$$= \frac{\left(s + \frac{1}{R_1 C_1}\right)\left(s + \frac{1}{R_2 C_2}\right)}{\left[s^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_1 C_2}\right)s + \frac{1}{R_1 R_2 C_1 C_2}\right]} \quad \text{.... (4)}$$

Comparing equations (4) and (5), we get

$$R_1 C_1 = \tau_1, \qquad R_2 C_2 = \tau_2$$

$$R_1 R_2 C_1 C_2 = \alpha \beta \tau_1 \tau_2 \qquad \Rightarrow \alpha \beta = 1$$

and $$\left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_1 C_2}\right) = \frac{1}{\beta \tau_1} + \frac{1}{\beta \tau_2}$$

$\therefore$ Equation (3) may be re-written as

$$\frac{E_o(s)}{E_i(s)}=\frac{\left(s+\frac{1}{\tau_1}\right)\left(s+\frac{1}{\tau_2}\right)}{\left(s+\frac{1}{\beta\tau_1}\right)\left(s+\frac{1}{\alpha\tau_2}\right)} \quad \text{.... (5)}$$

where, $\beta=\frac{z_{C1}}{p_{C1}}$ and $\alpha=\frac{p_{C2}}{z_{C2}}$

2. Write short notes on the following:
a) Lead-lag compensation **[WBUT 2008, 2009, 2013, 2019]**

OR,

Lead lag compensator **[WBUT 2014]**
b) Lead compensators **[WBUT 2012]**

Answer:

a) Lead-lag compensation:

A lead-lag compensator consists of a lead compensator cascaded with a lag compensator. Both lead compensators and lag compensators introduce a pole-zero pair into the open loop transfer function. The General form of the overall transfer function of such a compensator can be written as mentioned in equation (1)

$$G_C(s)=\underbrace{\left(\frac{s+\frac{1}{\tau_1}}{s+\frac{1}{\beta\tau_1}}\right)}_{\text{Contribution of Lag Compensator}}\cdot\underbrace{\left(\frac{s+\frac{1}{\tau_2}}{s+\alpha\tau_2}\right)}_{\text{Contribution of Lead Compensator}} \quad \text{......(1)}$$

where, $\beta>1$ and $\alpha<1$

The lead compensator section provides phase lead at high frequencies. This shifts the poles to the left, which enhances the responsiveness and stability of the system.
The lag compensator section provides phase lag at low frequencies reducing the 'steady state' error.

Lag-Lead Network

Fig. 1 shows an electrical circuit representing a Lag-Lead network

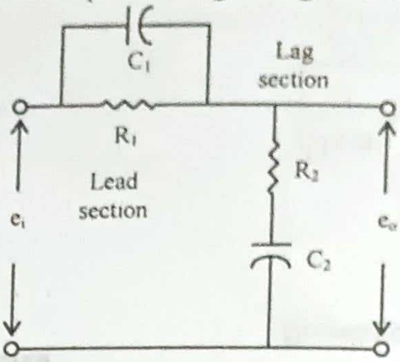


Fig: 1 Lag -Lead network

- ***Transfer Function:*** Applying potential divider rule

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2 + \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2} + \frac{R_1/sC_1}{R_1 + \frac{1}{sC_1}}} \quad \text{.... (2)}$$

After Simplification,

$$= \frac{(R_1 C_1 s+1)(R_2 C_2 s+1)}{(R_1 C_1 s+1)(R_2 C_2 s+1)+R_1 C_2 s}$$

$$= \frac{(R_1 C_1 s+1)(R_2 C_2 s+1)}{R_2 C_2 R_1 C_1 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2)s+1}$$

$$= \frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{\left[s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_1C_2}\right)s+\frac{1}{R_1R_2C_1C_2}\right]} \quad \text{.... (3)}$$

Comparing the above equations we get

$$R_1 C_1 = \tau_1, \qquad R_2 C_2 = \tau_2$$

$$R_1 R_2 C_1 C_2 = \alpha \beta \tau_1 \tau_2 \qquad \Rightarrow \alpha \beta = 1$$

and $$\left(\frac{1}{R_1 C_1}+\frac{1}{R_2 C_2}+\frac{1}{R_1 C_2}\right) = \frac{1}{\beta \tau_1}+\frac{1}{\beta \tau_2}$$

$\therefore$ Equation (1) may be re-written as

$$\frac{E_o(s)}{E_i(s)} = \frac{\left(s+\frac{1}{\tau_1}\right)\left(s+\frac{1}{\tau_2}\right)}{\left(s+\frac{1}{\beta\tau_1}\right)\left(s+\frac{1}{\alpha\tau_2}\right)} \quad \text{.... (4)}$$

where, $\beta = \frac{z_{C1}}{p_{C1}}$ and $\alpha = \frac{p_{C2}}{z_{C2}}$

b) Lead Compensators:

In phase lead compensation; the location of zero is closer to the origin of s-plane than that of the pole of the compensating network.

The transfer function of a lead compensator is given by

$$G_c(s) = \frac{s+\frac{1}{\tau}}{s+\frac{1}{\alpha\tau}} \quad \text{.... (1)} \quad \alpha < 1,\ \tau > 0$$

Sinusoidal transfer function of the lead network is given by

$$G_c(j\omega) = \frac{j\omega+\frac{1}{\tau}}{j\omega+\frac{1}{\alpha\tau}} \quad \text{.... (2)}$$

$\therefore \quad \angle G_c(j\omega) = \tan^{-1}\omega\tau - \tan^{-1}\omega\alpha\tau$ .... (3)

$\because \quad \omega\tau > \omega\alpha\tau \quad \therefore \angle G_c(j\omega)$ is positive.

So, under steady state conditions, output of this network leads the input and hence such network is known as a lead network.

Bode Plots

Fig. (a) shows the bode plots of a phase lead compensator.

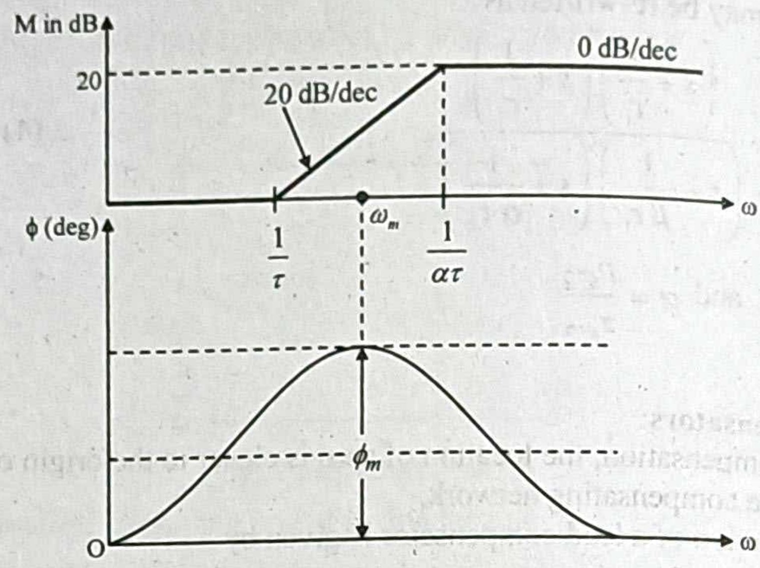


Fig: (a) Bode plots

The magnitude plot says that the Lead network behaves as a high pass filter.

Realization of Lead Network

The LEAD network is shown in the Fig. 1

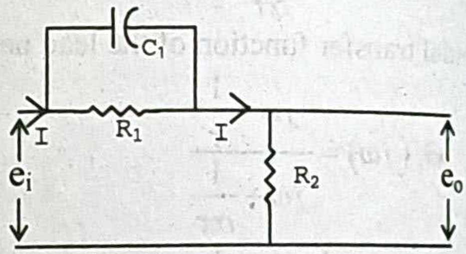


Fig: 1 Lead network

The "Transformed network of the above network (Fig.1) is shown in Fig. 2

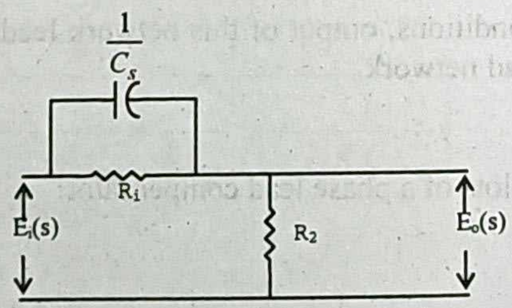


Fig: 2 Transformed lead network

Applying potential divider rule we get

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2}{\dfrac{R_1 \cdot \dfrac{1}{Cs}}{R_1 + \dfrac{1}{Cs}} + R_2} = \frac{R_2}{\dfrac{\dfrac{R_1}{Cs}}{\dfrac{R_1 Cs + 1}{Cs}} + R_2}$$

$$= \frac{R_2}{R_1 + R_2} \cdot \frac{(R_1 C s + 1)}{\left(\frac{R_1 R_2}{R_1 + R_2}\right) C s + 1} \quad \text{.... (1)}$$

Substituting $R_1 C = \tau$, and $\frac{R_2}{R_1 + R_2} = \alpha(<1)$,

$G_C(s)$ may be written in two forms

$$G_C(s) = \frac{\alpha(s\tau + 1)}{(\alpha\tau s + 1)} \quad \text{.... (2) Form I}$$

$$= \frac{\alpha\tau\left(s + \frac{1}{\tau}\right)}{\alpha\tau\left(s + \frac{1}{\alpha\tau}\right)} = \frac{\left(s + \frac{1}{\tau}\right)}{s + \frac{1}{\alpha\tau}} \quad \text{.... (2a) Form II}$$

Form I is suitable for frequency domain design where factor α gets cancelled by an amplifier having gain $A = \frac{1}{\alpha}$.

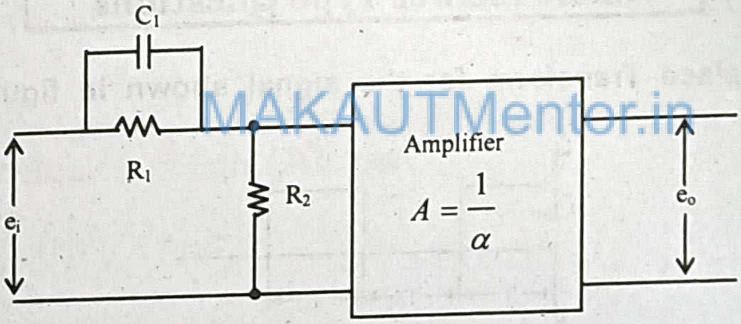


Fig: 3 Lead network

Form II is suitable for Root Locus design.

Use of Lead Compensator

We know that higher the order of the system lower is its stability. The system which are type-2 or higher, are usually unstable i.e., bear lower margin of stability.

As the Lead compensator increases the margin of stability. So we will use Lead Compensator for such system having lower degree of stability.

MISCELLANEOUS

Multiple Choice Type Questions

1. The transfer function of a ZOH is given by [WBUT 2014]

a) $G_{h0}(s) = \frac{1-e^{-Ts}}{s}$
b) $G_{h0}(s) = \frac{1-e^{-T}}{s}$
c) $G_{h0}(s) = 1-e^{-Ts}$
d) $G_{h0}(s) = s\left(1-e^{-Ts}\right)$

Answer: (a)

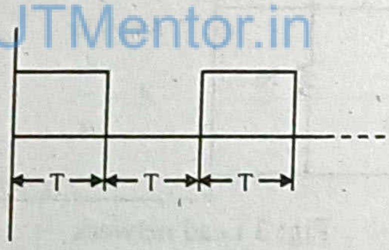
2. If the z-transformation of a function is $\frac{z\sin\omega T}{z^2 - 2z\cos\omega T + 1}$. Its corresponding Laplace transform will be [WBUT 2014]

a) $\frac{s}{s^2+\omega^2}$
b) $\frac{\omega}{s^2+\omega^2}$
c) $\frac{1}{s^2+\omega^2}$
d) $\frac{s+\omega}{s^2+\omega^2}$

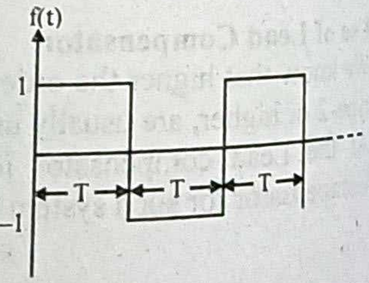
Answer: (b)

Short Answer Type Questions

1. Obtain the Laplace Transform for the signal shown in figure below : [WBUT 2011]



Answer:



$$f(t) = u(t) - 2u(t-T) + 2u(t-2T) \cdots\cdots$$

$$L^{-1}f(t) = F(s) = \frac{1}{s}\left[1 - 2e^{-Ts} + 2e^{-2Ts} - 2e^{-3Ts} \cdots\cdots\right]$$

$$= \frac{1}{s}\left[1 - 2e^{-Ts}\cdot\frac{1}{1+e^{-Ts}}\right] = \frac{1}{s}\left(\frac{e^{Ts}-1}{e^{Ts}+1}\right) = \frac{1}{s}\cdot\tanh\left(\frac{Ts}{2}\right)$$

2. State BIBO stability criterion. Show that for a bounded input-bounded output stable system $\int_0^\infty g(\tau)d\tau$ is finite, where $g(t)$ is the impulse response of the system. [WBUT 2014]

Answer:
The output remains bounded when the system is excited by a bounded input. This is called the Bounded Input Bounded Output (BIBO) stability criterion.
Let us consider a transfer function defined as

$$G(s)=\frac{C(s)}{R(s)}=\frac{b_0s^m+b_1s^{m-1}+b_2s^{m-2}+......+b_{m-1}s+b_m}{a_0s^n+a_1s^{n-1}+a_2s^{n-2}+......+a_{n-1}s+a_n} \quad .... (1)$$

We know, for impulse response, $R(s)=1$ and therefore, $C(s)=G(s)$. Taking the inverse Laplace transform, $c(t)=g(t)$. This is true only for the impulse input. But it can be shown that for any input $r(t)$, the output $c(t)$ can be written in the form of convolution $c(t)=\int_0^\infty g(\tau)r(t-\tau)d\tau$ .... (2)

where $g(\tau)$ is the impulse response of the system. If $r(t)$ remains bounded (i.e., finite) and $c(t)$ has to remain bounded for all t, we find from Eqn. (2) that it is necessary

$$\int_0^\infty g(\tau)d\tau = \text{finite} \quad .... (3)$$

We derived Eqn. (3) intuitively. Mathematically, it can be derived as follows. we take absolute value on both sides of Eqn. (2) to get

$$|c(t)|=\left|\int_0^\infty g(t)r(t-\tau)d\tau\right| \quad .... (4)$$

A theorem states that the absolute value of an integral is no greater than the integral of the absolute value of the integrand. Therefore, it follows from Eqn. (4) that

$$|c(t)|\le\int_0^\infty |g(t)r(t-\tau)|d\tau \quad .... (5)$$

$$\le\int_0^\infty |g(\tau)||r(t-\tau)|d\tau$$

If $r(t)$ is bounded, it means $|r(t)|\le M<\infty$ for $t\ge t_0$ .... (6)
Substitution of Eqn. (6) in Eqn. (5) implies

$$|c(t)|\le\int_0^\infty M|g(\tau)|d\tau$$

$$\le M\int_0^\infty |g(\tau)|d\tau \quad .... (7)$$

If now $c(t)$ has to remain bounded, it means

$$\left|c(t)\right| \le N < \infty \text{ for } t \ge t_0 \quad \text{.... (8)}$$

Combining Eqn. (7) and Eqn. (8), we get

$$\int_0^\infty \left|g(\tau)\right| d\tau \le P < \infty \quad \text{.... (9)}$$

Eqn. (9) is the same as that we arrived at Eqn. (3) from intuitive reasoning.

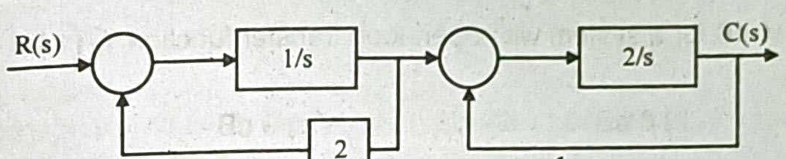
QUESTION 2015

Group – A
(Multiple Choice Type Questions)

1. Answer any *ten* questions.

i) A servomechanism is
- a) an automatic regulating system
- b) a position control system
- c) a process control system
- ✓d) a closed-loop system

ii) The transfer function of a system shown in the block-diagram is



- ✓a) $\dfrac{2}{(s+2)^2}$
- b) $\dfrac{1}{s^2(s+2)}$
- c) $\dfrac{2}{s^2+2}$
- d) $\dfrac{s}{s^2+2}$

iii) A second-order feedback system has two closed loop poles at the same location in the S-plane and has no finite zeros. The nature of unit step response of the system is
- a) under damped
- b) over damped
- ✓c) critically damped
- d) oscillatory

iv) A negative feedback control system has open loop transfer function $G(s)H(s)=\dfrac{k}{s^2(s+a)}$.

The closed loop system is
- a) unstable
- b) stable
- ✓c) marginally stable
- d) conditionally stable

v) A signal flow graph is used to determine the
- a) steady state error in the system
- b) stability of the system
- ✓c) transfer function of the system
- d) dynamic error co-efficient

vi) As compared to an open loop system, a closed loop system is
- a) more stable and more accurate
- ✓b) more stable and less accurate
- c) less stable and more accurate
- d) less stable and less accurate

vii) The Nyquist plot of a system encloses the point (–1, 0). The gain margin of the system is
- a) less than zero
- ✓b) greater than zero
- c) zero
- d) infinite

viii) The unit step response of a second-order system is

$$C(t)=1-1.125e^{-3t}\sin(4t-0.927).$$

The damping ratio and the damped frequency in rad/s are respectively
- a) 0.5, 3
- b) 0.5, 4
- ✓c) 0.6, 5
- d) none of these

ix) The addition of a PD controller in cascade with the plant
✓a) improves damping
b) reduces the steady state error
c) increases undamped frequency
d) increase the order of the system

x) The transfer function of a phase-lead compensator is $G_c(s) = \frac{1+Ts}{1+0.333Ts}$

The maximum phase contribution from the compensator is
a) 20°
b) 30°
c) 45°
✓d) 60°

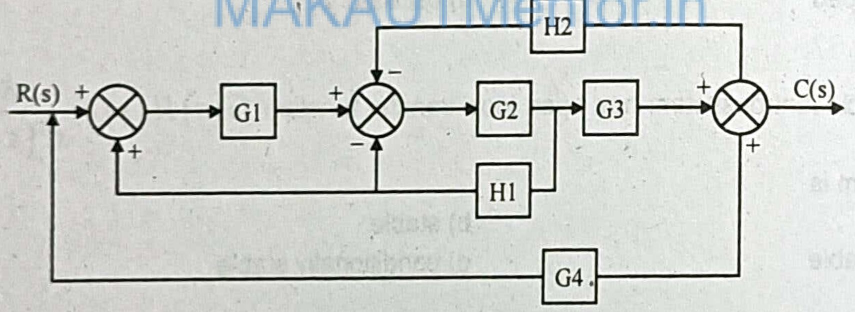
xi) The gain margin in dB for a system with open-loop transfer function $G(s) = \frac{2\sqrt{2}}{s(s+2)^2}$ is
a) 0
b) 6 dB
✓c) 9 dB
d) 12 dB

xii) The open-loop transfer function of a negative feedback system is $G(s)H(s) = \frac{k}{s(s+1)(s+2)}$.

The root crosses the imaginary axis when the value of k is
a) 8
b) 6
✓c) 4
d) 2

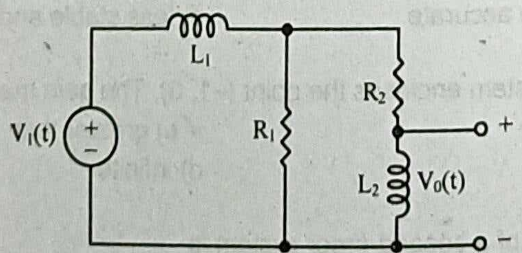
Group – B
(Short Answer Type Questions)

2. Obtain the transfer function C/R of the block diagram shown using Mason's Gain Formula.



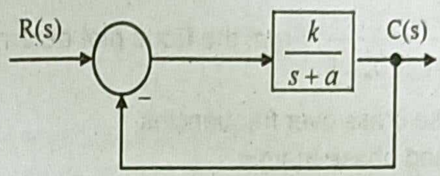
See Topic: SIGNAL FLOW GRAPH, Short Answer Type Question No. 4.

3. Draw a block diagram representation for the system shown below, representing every element by a block. Also find the transfer function $\frac{V_0(s)}{V_i(s)}$.



See Topic: TRANSFER FUNCTION, Short Answer Type Question No. 2.

4. For a first order system shown below, find the time constant, rise time and setting time for step response, given $k=12$ and $a=4$



See Topic: TIME DOMAIN ANALYSIS, Short Answer Type Question No. 8.

5. A unity feedback control system has the open loop transfer function

$$G(s)=\frac{k}{s\left(s^2+4s+20\right)}$$

Specify the type of the system.
Find the static error constants and the corresponding steady state errors. Assume that the system is stable.
See Topic: TIME DOMAIN ANALYSIS, Short Answer Type Question No. 9.

6. The characteristic equation of a feedback system is

$$s^4+4s^3+16s^2+16s+48=0$$

Check whether the response is oscillatory or not. If so, determine the frequency of oscillation.
See Topic: STABILITY ANALYSIS AND ROUTH STABILITY CRIETERION, Short Answer Type Question No. 5.

Group – C
(Long Answer Type Questions)

7. The forward path and feedback path transfer functions of a negative feedback system are $G(s)=\frac{5}{s^2(s+2)}$ and $H(s)=(s+a)$ respectively. Sketch the root contour for the system with respect to the parameter. For what range of value of 'a' does the system remain stable?
See Topic: ROOT LOCUS, Long Answer Type Question No. 4.

8. a) A second order system is described by the differential equation.

$$\frac{d^2y(t)}{dt^2}+0.8\frac{dy(t)}{dt}+y(t)=x(t).$$

When $x(t)$ is the input and $y(t)$ is the output. Determine resonance frequency, peak resonance, cut off frequency and band width.
See Topic: FREQUENCY RESPONSE, Long Answer Type Question No. 2.

b) Sketch the polar plot of

$$G(s)=\frac{32}{(s+4)\left(s^2+4s+8\right)}$$

And find its points of intersection with real and imaginary axes.
See Topic: NYQUIST PLOT, Long Answer Type Question No. 6.

9. Construct the Bode plots for a unity feedback system whose open loop transfer function is given by $G(s)=\dfrac{10}{s(s+1)(1+0.02s)}$. From the Bode plot determine:
   (i) Gain and phase cross over frequencies
   (ii) Gain margin and phase margin
   (iii) Stability of the closed loop system.

See Topic: BODE PLOT, Long Answer Type Question No. 6.

10. A feedback control system has forward path gain $G(s)=\dfrac{2}{s(s-1)}$ and feedback path gain $H(s)=(s+1)$.

Draw the Nyquist diagram for the system and assess the stability of the closed loop system.

See Topic: NYQUIST PLOT, Long Answer Type Question No. 7.

11. Write short notes on any *three* of the following:
   a) PID controller
   b) Position encoders
   c) Servo motors
   d) Synchros
   e) Block diagram of speed control of dc motor.

a) See Topic: CONTROL ACTION, Long Answer type Question No. 3(a).
b) See Topic: COMPONENTS OF A CONTROL SYSTEM, Long Answer type Question No. 3.
c) See Topic: COMPONENTS OF A CONTROL SYSTEM, Long Answer type Question No. 5(c).
d) See Topic: COMPONENTS OF A CONTROL SYSTEM, Long Answer type Question No. 5(d).
e) See Topic: COMPONENTS OF A CONTROL SYSTEM, Long Answer Type Question No. 5(g).

QUESTION 2016

Group – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following:

i) A system has a single pole of origin. Its impulse response will be
   ✓a) constant b) ramp c) decaying in nature d) oscillatory

ii) Without affecting the steady state error, maximum overshoot can be decreased by
   ✓a) derivative control b) integral control
   c) gain adjustment d) proportional control

iii) The centre of the constant M-circles are defined by
   a) $\left[\dfrac{M^2}{1+M^2},0\right]$ ✓b) $\left[\dfrac{M^2}{1-M^2},0\right]$
   c) $\left[0,\dfrac{M^2}{1+M^2}\right]$ d) $\left[0,\dfrac{M^2}{1-M^2}\right]$

iv) The initial slope of Bode plot for a transfer function having single poles at origin is
a) 20 db/decade
b) – 40 db/decade
c) 40 db/decade
✓d) – 20 db/decade

v) The capacitance, in force-current analogy is analogous to
a) velocity
b) momentum
c) displacement
✓d) mass

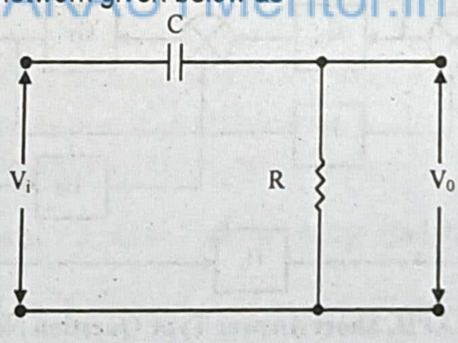
vi) The 'type' of a transfer function denotes the number of
a) zeros at origin
b) poles at infinity
✓c) poles at origin
d) finite poles

vii) The root locus diagram is
✓a) always symmetric about the real axis
b) always symmetric about the imaginary axis
c) never symmetric about the real axis
d) always asymmetric about both the real & imaginary axes

viii) The characteristic equation of a system is $s^2+2s+4=0$. The system is
a) critically damped
b) overdamped
c) undamped
✓d) underdamped

ix) If the gain margin of a certain feedback system is given as 20 db, the Nyquist plot will cross the negative real axis at the point
a) $S=-0.05$
b) $S=-0.2$
c) $S=0.5$
✓d) $S=-0.1$

x) The transfer function of the network given below as



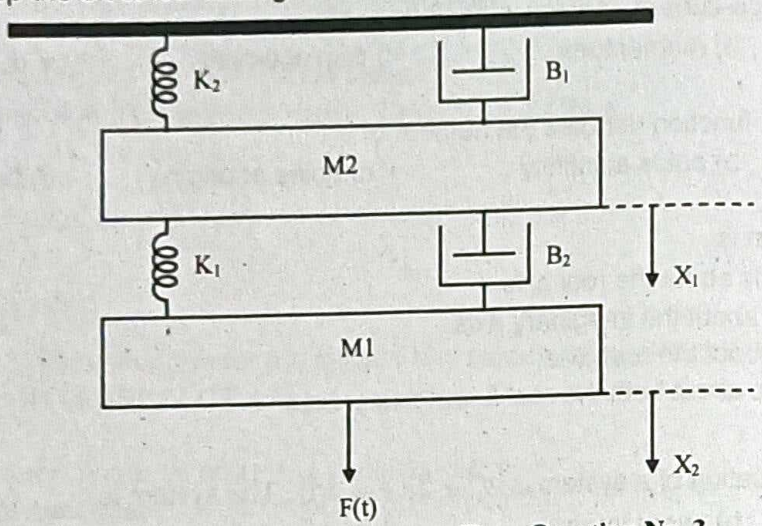
a) $\frac{1}{1+SRC}$
✓b) $\frac{SRC}{1+SRC}$
c) $\frac{RC}{1+SRC}$
d) $\frac{1+SRC}{1+RC}$

xi) For eliminating the steady state error, the control action required is
a) proportional control
b) proportional plus derivative control
c) proportional plus integral control
✓d) proportional, derivative & integral control

xii) The transfer function of a network is $\frac{1+0.3s}{2+s}$. It represents a
✓a) lag network
b) lead network
c) lag-lead network
d) proportional controller

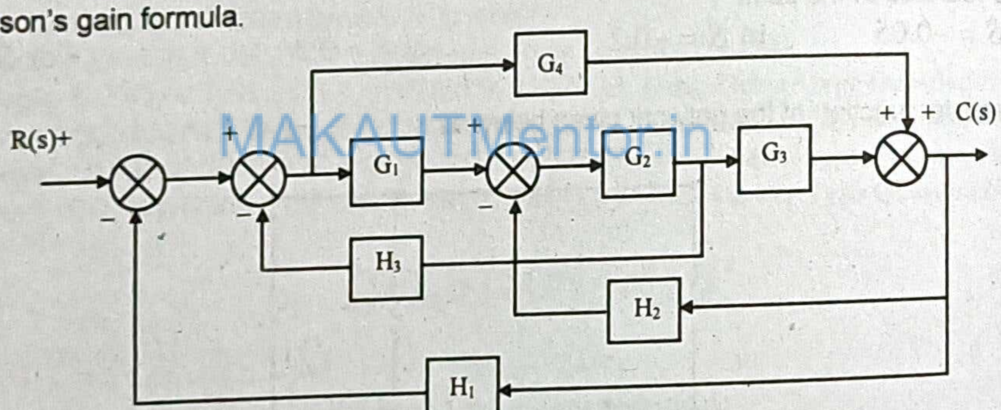
Group – B
(Short Answer Type Questions)

2. Determine the differential equation describing the complete dynamics of the mechanical system. Also develop the electrical analog circuit based on force – voltage analogy.



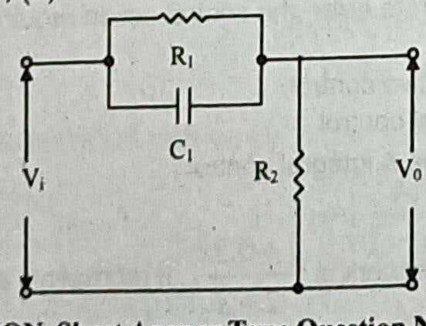
See topic: PHYSICAL SYSTEMS, Long Answer Type Question No. 3.

3. Find the overall transfer function of the system whose block diagram is given in figure below using Mason's gain formula.



See Topic: SINAL FLOW GRAPH, Short Answer Type Question No. 5.

4. Derive the transfer function $\frac{V_0(s)}{V_i(s)}$, for the electrical network shown below.



See Topic: TRANSFER FUNCTION, Short Answer Type Question No. 3.

5. The open loop transfer function of a unity feedback control system is given by

$$G(s)=\frac{k}{(s+2)(s+4)(s^2+6s+25)}.$$

By applying Routh's criterion determine the range of k for which the closed loop system will be stable and find the frequency of oscillation.

See Topic: STABILITY ANALYSIS & ROUTH STABILITY CRIETERION, Long Answer Type Question No. 1.b).

6. Consider the unit step response of a unity feedback system whose open loop transfer function $G(s)=\frac{1}{s(s+1)}$. Obtain the rise time, peak time, maximum overshoot & settling time.

See Topic: TIME DOMAIN ANALYSIS, Short Answer type Question No. 10.

Group – C
(Long Answer Type Questions)

7. a) Sketch the root loci of a system for the following open loop control system on a graph paper $G(s)=\frac{k}{s(s+3)(s^2+2s+2)}$ and comment on following factors: (i) number of root loci, (ii) number of asymptotes, (iii) angle of asymptotes and real axis intercept, (iv) angle of departure, (v) imaginary axis intercept.

b) Open loop transfer function of a system is given by $G(s)H(s)=\frac{k}{s(s+4)}$, check whether $s=-2+j2$ lies on root locus. If so, find system gain, k at given point.

a) See Topic: ROOT LOCUS, Long Answer Type Question No. 1.
b) See Topic: ROOT LOCUS, Long Answer Type Question No. 5.

8. a) The open loop transfer function of a unity feedback system is given by $G(s)=\frac{10}{s(1+s)(10+s)}$. Draw the Bode plot. Find phase crossover frequency, gain crossover frequency, gain margin & phase margin.

b) What is minimum phase system & non-minimum phase system? Give example.

See Topic: BODE PLOT, Long Answer type Question No. 7(a) & (b).

9. a) State the 'Principle of argument' & its extension to Nyquist criterion.

b) Draw the Nyquist plot & determine the stability condition for the open loop transfer function of the system $G(s)H(s)=\frac{(s+2)}{(s+1)(s-1)}$.

See Topic: NYQUIST PLOT, Long Answer Type Question No. 8(a) & (b).

10. a) Briefly discuss the necessity of PID controller in minimizing errors of a dynamic system response under step input. Show relevant graphical & mathematical expression.
b) Derive the transfer function for lead & lead-lag compensators with necessary passive equivalent circuits.

a) See Topic: CONTROL ACTION, Long Answer Type Question No. 2.
b) See Topic: COMPENSATORS, Long Answer Type Question No. 1.

11. a) Consider the transfer function $G(j\omega)=\frac{1}{j\omega(1+j\omega T)}$. Draw the polar plot of the function. Find the gain crossover frequency & phase margin of the transfer function.
b) A second order control system, having $\xi=0.4$ & $W_n=5$ rad/sec, is subject to a step input. Determine (i) transfer function, (ii) t_r, (iii) t_p, (iv) t_s for 2% tolerance, (v) M_p.

a) See Topic: NYQUIST PLOT, Long Answer Type Question No. 9.
b) See Topic: TIME DOMAIN ANALYSIS, Long Answer Type Question No. 4.

12. Write short notes on the following:
a) Liquid level control
b) Speed control of DC motor
c) DC tacho generator

a) See Topic: COMPONENTS OF CONTROL SYSTEM, Long Answer Type Question No. 1.
b) See Topic: COMPONENTS OF CONTROL SYSTEM, Long Answer Type Question No. 2.
c) See Topic: COMPONENTS OF CONTROL SYSTEM, Long Answer Type Question No. 5(c).

QUESTION 2017

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following:

i) A linear time-invariant system initially at rest, when subjected to an unit step input, gives a response $y(t)=te^{-t}$, the transfer function of the system is

a) $\frac{1}{(s+1)^2}$ b) $\frac{1}{s(s+1)^2}$ ✓c) $\frac{s}{(s+1)^2}$ d) $\frac{1}{s(s+1)}$

ii) The open loop transfer function of a feedback control system is $\frac{1}{(s+1)^3}$. The gain margin of the system is

a) 16 ✓b) 8 c) 4 d) 2

iii) A unity feedback system has three open loop poles at $(-2+j2)$ and 0. It has a single zero at (-4 ± 0). The angle of departure of the root locus branch starting from the pole $(-2-j2)$ is

a) 135° b) 225° ✓c) 0° d) –45°

iv) For a unit step input, a system with closed loop transfer function $\frac{20}{s^2+2s+5}$ has a steady state output of

a) 10 b) 5 c) 2 ✓d) 4

v) The transfer function of a system is its

a) square wave response b) step response
c) ramp response ✓d) impulse response

vi) The phase margin of the system for which the loop gain $GH(j\omega)=\frac{1}{(1+j\omega)^3}$ is

a) $-\pi$ b) π c) 0 ✓d) $\pi/2$

vii) The condition for stability of a closed loop system with characteristic equation $s^3+Bs^2+Cs+1=0$, with positive coefficients is

a) $B+C>1$ ✓b) $BC>1$ c) $B=C$ d) $B>C$

viii) If the gain of the open loop system is doubled, the gain margin

a) is not affected b) gets doubled
✓c) becomes half d) becomes 1/4th

ix) The function $\frac{1}{(1+sT)}$ has slope of

a) –6 dB/decade b) 6 dB/decade
✓c) –20 dB/decade d) 20 dB/decade

x) Phase margin of a system is used to specify

a) time response b) frequency response
c) absolute stability ✓d) relative stability

xi) The phase lead compensation is used to

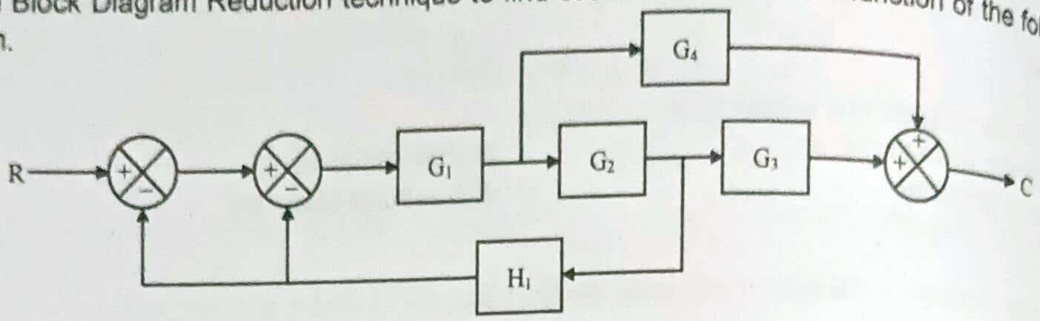
a) increase rise time and decrease overshoot
✓b) decrease both rise time and overshoot
c) increase both rise time and overshoot 20 dB/decade
d) decrease rise time and increase overshoot

xii) An a.c. servomotor is basically a

a) universal motor ✓b) two-phase induction motor
c) three-phase induction motor d) repulsion motor

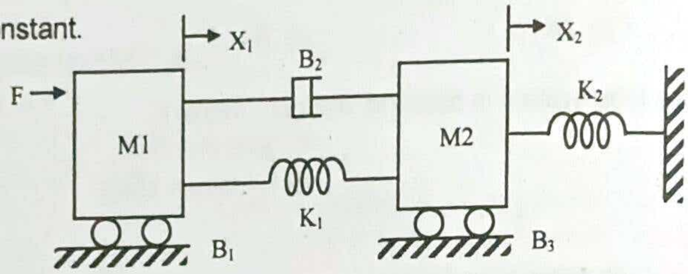
Group – B
(Short Answer Type Questions)

2. Use Block Diagram Reduction technique to find out the overall transfer function of the following system.



See topic: BLOCK DIAGRAM, Long Answer Type Question No. 1.

3. Consider the following mechanical translation system:
F denotes force, x denotes displacement, M denotes mass, B denotes friction coefficient and K denotes spring constant.



i) Write down the differential equations governing the above system.
ii) Draw the corresponding electrical equivalent circuit using force-voltage analogy scheme.

See topic: PHYSICAL SYSTEMS, Long Answer Type Question No. 1.

4. Sketch the time-domain response of $C(t)$ of a typical underdamped, second order system to a unit step input $r(t)$. On the above sketch indicate and define the following time domain specifications:

i) Maximum peak overshoot
ii) Rise time
iii) Settling time
iv) Steady state error.

See Topic: TIME DOMAIN ANALYSIS, Short Answer type Question No. 11.

5. The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{K}{s(1+sT_1)(1+sT_2)}.$$

Using the Routh-Hurwitz method, determine the necessary conditions for the system to be stable.

See Topic: STABILITY ANALYSIS & ROUTH STABILITY CRIETERION, Short Answer Type Question No. 6.

6. Define error coefficients corresponding to step, ramp and parabolic inputs. A unity feedback closed loop second order system has a transfer function $\frac{81}{s^2+0.6s+9}$, and it is excited by a step input of 10 units. Find out its steady state error.
See Topic: TIME DOMAIN ANALYSIS, Long Answer type Question No. 1

Group – C
(Short Answer Type Questions)

7. a) Explain what is meant by relative stability of a system. How do we specify relative stability in terms of (i) closed loop pole locations, (ii) gain margin and phase margin?
See Topic: FREQUENCY RESPONSE, Long Answer Type Question No. 3.

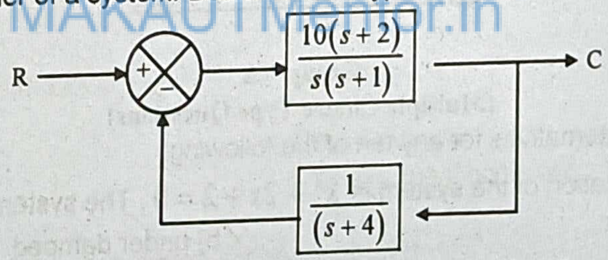
b) A unity feedback control system has open loop transfer function $G(s)=\frac{K}{s^3+s^2+s-3}$

i) Find the range of values of K so that the closed loop system is stable.
ii) Find the value of K for which the closed loop system will be marginally stable.
iii) Find the frequency of the undamped oscillation in such a case of marginal stability.

Answer: Wrong question

8. a) Explain the theory and operation of a two-phase servomotor and explain how a position control scheme can be made up by using the motor.
See Topic: INTRODUCTION, Long Answer Type Question No. 1.

b) Define the Type and Order of a system. Determine the type and order of the following system.



Determine the steady state error of the above system for the following input:

$$r(t)=0 \text{ for } t<0$$
$$=2+3t \text{ for } t\geq 0.$$

See Topic: TIME DOMAIN ANALYSIS, Long Answer Type Question No. 5.

9. a) A second order servo system has poles at $(-1\pm j2)$ and a zero at $(-1+j0)$. Its steady state output for a unit step input is 2.

i) Determine its transfer function.
ii) What is its peak overshoot for a unit step input?

b) Consider the open loop transfer function of a unity feedback system

$$G(s)=\frac{K(s+3)}{s(s^2+2s+2)(s+5)(s+6)}.$$

Draw the root locus diagram of the system on a graph paper and indicate on that diagram,

i) At what points will the imaginary axis be crossed by the root loci and what is the corresponding value of K ?

ii) Is $(-10+j0)$ a point on the root loci? Explain with valid reasons.

See Topic: ROOT LOCUS, Long Answer Type Question No. 6(a) & (b).

10. a) Explain the meaning and significance of phase margin and gain margins of a control system. How will you obtain the values of these margins from Bode plots?

b) Sketch the Bode plot for the following function and find out the approximate values of the gain margin and the phase margin

$$G(s)=\frac{10(s+2)}{s(s+6)(s+10)}.$$

See Topic: BODE PLOT, Long Answer Type Question No. 8.

11. State and explain the Nyquist stability criteria. Explain how gain and phase margins can be obtained from the Nyquist plot of a system. Sketch the Nyquist plot on a plain paper for the following transfer function and hence comment on the stability of the system:

$$G(s)=\frac{10}{s(1+s)(1+0.5s)}$$

See Topic: NYQUIST PLOT, Long Answer Type Question No. 10.

QUESTION 2018

Group – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following:

i) The characteristic equation of the system is $s^2+2s+2=0$. The system is

a) critically damped ✓b) under damped

c) over damped d) undamped

ii) The transfer function $G(s)$ of a PID controller is

✓a) $k\left[1+\frac{1}{T_i s}+T_d s\right]$ b) $k\left[1+T_i s+T_d s\right]$

c) $k\left[1+\frac{1}{T_i s}+\frac{1}{T_d s}\right]$ d) $k\left[1+T_i s+\frac{1}{T_d s}\right]$

iii) The Bode Plot of $(1+JWT)$ has
✓a) Slope of 20 dB/decade and phase angle $+\tan^{-1}(WT)$
b) Slope of –20 DB/decade & phase angle $+\tan^{-1}(WT)$
c) Slope of 20 dB/decade & phase angle $-\tan^{-1}(WT)$
d) Slope of –40/dB decade & phase angle $-\tan^{-1}(WT)$

iv) The frequency of which Nyquist diagram crosses the negative real axis is known as
a) gain crossover frequency
✓b) phase crossover frequency
c) Natural frequency
d) break away point

v) Addition of zero to the closed loop transfer function
a) increase rise time
✓b) decrease rise time
c) increase overshoot
d) has no effect

vi) The value of ξ for a second order system is zero. The step response will be
a) over damped
b) critically damped
c) under damped
✓d) sustained oscillatory

vii) The root loci of a system have three asymptotes. The system can have
✓a) five poles & two zeros
b) three poles & one zero
c) four poles & two zeros
d) six poles & two zeros

viii) The transfer function of a network is $\frac{1+0.3s}{2+s}$. It represents a
✓a) lag network
b) lead network
c) lead – lag network
d) proportional controller

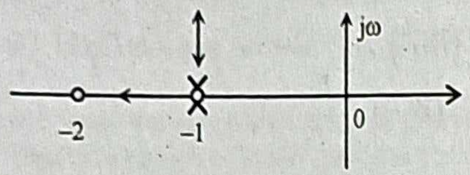
ix) For eliminating steady state error, the control action required is
a) Proportional control
b) Proportional plus derivative control
✓c) Proportional plus integram control
d) Proportional, derivative & integral control

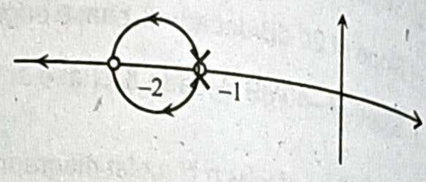
x) The characteristic equation $1+G(s)H(s)=0$ of a system is given by

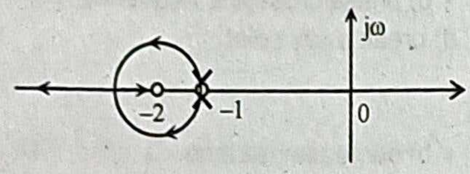
$$s^4+6s^3+11s^2+6s+K=0$$

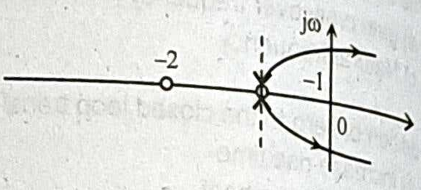
For the system to remain stable, the value of gain K should be
a) zero
✓b) greater than zero but less than 10
c) greater than 10 but less than 20
d) greater than 20 but less than 30

xi) Given a unity feedback system with open-loop transfer function $(s)=\frac{K(s+2)}{(s+1)^2}$. The correct root-locus plot of the system is

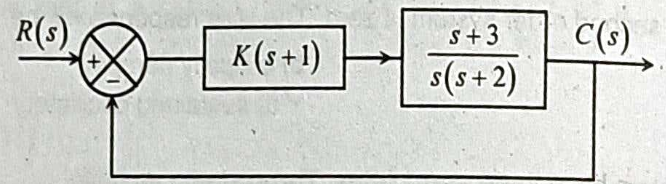
a) 

b) 

✓c) 

d) 

xii) For the system in the given figure the characteristic equation is

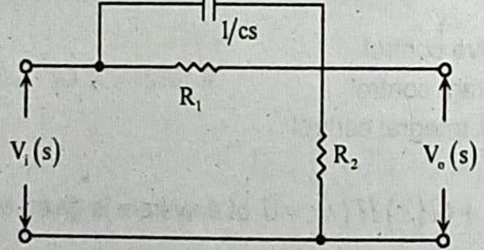


✓a) $1+\frac{K(s+1)(s+3)}{s(s+2)}=0$ b) $1+\frac{K(s-1)(s-3)}{s(s-2)}=0$

c) $K(s+1)(s+3)=0$ d) $s(s+2)=0$

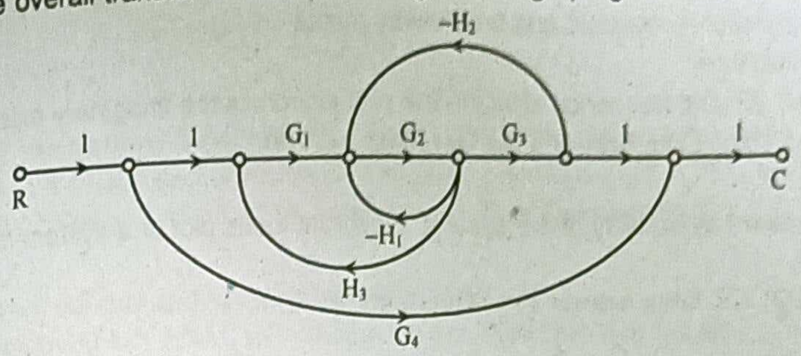
Group – B
(Short Answer Type Questions)

2. Find the transfer function of the system shown in the figure:



See Topic: TRANSFER FUNCTION, Short Answer Type Question No. 4.

3. Determine the overall transfer function of the signal flow graph given below using Mason's gain formula

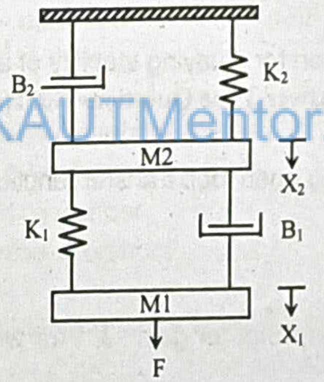


See Topic: TRANSFER FUNCTION, Short Answer Type Question No. 6.

4. A unity feedback system has an open loop transfer function $G(s)=\dfrac{25}{s(s+8)}$. Find its damping ratio, natural frequency, rise time, over shoot & time required to reach the peak output.

See Topic: TIME DOMAIN ANALYSIS, Short Answer Type Question No. 12.

5. Consider the following mechanical translational system. F denotes force, x denotes displacement, M denotes mass, B denotes friction coefficient and K denotes spring constant.



a) Write down the differential equations governing the above system.
b) Draw the corresponding electrical equivalent circuit using force-voltage analogy scheme.

See Topic: PHYSICAL SYSTEMS, Short Answer Type Question No. 2.

6. A unity feedback system has $G(s)=\dfrac{180}{s(s+6)}$ & $r(t)=4t$.

Determine: (a) Steady state error
(b) The value of K to reduce error by 6%

See Topic: TIME DOMAIN ANALYSIS, Short Answer Type Questions No. 2.

Group – C
(Long Answer Type Questions)

7. a) The open loop transfer function of a unity feedback control system is given by

$$G(s)=\frac{K}{s(s+1)(s+3)(s+5)}$$

Sketch the root locus plot of the system by finding the following:

i) angle of asymptotes, centroid and breakaway points
ii) angle of departure
iii) the value of K and frequency at which the root loci cross the imaginary axis.

See Topic: ROOT LOCUS, Long Answer Type Questions No. 7(a).

b) Explain what is meant by stability of the system. How root locus plot of a system is useful to find stability?
See Topic: ROOT LOCUS, Long Answer Type Questions No. 7(b).

8. a) What are the advantages of Bode diagram?
See Topic: BODE PLOT, Long Answer Type Questions No. 9(a).

b) Sketch the asymptotic Bode plot for the following open loop transfer function with unit feedback.

$$G(s)H(s)=\frac{20(s+10)}{s(s+20)(s^2+s+1)}$$

Calculate the gain and phase cross-over frequency, gain margin and phase margin of the Bode plot. Also determine the closed loop stability of the system.
See Topic: BODE PLOT, Long Answer Type Questions No. 9(b).

9. a) State and explain Nyquist criterion for studying stability of a control system.
See Topic: NYQUIST PLOT, Long Answer Type Questions No. 1(a).

b) For a unity feedback system having open loop transfer function

$$G(s)=\frac{K}{s(s^2+s+4)}.$$

Determine using Nyquist criterion the range of gain 'K' for which the closed loop system will be stable.
See Topic: NYQUIST PLOT, Long Answer Type Questions No. 1(b).

10. a) Obtain the equations for the armature controlled DC servomotor and find the transfer function of the DC servomotor.
See Topic: COMPONENTS OF A CONTROL SYSTEM, Long Answer Type Questions No. 2.

b) Show that the transfer function of a two-phase induction motor can be written in form

$$\frac{\theta_m(s)}{V_z(s)}=\frac{K_m}{s(1+s\tau_m)}$$

What are the expressions for K_m and τ_m and what are they called?
See Topic: TRANSFER FUNCTION, Long Answer Type Questions No. 1.

11. a) Derive the expression for the time response of a first order system subjected to unit step input.
See Topic: TIME DOMAIN ANALYSIS, Long Answer Type Question No. 6(a).

b) The open loop transfer function of a unity feedback system is given by

$$G(s)=\frac{K}{s(Ts+1)}$$

where K and T are positive constants. By how much should the amplifier gain be reduced so that the peak overshoot of unit step response of the system is reduced from 75% to 25%?
See Topic: TIME DOMAIN ANALYSIS, Long Answer Type Question No. 2.

c) Define position, velocity and acceleration error constants.
See Topic: TIME DOMAIN ANALYSIS, Long Answer Type Question No. 6(b).

QUESTION 2019

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following:

i) If the gain of an open loop system is doubled, the gain margin
a) is not affected b) gets doubled ✓c) becomes half d) become 1/4th

ii) Feedback control system is basically
a) High pass filter ✓b) Low pass filter c) Band pass filter d) Band stop filter

iii) The phase lead compensation is used to
a) increase rise time and decrease overshoot
b) decrease both rise time and overshoot
✓c) increase both rise time and overshoot
d) decrease rise time and increase overshoot

iv) For type-1, second order system the resonance peak will occur when the system gain is at the
a) underdamping ✓b) critical damping value
c) overdamping value d) none of these

v) A linear time invariant system obeys
a) the principle of superposition b) the principle of homogeneity
✓c) both of the principles in (a) and (b) d) none of these

vi) If the maximum overshoot is 100%, the damping ratio is
a) 1 ✓b) 0 c) 0.5 d) infinite

vii) If the gain k of the system increases, the steady state error of the system
✓a) decreases b) increases
c) may increase or decrease d) remains unchanged

viii) By the use of PD controller to a second order system, the rise time
✓a) decreases b) increases c) remains same d) has no effect

ix) The open loop transfer function of a unity feedback control system is $G(s)\,H(s) = 100(1+2s)/s^2(1+5s)$.

If the system is subjected to an input $r(t)=1+t+t^2/2(t\geq 0)$, the steady state error of the system will be

a) 1 b) 0.1 ✓c) 0.01 d) 100

x) The characteristics equation of a system is $s^2+2s+4=0$. The system is

a) critically damped b) overdamped

✓c) undamped d) under-damped

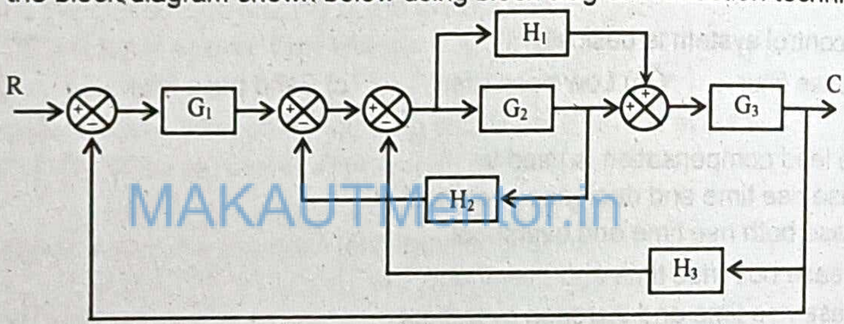
xi) A potentiometer converts linear/rotational displacement into

a) current b) power ✓c) voltage d) torque

Group – B

(Short Answer Type Questions)

2. Find C/R for the block diagram shown below using block diagram reduction techniques.

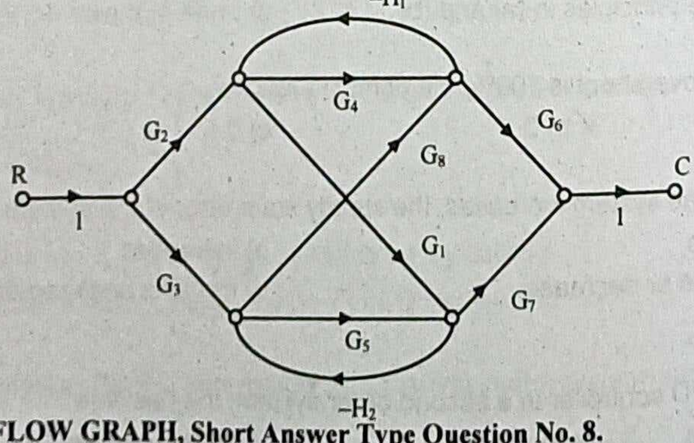


See Topic: SIGNAL FLOW GRAPH, Short Answer Type Question No. 7.

3. Derive the transfer function of armature controlled DC Motor.

See Topic: BLOCK DIAGRAM, Short Answer Type Question No. 2.

4. Evaluate the transfer function using Mason's gain formula of the equivalent signal-flow graph shown in figure.



See Topic: SIGNAL FLOW GRAPH, Short Answer Type Question No. 8.

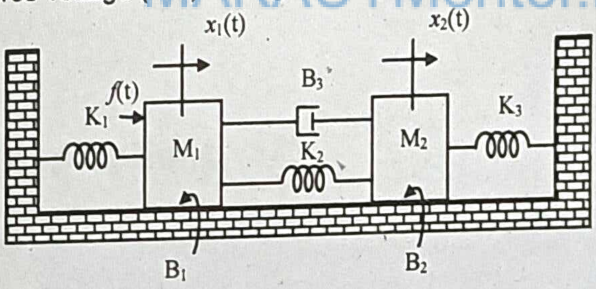
5. A feedback system has an open loop transfer function $G(s)H(s) = ke^{-s}/s(s^2+2s+1)$. Determine by use of Routh-Hurwitz criterion the maximum value of *k* for the closed loop system to be stable. Also find the frequency of sustained oscillations.
See Topic: STABILITY ANALYSIS AND ROUTH STABILITY CRITERION, Short Answer Type Question No. 7.

6. A unity feedback control system has an open loop transfer $G(s) = 5/s(s+1)$ Find the rise time, percentage overshoot, peak time & settling time (2% error) for a step input of 10 units.
See Topic: TIME DOMAIN ANALYSIS, Short Answer Type Question No. 13.

7. For a closed loop system with $G(s) = 15/(s+2)(s+5)$ and $H(s) = 1$ calculate the generalized error coefficient & find error series for input of $3 + 8t + 5t^2/2$.
See Topic: TIME DOMAIN ANALYSIS, Short Answer Type Question No. 14.

Group – C
(Long Answer Type Questions)

8. a) Define transfer function of a system.
b) Obtain the differential equations describing the dynamics of the mechanical translational system and find the transfer function $X_2(s)/F(s)$ for the system shown in figure; *f* denotes the force, *x* denotes displacement, *M* denotes mass, *B* denotes friction co-efficient and *K* denotes spring constant. Also draw its force-voltage (F-V) analogous circuit.



See Topic: PHYSICAL SYSTEMS, Long Answer Type Question No. 4.

9. Sketch the root locus diagram as *k* is varied from zero to infinity for the system whose open loop transfer function is given by

$$G(s)H(s) = k/\left[s(s+4)(s^2+4s+20)\right]$$

Evaluate the value of *k* at the point where the root locus crosses the imaginary axis. Also determine the frequency at this point.
See Topic: ROOT LOCUS, Long Answer Type Question No. 8.

10. a) Construct the Bode plot for a unity feedback control system having

$$G(s) = 36(0.2s+1)/s^2(0.05s+1)(0.01s+1)$$

b) From the plot obtain the gain margin, phase margin, gain cross over frequency, phase cross over frequency.
c) Comment on the closed loop stability of the system.

See Topic: BODE PLOT, Long Answer Type Question No. 10.

11. a) State and explain Nyquist criteria for study of control system.
b) The open loop transfer function of closed loop system is

$$G(s)H(s) = 120/[s(s+3)(s+5)]$$

Draw the Nyquist plot and hence find out whether the system is stable or not.
c) What are the advantages of Nyquist plot?
See Topic: NYQUIST PLOT, Long Answer Type Question No. 11.

12. Write the short notes any *three* of the following:
   a) PID Controller
   b) Led-lag Compensator
   c) Nichols Chart
   d) Synchros
   e) D.C. & A.C. tacho-generators

a) See Topic: CONTROL ACTION, Long Answer Type Question No. 3(a).
b) See Topic: COMPENSATORS, Long Answer Type Question No. 2(a).
c) See Topic: NICHOLS CHART, Long Answer Type Question No. 1.
d) See Topic: COMPONENTS OF A CONTROL SYSTEM, Long Answer Type Question No. 5(d).
e) See Topic: COMPONENTS OF A CONTROL SYSTEM, Long Answer Type Question No. 5(b) & (e).