ELECTRICAL ENGINEERING

ELECTROMAGNETIC FIELD THEORY [EMFT]

NEW SYALLABUS FOR BTECH

CREDIT=3

ALL SYALLABUS COVERED

ELECTROMAGNETIC FIELD THEORY (c=3)

1) Introduction :—

→ co-ordinate systems and transformation, cartesian coordinates, circular cylinder coordinates, spherical coordinates & their transformation. Differential length, area and Volume in different coordinate systems. Solution of problems.

2. Introduction to vector calculas :—

DEL operator, Gradient of a scalar, Divergence of a vector & Divergence theorem, curl of a vector & strokes theorem, Laplacian of a scalar, classification of vector fields, Helmholtz's theorem. Solution of problems.

3. Electrostatic field :—

Coulomb's law, field intensity, Gauss's law, electric potential and Potential gradient, Relation b/w E and v, an electric dipole and flux lines. Energy density in electrostatic field. Boundary conditions : Dielectric – dielectric, conductor – dielectric, conductor – free space. Poisson's and Laplace's equation, General procedure for solving poisson's and Laplace's equation. Solution of problems.

4) Magneto Static field :—

Bio-savert law, Ampere's circuit law, Magnetic flux density, magnetic static and vector potential, forces due to magnetic field, Magnetic torque and moments, Magnetisation in material, Magnetic boundary condition, ~~Induc~~ Inductor and Inductances, Magnetic energy, force on magnetic material. Solution of problems.

5) Electromagnetic fields:-

faraday's law, Transformer and motional emf. Displacement current, Maxwells equations, Time varying potential, Time harmonic fields. Solution of problems.

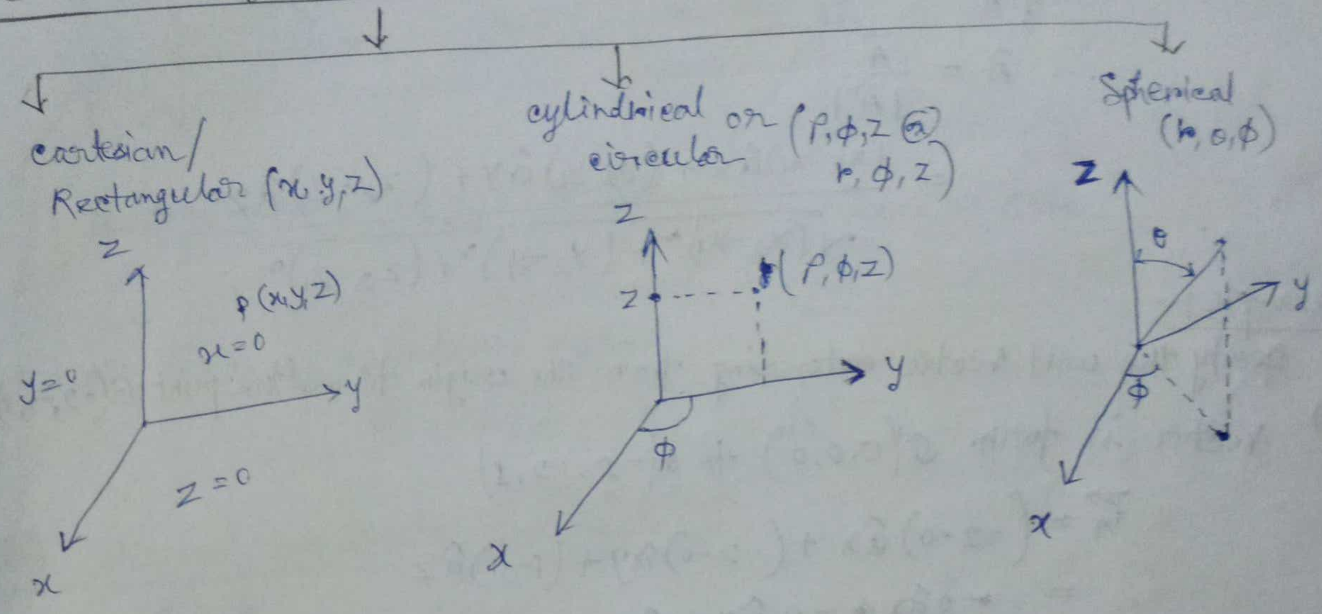
6) Electromagnetic wave propagation:-

wave equation, wave propagation in lossy dielectric, plane waves in loss less dielectric, plane wave in free space, plane wave in good conductor, skin effect, skin depth, power & poynting vector, Reflection of a plane wave at normal incidence, reflection of a plane wave at oblique incidence, polarisation. solution of problems

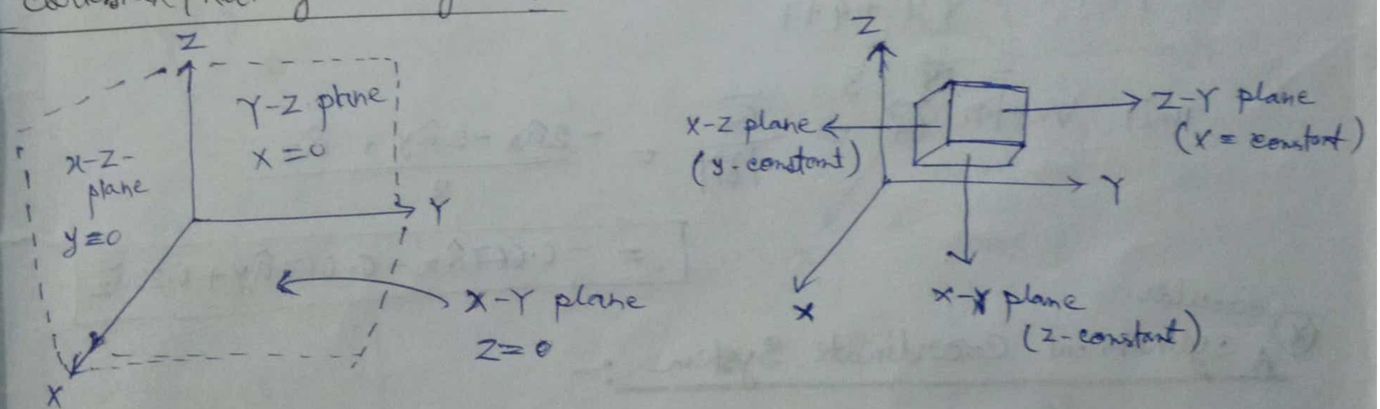
7) Transmission Line:-

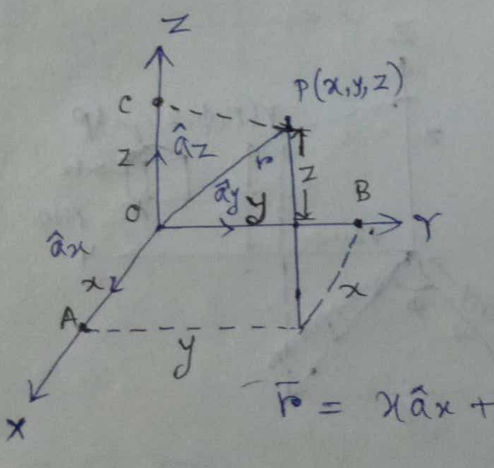
Concept of lump & distributed parameters. Line parameters, transmission line equation & solution, physical significance of solutions, propagation constants, characteristics impedance, wavelength, velocity of propagation, solution of problems, ① Introduction

co-ordinate system :—



cartesian/Rectangular system :—





position vector

$\bar{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$

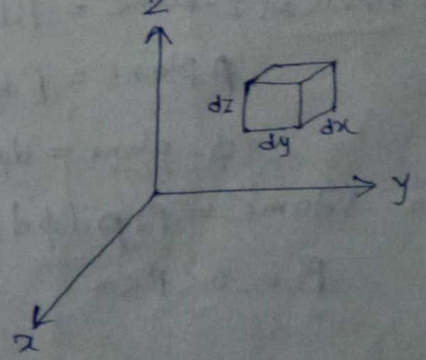
length. $d\vec{L} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$

Area ds in x-direction/x-plane $= dy\,dz$

y " $= dx\,dz$

Z " $= dx\,dy$

Volume $= dx\,dy\,dz$



Representation of vector from (x_1, y_1, z_1) to (x_2, y_2, z_2)

$$\vec{A} = (x_2 - x_1)\hat{a}x + (y_2 - y_1)\hat{a}y + (z_2 - z_1)\hat{a}z.$$

unit Vector of $\vec{A}$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

$$= \frac{(x_2 - x_1)\hat{a}x + (y_2 - y_1)\hat{a}y + (z_2 - z_1)\hat{a}z}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

example:-

Specify the unit vector extending from the origin toward the point $G(-2,-2,1)$

⇒ Vector is from $O(0,0,0)$ to $G(-2,-2,1)$

$$\vec{G} = (-2-0)\hat{a}x + (-2-0)\hat{a}y + (1-0)\hat{a}z$$

$$= -2\hat{a}x - 2\hat{a}y + \hat{a}z$$

$$|\vec{G}| = \sqrt{4+4+1}$$

$$= 3$$

$$\text{unit vector } \vec{G} = \frac{\vec{G}}{|\vec{G}|} = \frac{-2\hat{a}x - 2\hat{a}y + \hat{a}z}{3}$$

$$\boxed{= -0.667\hat{a}x - 0.667\hat{a}y + 0.33\hat{a}_z}$$

⊛ circular cylindrical Coordinate System :-

$x = \rho\cos\phi$

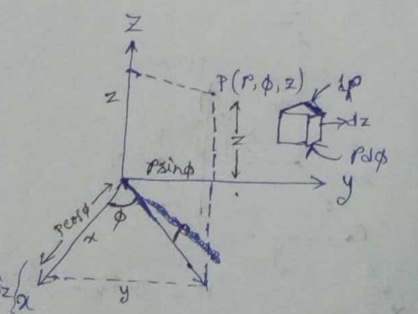
$y = \rho\sin\phi$

$z = z$

$\rho = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}(y/x)$

$z = z$



∴ length $\vec{dl} = d\rho\hat{a}_\rho + \rho d\phi\hat{a}_\phi + dz\hat{a}_z$

∴ Area at z-plane $= \rho d\rho d\phi$

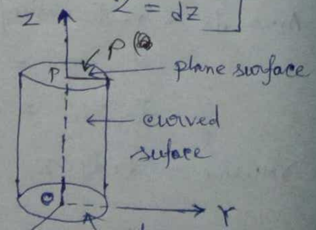
ρ-plane $= \rho d\phi dz$

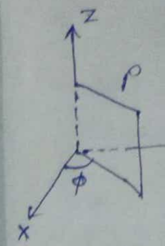
φ-plane $= d\phi dz$

∴ Volume $= \rho d\rho d\phi dz$

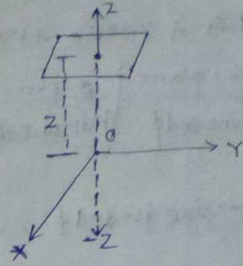
$\rho = 0 < \rho < \alpha$

$$\left[\begin{array}{l} \rho = d\rho \\ \phi = \rho d\phi \\ z = dz \end{array}\right]$$





2. $0 \le \phi \le 360°$



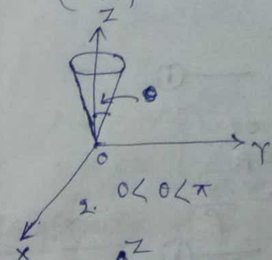
3. $-\alpha < z < \alpha$

vector form of cylindrical system

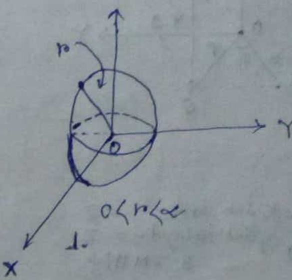
$$\vec{r} = \rho\hat{a}\rho + \phi\hat{a}\phi + z\hat{a}z$$

⊛ Spherical coordinates System :-

(r, θ, ϕ)

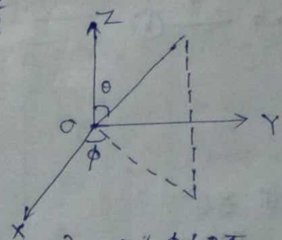


2. $0 < \theta < \pi$

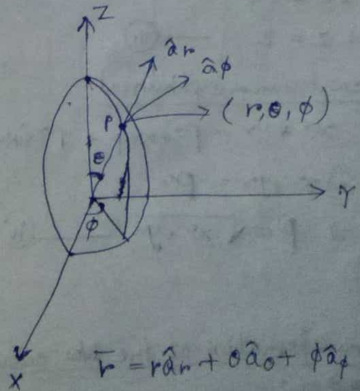


$0 < r < \alpha$

1.



3. $0 < \phi < 2\pi$



$\vec{r} = r\hat{a}_r + \theta\hat{a}_\theta + \phi\hat{a}_\phi$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

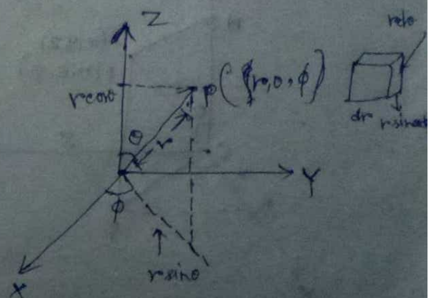
$z = r\cos\theta$

} spherical to rectangular

$r = \sqrt{x^2 + y^2 + z^2}$

$\theta = \cos^{-1}\frac{z}{\sqrt{x^2 + y^2 + z^2}}$

$\phi = \tan^{-1} y/x$



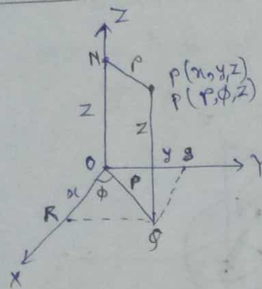
length $\vec{dl} = dr\hat{a}_r + r d\theta \hat{a}_\theta + r\sin\theta\, d\phi\, \hat{a}_\phi$

Area =	r-plane	θ-plane	φ-plane
	$r^2 \sin\theta\, d\theta\, d\phi$	$r\, dr \sin\theta\, d\phi$	$r\, dr\, d\theta$

Volume $= r^2 \sin\theta\, dr\, d\theta\, d\phi$

(*) Transformation :-

transformation of the coordinate b/w cartesian & cylinderical coordinate system :-



$\angle R = 90°$

$\triangle ORQ$

$\cos\phi = \frac{OR}{OQ}$

$\Rightarrow \cos\phi = \frac{x}{\rho}$

$\Rightarrow x = \rho\cos\phi$ —— (i)

$\therefore \sin\phi = \frac{RQ}{OR} = \frac{OS}{OR}$

$\Rightarrow \sin\phi = \frac{y}{\rho}$

$\Rightarrow y = \rho\sin\phi$ —— (ii)

ON = cartesian system = Z

PQ = cylindrical system = Z

& ON ∥ PQ

ON = PQ

$\Rightarrow z = z$ —— (iii)

cartesian → cylindrical.

$x^2 + y^2 = \rho^2\cos^2\phi + \rho^2\sin^2\phi$

$\Rightarrow x^2 + y^2 = \rho^2$

$\Rightarrow \rho = \sqrt{x^2 + y^2}$ —— (iv)

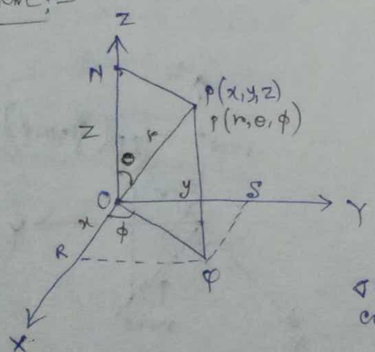
∴ (ii) ÷ (i)

$\frac{\rho\sin\phi}{\rho\cos\phi} = y/x$

$\Rightarrow \tan\phi = y/x$

$\Rightarrow \phi = \tan^{-1}(y/x)$ —— (v)

transformation of the coordinate b/w cartesian & spherical coordinate system :-



$\angle OPN$

$\cos\theta = \frac{ON}{OP} = \frac{z}{r}$

$\Rightarrow z = r\cos\theta$ —— (i)

$\sin\theta = \frac{NP}{OP} = \frac{NP}{r}$

$NP = r\sin\theta = OQ$

$\angle ORQ$

$\cos\phi = \frac{OR}{OQ} = \frac{x}{r\sin\theta}$

$\Rightarrow x = r\sin\theta\cos\phi$

$\sin\phi = \frac{RQ}{OQ} = \frac{OS}{OQ}$

$\Rightarrow \sin\phi = \frac{y}{r\sin\theta}$

$\Rightarrow y = r\sin\phi\sin\theta$ —— (iii)

c → spherical

$x^2 + y^2 + z^2 = r^2\cos^2\theta + r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi$

$= r^2\sin^2\theta(1) + r^2,$

$x^2 + y^2 + z^2 = r^2$

$\Rightarrow r = \sqrt{x^2 + y^2 + z^2}$ —— (iv)

(iii) ÷ (i)

$\frac{r\sin\theta\sin\phi}{r\sin\theta\cos\phi} = y/z$

$\Rightarrow \tan\phi = y/z$

$\Rightarrow \phi = \tan^{-1}(y/z)$ —— (v)

$\cos\theta = \frac{z}{r}$

$\therefore \theta = \cos^{-1}(z/r)$

$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$ —— (vi)

transformation of a Vector from cartesian to cylindrical coordinate system:

$\bar{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$ (cartesian)

$$\begin{vmatrix} A_\rho \\ A_\phi \\ A_z \end{vmatrix} = \begin{vmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix}$$

$A_\rho = A_x\cos\phi + A_y\sin\phi + 0$

$A_\phi = -A_x\sin\phi + A_y\cos\phi + 0$

$A_z = 0 + 0 + A_z$

$= A_z$

$\rho = \sqrt{x^2 + y^2}$, $\phi = \tan^{-1} y/x$, $z = z$

$\bar{A} = A_\rho\hat{a}_\rho + A_\phi\hat{a}_\phi + A_z\hat{a}_z$

~~stream~~

Transformation of a vector from cylindrical to cartesian coordinate system :—

$$\vec{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

$$\begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix} = \begin{vmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} A_\rho \\ A_\phi \\ A_z \end{vmatrix}$$

cartesian system constant; cylindrical system constant

$$A_x = A_\rho \cos\phi - A_\phi \sin\phi$$

$$A_y = A_\rho \sin\phi + A_\phi \cos\phi$$

$$A_z = A_z$$

cylindrical — cartesian

$$x = \rho\cos\phi, \quad y = \rho\sin\phi, \quad z = z$$

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

Transformation of vector from spherical to cartesian coordinate :—

$$\vec{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi \quad \text{(spherical)}$$

$$\begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix} = \begin{vmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{vmatrix} \cdot \begin{vmatrix} A_r \\ A_\theta \\ A_\phi \end{vmatrix}$$

$$A_x = A_r(\sin\theta\cos\phi) + A_\theta(\cos\theta\cos\phi) - A_\phi \sin\phi$$

$$A_y = \sin\theta\sin\phi\, A_r + \cos\theta\sin\phi\, A_\theta + \cos\phi\, A_\phi$$

$$A_z = \cos\theta\, A_r - \sin\theta\, A_\theta$$

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi \quad z = r\cos\theta$$

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

2. Introduction to Vector calculas

Vector calculas
- Vector Differentiation
  - Gradient of a scalar (DEL operator)
  - Divergence of a vector
  - curl of a vector
  - Laplace of a scalar
- Vector Integration
  - Line integrals
  - Surface integral
  - Volume Integral

DEL operator :—

Rectangular $\nabla = \frac{\delta}{\delta x} a_x + \frac{\delta}{\delta y} a_y + \frac{\delta}{\delta z} \cdot a_z$

cylindrical $\nabla = \frac{\delta}{\delta \rho} a_\rho + \frac{1}{\rho}\frac{\delta}{\delta \phi} a_\phi + \frac{\delta}{\delta z} a_z$

spherical $\nabla = \frac{\delta}{\delta r} a_r + \frac{1}{r}\frac{\delta}{\delta \theta} a_\theta + \frac{1}{r\sin\theta}\frac{\delta}{\delta \phi} a_\phi$

$$\nabla = \frac{1}{h_1}\frac{\delta}{\delta u} \cdot a_u + \frac{1}{h_2}\frac{\delta}{\delta v} a_v + \frac{1}{h_3}\frac{\delta}{\delta w} a_w$$

i) Gradient of scalar (∇V)

ii) Divergence of a vector $(\nabla \cdot A)$

iii) curl of a vector $(\nabla \times A)$

iv) Laplace of a scalar $\boxed{\nabla \cdot (\nabla V) = \nabla^2 V}$

Gradient of scalar function :—

for Rectangular/cartesian coordinate :—

$$\nabla V = \frac{\delta V}{\delta x} a_x + \frac{\delta V}{\delta y} a_y + \frac{\delta V}{\delta z} a_z$$

for circular cylindrical coordinate :

$$\nabla V = \frac{\delta V}{\delta \rho} a_\rho + \frac{1}{\rho}\frac{\delta V}{\delta \phi} a_\phi + \frac{\delta V}{\delta z} a_z$$

for spherical coordinate :—

$$\nabla V = \frac{\delta V}{\delta r} \cdot a_r + \frac{1}{r}\frac{\delta V}{\delta \theta} a_\theta + \frac{1}{r\sin\theta}\frac{\delta V}{\delta \phi} a_\phi$$

V = scalar function

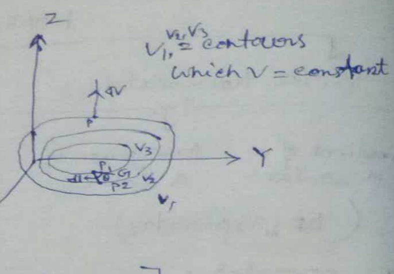
∇V = Vector function

If A & B are two scalars then,

$$\boxed{\nabla(A \pm B) = \nabla A \pm \nabla B}$$

$$\boxed{\nabla(A \cdot B) = A(\nabla B) + B(\nabla A)}$$

→ It is a Vector that represents both magnetic & Direction represents of maximum rate of change of scalar.



$$dv = \frac{dv}{dx}dx + \frac{dv}{dy}dy + \frac{dv}{dz}dz$$

$$dv = \left(\frac{\delta v}{dx}a_x + \frac{\delta v}{\delta y}a_y + \frac{\delta y}{dz}a_z\right)\cdot(dx a_x + dy a_y + dz a_z)$$

$$\left[a_x a_x = 1, a_y a_y = 1, a_z a_z = 1\right]$$

$$dv = (G)\cdot dl \quad \left[G = \frac{dv}{dx}a_x + \frac{dv}{dy}a_y + \frac{\delta v}{dz}a_z \quad dl = dx a_x + dy a_y + dz a_z\right]$$

$\Rightarrow dv = G\cdot dl\cos\theta$

$\Rightarrow \frac{dv}{dl} = G\cos\theta \quad [\theta = 0°]$

$\Rightarrow \frac{dv}{dl} = G\cdot\cos 0°$

$\Rightarrow \left(\frac{dv}{dl}\right) = G$ = maximum $\quad$ [G = Gradient]

ⓘ $\nabla(U+V) = \nabla U + \nabla V$

i) $\nabla(UV) = U\nabla V + V\nabla U$

iii) $\nabla(U/V) = \frac{V\nabla U - U\nabla V}{V^2}$

iv) $\nabla(V^n) = nV^{n-1}\nabla(V)$

ⓘⓘ Divergence of a Vector :-

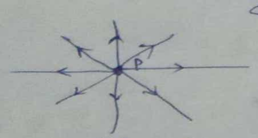
Divergence of any vector field A & any point P is outward flux per unit volume.

$$\oint_S A\cdot ds$$

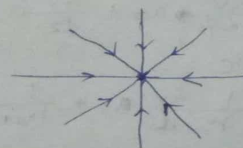
$$\operatorname{div}(A) = \frac{\oint A\cdot ds}{\Delta V}$$

ΔV

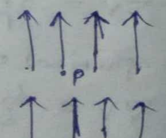
$$\Rightarrow \operatorname{div}(A) = \lim_{\Delta V\to 0}\frac{\oint_S A\cdot ds}{\Delta V}$$



positive Divergence
(Divergence



Negative divergence (
(convergence)



0- divergence

→ total outflow = strength(f) × Area(A) = constant

⇒ div(A) = 0 this is called Solenoidal Vector.

Rectangular

$$\nabla\cdot A = \frac{\delta}{\delta x}A_x + \frac{\delta}{\delta y}A_y + \frac{\delta}{\delta z}A_z \text{ (scalar vector)}$$

cylindrical

$$\nabla\cdot A = \frac{1}{\rho}\left[\frac{\delta}{\delta\rho}(A\rho\rho) + \frac{\delta}{\delta\phi}(A\phi) + \frac{\delta}{\delta z}(A_z\rho)\right]$$

spherical

$$\nabla\cdot A = \frac{1}{r^2\sin\theta}\left[\frac{\delta}{\delta r}(A_r r^2\sin\theta) + \frac{\delta}{\delta\theta}(A_\theta r\sin\theta) + \frac{\delta}{\delta\phi}(A_\phi r)\right]$$

Divergence theorem :-

$$\oint_S A\cdot ds = \int_V (\nabla\cdot A)\,dv$$

It states that the total outward flux of a vector field A at the closed surface S is the same as volume integral of divergence of A.

(*) $A = 4xy a_x - 3y a_z$, determine flux of A out of surface enclosed by $0\le x\le 1,\ 0\le y\le 1,\ -1\le z\le 1$

$$\oint A\cdot ds = \int(\nabla\cdot A)\,dv$$

$$\nabla\cdot A = \frac{\delta A_x}{\delta x} + \frac{\delta A_y}{\delta y} + \frac{\delta A_z}{\delta z}$$

$$= \frac{\delta}{\delta x}(4xy) + 0 + \frac{\delta}{\delta z}(-3y)$$

$$= 4y + 0 + 0$$

$$= 4y$$

$$\oint A\cdot ds = \int_{-1}^{1}\int_0^1\int_0^1 4y\,dx\,dy\,dz$$

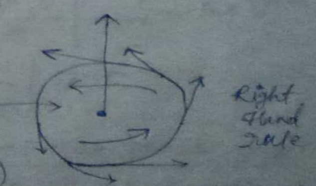
$$= 4\left(\frac{y^2}{2}\right)_0^1 (x)_0^1 (z)_{-1}^1$$

$$= 4\times\frac{1}{2}\times(1)(1+1)$$

$$= 4\times\tfrac{1}{2}\times 1\times 2$$

$$= 4 \text{ (Ans)}$$

ⓘⓘⓘ curl of a Vector :-



closed path 'L' ≃ open surface (unit area ΔS)

Right Hand Rule

"The curl of $\vec{A}$ is an axial (or rotational) Vector where magnitude is the maximum circulation of $\vec{A}$ per unit area as the area tends to zero and whose direction is the normal direction of the area when the area S is oriented to make the circulation maximum".

→ circulation of $\vec{A}$ around a closed path $L = \oint_L \vec{A}\cdot\vec{dl}$

$$\text{curl } \vec{A} = \nabla\times\vec{A} = \left(\lim_{\Delta s\to 0} \frac{\oint_L \vec{A}\cdot\vec{dl}}{\Delta s}\right)_{max} \vec{a_n} \text{ (normal area)}$$

→ ~~At~~ maximum circulation when occoured A at a time

A = 0.

Δs = Area

a_n = normal area

[total circulation = (length × strength)]

cartesian :—

$$\nabla\times\vec{A} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla\times\vec{A} = \left[\frac{\delta A_z}{\delta y} - \frac{\delta A_y}{\delta z}\right]\vec{a_x} + \left[\frac{\delta A_x}{\delta z} - \frac{\delta A_z}{\delta x}\right]\vec{a_y} + \left[\frac{\delta A_y}{\delta x} - \frac{\delta A_x}{\delta y}\right]\vec{a_z}$$

cylindrical :—

$$\nabla\times\vec{A} = \frac{1}{\rho}\begin{vmatrix} \vec{a_\rho} & \rho\vec{a_\phi} & \vec{a_z} \\ \frac{\delta}{\delta\rho} & \frac{\delta}{\delta\phi} & \frac{\delta}{\delta z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\nabla\times\vec{A} = \left(\frac{1}{\rho}\frac{\delta A_z}{\delta\phi} - \frac{\delta A_\phi}{\delta z}\right)\vec{a_\rho} + \left(\frac{\delta A_\rho}{\delta z} - \frac{\delta A_z}{\delta\rho}\right)\vec{a_\phi} + \frac{1}{\rho}\left(\frac{\delta(\rho A_\phi)}{\delta\rho} - \frac{\delta A_\rho}{\delta\phi}\right)\vec{a_z}$$

spherical :—

$$\nabla\times\vec{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{a_r} & r\vec{a_\theta} & r\sin\theta\,\vec{a_\phi} \\ \frac{\delta}{\delta r} & \frac{\delta}{\delta\theta} & \frac{\delta}{\delta\phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

$$\nabla\times\vec{A} = \frac{1}{r\sin\theta}\left[\frac{\delta(A_\phi\sin\theta)}{\delta\theta} - \frac{\delta A_\theta}{\delta\phi}\right]\vec{a_r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\delta A_r}{\delta\phi} - \frac{\delta(rA_\phi)}{\delta r}\right]\vec{a_\theta} + \frac{1}{r}\left[\frac{\delta(rA_\theta)}{\delta r} - \frac{\delta A_r}{\delta\theta}\right]\vec{a_\phi}$$

properties of curl :—

i) the divergence of the curl of a vector field vanishes

$$\boxed{\nabla\cdot(\nabla\times\vec{A}) = 0}$$

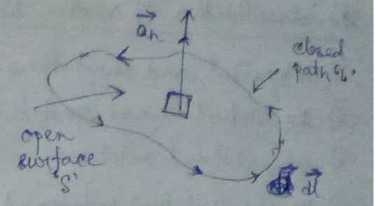
ii) the curl of the gradient of a scalar field vanishes

$$\boxed{\nabla\times(\nabla V) = 0}$$

(*) Strokes theorem :—

$$\oint_L \vec{A}\cdot\vec{dl} = \int_S (\nabla\times\vec{A})\cdot\vec{ds}$$

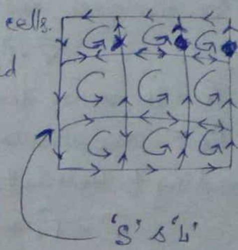
circulation of $\vec{A}$ around closed path 'L'



" Stoke's theorem states that the circulation of $\vec{A}$ around a closed path is equal to the ~~sufa~~ surface integral of the curl $\vec{A}$ over the open surface 'S' ~~obtained~~ bounded by 'L' provided $\vec{A}$ of $\nabla\times\vec{A}$ are continuous on 'S'."

→ surface 'S' subdivided into a large no. of cells.

→ Let, k^{th} cell has surface area ΔS_K bounded by L_K



$\oint \vec{A}\cdot\vec{dl} \simeq$ circulation around L_K

$$\oint \vec{A}\cdot\vec{dl} = \sum_K \oint \vec{A}\cdot\vec{dl} = \sum_K \frac{\oint_{L_K} \vec{A}\cdot\vec{dl}}{\Delta S_K} \Delta S_K$$

→ cancellation of every interior path, so sum of the line integrals around all L_K's is same as the line integral around the curve 'L'.

→ taking limit $\left(\lim_{\Delta S_K\to 0}\right)$ on right hand side,

$$\oint_L \vec{A}\cdot\vec{dl} = \int_S (\nabla\times\vec{A})\cdot\vec{ds}$$

$$\text{since, } \nabla\times\vec{A} = \left(\lim_{\Delta S\to 0} \frac{\oint_L \vec{A}\cdot\vec{dl}}{\Delta S}\right)\vec{a_n}$$

(iv) Laplacian of a Scalar (∇^2)

It can operate both on scalar as well as vector fields

$$\nabla^2 F = \vec{\nabla}\cdot\vec{\nabla}F = \left(\frac{\delta}{\delta x}\hat{a}_x + \frac{\delta}{\delta y}\hat{a_y} + \frac{\delta}{\delta z}\hat{a}_z\right)\cdot\left(\frac{\delta F}{\delta x}\hat{a}_x + \frac{\delta F}{\delta y}\hat{a}_y + \frac{\delta F}{\delta z}\hat{a}_z\right)$$

$$= \frac{\delta^2 F}{\delta x^2} + \frac{\delta^2 F}{\delta y^2} + \frac{\delta^2 F}{\delta z^2}$$

properties :—

i) $\nabla^2 V = 0$ → laplace's equation.

V = harmonic

ii) laplacian of a scalar & vector.

⊛ classification of vector fields :—

i) Solenoidal and irrotational vector fields $(\vec{\nabla}\cdot\vec{F}=0,\ \vec{\nabla}\times\vec{F}=0)$

ii) Non solenoidal and irrotational vectors $(\vec{\nabla}\cdot\vec{F}\neq 0,\ \vec{\nabla}\times\vec{F}=0)$

iii) Solenoidal and rotational " " $(\vec{\nabla}\cdot\vec{F}=0,\ \vec{\nabla}\times\vec{F}\neq 0)$

iv) Nonsolenoidal and rotational vector fields $(\vec{\nabla}\cdot\vec{F}\neq 0,\ \vec{\nabla}\times\vec{F}\neq 0)$

⊛ Helmholtz's theorem :—

A vector field is uniquely described within a region by its divergence and ~~cctt~~ curl.

If the divergence and curl of any vector $\vec{F}$ are given,

$$\vec{\nabla}\cdot\vec{F}=\rho_v \text{ and } \vec{\nabla}\times\vec{F}=\vec{\rho_s}$$

ρ_v = the source density of $\vec{F}$

$\vec{\rho_s}$ = the circulation density of $\vec{F}$ both vanishing at infinity then,

According to Helmholtz theorem,

$$\vec{F}=\vec{F_c}+\vec{F_i} \quad \left[\text{where, } \vec{\nabla}\cdot\vec{F_s}=0 \text{ (solenoidal)}\right.$$
$$\vec{\nabla}\times\vec{F_i}=0 \text{ (irrotational)}$$

$\therefore \vec{F_s}=\vec{\nabla}\times\vec{A}$ and $\vec{F_i}=\vec{\nabla}\phi$ where

$\vec{A}$ & ϕ are vector and scalar quantity.

$$\therefore \vec{F}=\vec{\nabla}\phi+\vec{\nabla}\times\vec{A}$$

i) Vector addition = $(\vec{a}+\vec{b})$ → triangle/parallelogram law.

ii) Vector subtraction = $\vec{a}+(-\vec{b})$ " "

iii) multification :—

with scalar — with vector

$a(\vec{A}+\vec{B})$

$=a\vec{A}+a\vec{B}$

dot product → $\vec{A}\cdot\vec{B}$

cross product → $\vec{A}\times\vec{B}=AB\sin\theta$

Scalar/vector triple product :—

Scalar triple product :— $\vec{A}(\vec{B}\times\vec{c})$
$=\vec{B}(\vec{c}\times\vec{A})-\vec{c}\cdot(\vec{A}\times\vec{B})$

Vector triple product :—

$\vec{A}\times(\vec{B}\times\vec{c})$
$=\vec{B}\cdot(\vec{A}\cdot\vec{c})-\vec{c}(\vec{A}\cdot\vec{B})$ $\boxed{BAC-CAB}$

Dot product :—

$\vec{A}\cdot\vec{B}$

$\vec{A}\cdot\vec{B}=AB\cos\theta$ [θ = angle b/w $\vec{A}$ and $\vec{B}$)

$A=|\vec{A}|,\ B=|\vec{B}|$

Example

$\vec{A}=-7\hat{a}_x+12\hat{a}_y+3\hat{a}_z$

$\vec{B}=74\hat{a}_x-2\hat{a}_y+16\hat{a}_z$

$\vec{A}\cdot\vec{B}=-28-24+48$
$=-52+48$
$=-4$

$|\vec{A}|=\sqrt{49+144+9}=\sqrt{202}$

$|\vec{B}|=\sqrt{16+4+256}=\sqrt{276}$

$\therefore \cos\theta=\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}=\frac{4}{\sqrt{202\times 276}}$

$\Rightarrow \theta=\cos^{-1}\left(\frac{-4}{\sqrt{202\times 276}}\right)$

$\Rightarrow \theta=90.97°$

cross product :—

$\vec{A}$ & $\vec{B}$

$\vec{A}\times\vec{B}=AB\sin\theta\,\hat{a}_n$ [$\hat{a}_n$ = Unit vector normal to the plane containing $\vec{A}$ & $\vec{B}$]

example

$\vec{A}=8\hat{a}_x+3\hat{a}_y-10\hat{a}_z$, $B=-15\hat{a}_x+6\hat{a}_y+17a_z$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

a_x	a_y	a_z
8	3	−10
−15	6	17

$=\hat{a}_x(A_yB_z-B_yA_z)+a_y(A_zB_x-A_xB_z)+a_z(A_xB_y-B_xA_y)$

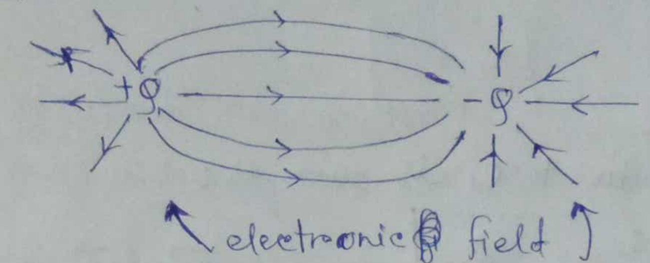
$=\hat{a}_x(51+60)+\hat{a}_y(150-136)+\hat{a}_z(48+45)$

$=111\hat{a}_x+14\hat{a}_y+\hat{a}_z 93$

3. Electrostatic Field

electrostatic field :-

Electrostatic ~~field~~ is a science related to the electric charges which are static i.e are at rest".



(+) = source point
(-) = sink point

→ {Due to static electric ~~field~~ charge does not vary with time and called time Invariant} called static electric field

Fundamental Law's of electrostatic field are :—

i) Coulomb's law :- Any charge distribution

ii) Gauss's Law :- charge distribution is symmetrical.

Note :- concept of electric field Intensity will be introduced & applied to care involving point, Line, surface & volume charge.

" we assume that electric field is in a vaccum or free space".

Coulomb's law & field Intensity :—

Coulomb's law states that the force F b/w two point charges q_1 & q_2 is

$$F \alpha \frac{q_1 q_2}{R^2}$$

q_1 ←— R —→ q_2

or $\boxed{F \alpha K \frac{q_1 q_2}{R^2}}$ K = called proportionality constant.

SI unit :— charges (q_1 & q_2) = coulomb (c).
distance (R) = meter (m)
force (F) = Newton (N)

$$\boxed{K = \frac{1}{4\pi\epsilon_0} \simeq 9 \times 10^9 \text{ m/F}}$$

ϵ_0 = permitivity of free space (in farads/meter)

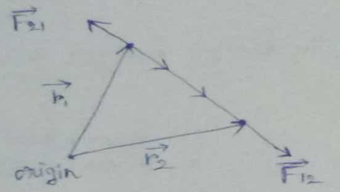
$$\begin{cases} \epsilon_0 = 8.854 \times 10^{-12} \text{ F/m} \\ \epsilon_0 \approx \frac{10^{-9}}{36\pi} \text{ F/m} \end{cases}$$

$$\therefore \boxed{F = \frac{q_1 q_2}{4\pi\epsilon_0 R^2}}$$

Note :- F b/w two point charges q_1 & q_2 is along the line joining them.

Coulomb Vector force on point charges q_1 & q_2

$\vec{r_1}$ & $\vec{r_2}$ are position vectors



force on q_2 due to q_1 is given as :

$$\boxed{\vec{F_{12}} = \frac{q_1 q_2}{4\pi\epsilon_0 R^2}\vec{a}_{12}}$$

$\vec{F_{12}} = \vec{r_2} - \vec{r_1}$

$|\vec{F_{12}}| = R$

or, $\vec{F_{12}} = \frac{q_1 q_2}{4\pi\epsilon_0 R^3}\vec{R_{12}}$

$\vec{a}_{R_{12}} = \frac{\vec{R_{12}}}{R}$

or, $\vec{F_{12}} = \frac{q_1 q_2(\vec{r_2}-\vec{r_1})}{4\pi\epsilon_0|\vec{r_2}-\vec{r_1}|^3}$

Similarly,

$\vec{F_{21}} = \frac{q_1 q_2}{4\pi\epsilon_0 R^2}\cdot\vec{a}_{R21}$

; Since $a_{R21} = -a_{R12}$

So. $\boxed{\vec{F_{21}} = -\vec{F_{12}}}$

Note :-

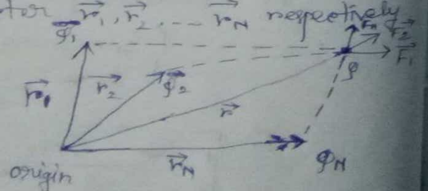
i) Like charges (charges of same sign) repel each other while unlike charges attract.

For like charges, $q_1 q_2 > 0$ } q_1 & q_2 must be
unlike charges $q_1 q_2 < 0$ } static (at rest)

ii) R = distance b/w q_1 & q_2 must be larged compared with the dimensions of q_1 & q_2 called point charges.

special case :-

Let, there are N charges $q_1, q_2 \dots q_N$ with position Vector $\vec{r_1}, \vec{r_2}, \dots \vec{r_N}$ respectively



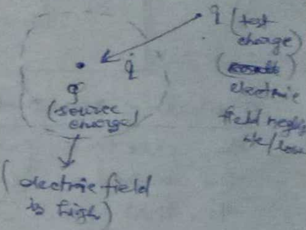
Hence,

$$\vec{F} = \frac{q q_1(\vec{r}-\vec{r_1})}{4\pi\epsilon_0|\vec{r}-\vec{r_1}|^3} + \frac{q q_2(\vec{r}-\vec{r_2})}{4\pi\epsilon_0|\vec{r}-\vec{r_2}|^3} \dots \dots + \frac{q q_N(\vec{r}-\vec{r_N})}{4\pi\epsilon_0|\vec{r}-\vec{r_N}|^3}$$

$$\boxed{\vec{F} = \frac{q}{4\pi\epsilon_0}\sum_{K=1}^{N}\frac{q_K(\vec{r}-\vec{r_K})}{|\vec{r}-\vec{r_K}|^3}}$$

electric field Intensity :-

The electric field intensity at any point is the forced experienced by a unit positive charge placed at that point.



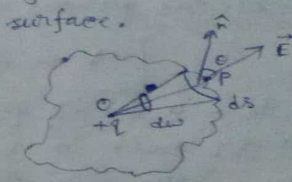
→ $E = \frac{F}{q}$

→ S.I unit of ~~elect~~ N/c or Nc^{-1}

→ electric field intensity is a Vector quantity.

Gauss's law :-

It states that the total electric ~~field~~ flux through a closed surface is $\frac{1}{\epsilon_0}$ times the net charge enclosed with in the surface.



mathematically,

$$\phi_E = \oint_S \vec{E}\cdot\vec{ds}$$
$$= \frac{1}{\epsilon_0}\times q$$

proof :- consider a single point charge +q at point o with in the closed surface (S). Let P be a point on the closed surface S

Imagine a small area 'ds' around P.

Let $op = r$

the magnetude of electric field intensity at point P is given by

$$E = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r^2} \quad \text{—— (i)}$$

Small electric flux through small area ds is given by

$$d\phi_E = \vec{E}\cdot\vec{ds}$$

$$d\phi_E = E ds \cos\theta \quad \text{—— (ii)} \quad [\theta = \text{is angle b/w } \vec{E} \text{ \& } \hat{n}]$$

total electric flux through closed surface S is given by

$$\phi_E = \oint_S d\phi_E = \oint E ds\cos\theta \quad \text{—— (iii)}$$

using (i) & (ii) we get,

$$d\phi_E = E ds\cos\theta$$
$$= \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r^2}ds\cos\theta$$

$$\phi_E = \frac{q}{4\pi\epsilon_0}\oint_S\frac{ds\cos\theta}{r^2} \quad \text{—— (iv)}$$

But, $\frac{ds\cos\theta}{r^2} = d\omega$ = small solid angle made by ds at o $\left[d\omega = \frac{ds\cos\theta}{r^2}\right]$

equation (4) becomes

$$\phi_E = \frac{q}{4\pi\epsilon_0}\oint_S d\omega \quad \text{—— (v)}$$

But $\oint_s d\omega = 4\pi$ = total solid angle made by closed surface at o

equation (5) becomes

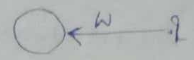
$$\phi_E = \frac{1}{4\pi\epsilon_0}(4\pi)$$

$$\boxed{\Rightarrow \phi_E = \frac{q}{\epsilon_0}}$$ — [proved]

(*) Electric potential (V) —

the electric potential at a point in an electric field is defined as the workdone in moving a unit positive test-charge from infinity to that point in the electric field. It is denoted by V.

electric potential = $\frac{\text{Work done}}{\text{charge}}$



$$\boxed{V = \frac{W}{q}}$$

→ It is a scalar quantity.

→ S.I unit = V

Also = $V = \frac{W(J)}{q(C)}$

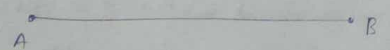
$$\boxed{\Rightarrow 1V = J/C}$$

→ $V = \frac{[W]}{[q]} = \frac{[ML^2T^{-2}]}{[AT]}$

$\Rightarrow V = [ML^2T^{-3}A^{-1}]$

(*) potential Difference :-

The work done by an external agent in carrying a unit positive test-charge from one point to the other in an electric field is called the potential difference b/w these two points.

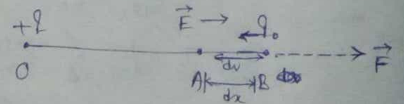


$V_A - V_B = \frac{W}{q}$

$\Rightarrow \Delta V = \frac{W}{q}$

⇒ workdone = potential difference × charge

(*) Relation b/w $\vec{E}$ & V :-



the force required to move the charge q_0 from B to A against the electric field $\vec{E}$ is

$F = -q_0E$

workdone to move the charge from B to A by a distance dx is

$dw = F.dx$

$dw = -q_0 E\,dx$

or, $\frac{dw}{q_0} = -Edx$ — (i)

If dv is the potential difference b/w points A & B. then by the definition

$dv = \frac{dw}{q_0}$

$\Rightarrow dv = -Edx \quad \left[\frac{dw}{q_0} = -Edx\right]$

$\Rightarrow \boxed{E = -\frac{dv}{dx}} \quad \left[\frac{dv}{dx} = \text{potential gradient}\right]$

potential Gradient :-

$E = \frac{dv}{dx}$ ~~potential Gradient~~

unit = V/m

The rate of change of potential with respect to the distance is called the potential gradient.

potential Gradient $= \frac{dv}{dx} = \lim_{\Delta x \to 0}\left(\frac{\Delta V}{\Delta x}\right)$

distance b/w two points = $\vec{dx}$

the work done by an external electric field $\vec{E}$ in moving a unit positive charge from one point to the other is given by,

$dw = dv = -\vec{E}.\vec{dx}$

$\therefore \frac{\delta v}{\delta x}\delta x + \frac{\delta v}{\delta y}\delta y + \frac{\delta v}{\delta z}dz = -\vec{E}.\vec{dx}$

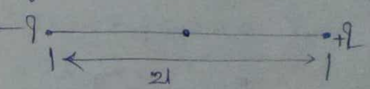
$\Rightarrow \left(\frac{\delta v}{\delta x}\hat{i} + \frac{\delta v}{\delta y}\hat{j} + \frac{\delta v}{\delta z}\hat{k}\right).(dx\hat{i} + dy\hat{j} + \delta z\hat{k}) = -\vec{E}.\vec{dx}$

$\Rightarrow \nabla\vec{V}.\vec{dx} = -\vec{E}.\vec{dx}$

$$\boxed{\vec{E} = -\vec{\nabla}V}$$

Electric Dipole :—

It is an arrangement in which two equal and opposite charges are separated by a certain distance.



2l = dipole length

Electric Dipole Moment :—

Electric Dipole moment measure strength of electric dipole. It is given by product of magnitude of any charge and dipole length.

It is denoted by = P

It is a vector quantity.

It is given by, $\boxed{P = q \times 2l}$

electric flux (ϕ) :-

The total no of electric field lines passing normally through an area is called electric flux related to that area.

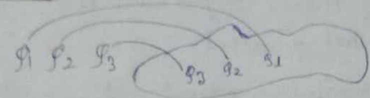
$$\phi_E = \int_S \vec{E}\cdot\vec{ds} :$$

which is the measure of no. of field lines passing through the surface S.

(1) Flux through any closed surface is measure total charge inside the surface.

(i') charge outside the surface will contribute nothing to the total flux, since its field lines pass in one side of out from the ~~otherside~~ otherside.

Energy density in electrostatic field :-



- energy density in electrostatic field

$W = W_1 + W_2 + W_3$

$W = 0 + \rho_2 V_{21} + \rho_3 [V_{31} + V_{32}]$ —— (i)

$W = 0 + \rho_2 V_{23} + \rho_1 [V_{12} + V_{13}]$ —— (ii)

$2W = \rho_1 [V_{12} + V_{13}] + \rho_3 [V_{31} + V_{32}] + \rho_2 (V_{21} + V_{23})$

$\Rightarrow \boxed{2W = \rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3}$

$\Rightarrow W = \frac{1}{2}[\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3]$

$W = \frac{1}{2}\sum_{K=1}^{n} \rho_K V_K$

$W = \frac{1}{2}\int \rho_L V dL$

$W = \frac{1}{2}\int \rho_S V ds$

$W = \frac{1}{2}\int \rho_V V dv$

$W = \frac{1}{2}\int \rho_V V dv$

$W = \frac{1}{2}\int (\nabla\cdot D) V\, dv \quad [\nabla\cdot D = \rho_V]$

$\nabla\cdot(DV) = D\nabla V + V\nabla D$

$\nabla\cdot(DV) - D\nabla V = V(\nabla\cdot D)$

$\rightarrow W = \frac{1}{2}\int \nabla\cdot(DV)\,dv - \frac{1}{2}\int D\nabla V\, dv \qquad \oint_S A\, ds = \int_V \nabla\cdot A\, dv$

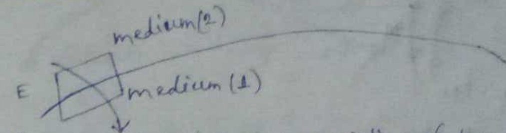
$W = \frac{1}{2}\int \underset{\text{negligible}}{DV\, ds} + \frac{1}{2}\int DE\, dv \qquad [E = -\nabla V]$

$\Rightarrow W = \frac{1}{2}\int DE\, dv$

$$\boxed{W = \frac{1}{2}\int \epsilon_0 E^2 dv}$$

unit = J/m^3

Boundary Conditions:



If electric field $\vec{E}$ exists in a region consisting of two different media, the condition that the field must satisfy at the interface seperating the media are called Boundary conditions.

→ the Boundary conditions are used in determining the field on one side of the boundary - if the field on the other side is known.

types of interface of boundary condition :-

i) Dielectric (ϵ_{r1}) - Dielectric (ϵ_{r2})
ii) Conductor - Dielectric Boundary
iii) conductor - freespace boundary

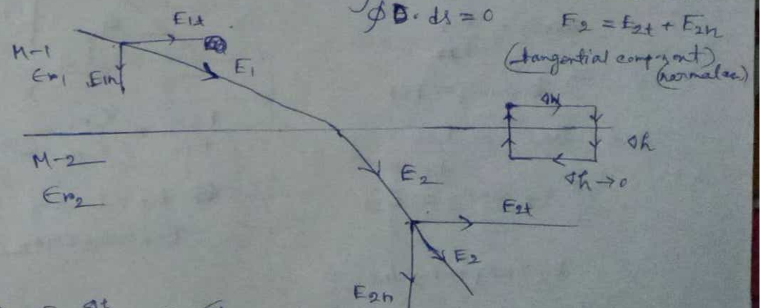
i) Dielectric - Dielectric :-

$\oint E\cdot dl = 0 \qquad E_1 = E_{1t} + E_{1n}$

$\oint D\cdot ds = 0 \qquad E_2 = E_{2t} + E_{2n}$

(tangential component) (normal)

(a)



$E_{1t}\Delta w + E_{1n}\frac{\Delta h}{2} + E_{2n}\frac{\Delta h}{2} - E_{2t}\Delta w - E_{2n}\Delta h/2 - E_{1n}\frac{\Delta h}{2} = 0$

$\Rightarrow E_{1t}\Delta w - E_{2t}\Delta w = 0$

$\Rightarrow (E_{1t} - E_{2t}) = 0$

$\boxed{\Rightarrow E_{1t} = E_{2t}}$

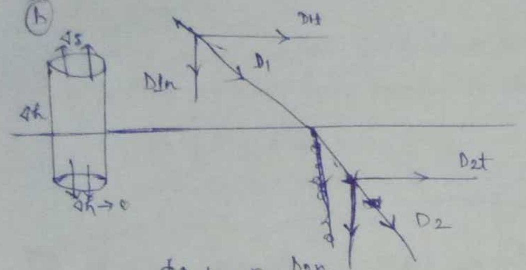
tangential component is always same.
tangential component is continuous of Electric field interface.

$D = \epsilon E$

$\Rightarrow E = \frac{D}{\epsilon}$

$\frac{D_{1t}}{\epsilon_0\epsilon_{r1}} = \frac{D_{2t}}{\epsilon_0\epsilon_{r2}} \qquad \boxed{\Rightarrow \frac{D_{2t}}{D_{1t}} = \frac{\epsilon_{r2}}{\epsilon_{r1}}}$

(b)



$\oint D \cdot ds = Q$

$\oint D \cdot ds = Q$

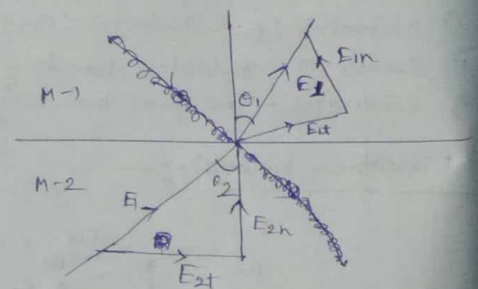
$-D_{1n}\Delta s + D_{2n}\Delta s = \rho_s \Delta s$

$$\boxed{D_{2n} - D_{1n} = \rho_s}$$

$D_{2n} - D_{1n} = 0$ (charge free)
$D_{2n} = D_{1n}$
$\epsilon_0 \epsilon_{r2} E_{2n} = \epsilon_0 \epsilon_{r1} E_{1n}$

$$\boxed{\Rightarrow \frac{E_{2n}}{E_{1n}} = \frac{\epsilon_{r1}}{\epsilon_{r2}}}$$

(c) relation of angl



$E_{1t} = E_{2t}$

$\sin\theta_1 = \frac{E_{1t}}{E_1}$

$\Rightarrow E_1 \sin\theta_1 = E_{1t}$

and $\sin\theta_2 = \frac{E_{2t}}{E_2}$

$\Rightarrow E_2 \sin\theta_2 = E_{2t}$

$\frac{E_{2n}}{E_{1n}} = \frac{\epsilon_{r1}}{\epsilon_{r2}} \rightarrow E_{1n} = E_{2n} \cdot \frac{\epsilon_{r2}}{\epsilon_{r1}}$

$E_{1n} = E_1 \cos\theta_1$

$E_{2n} = E_2 \cos\theta_2$

$E_1 \sin\theta_1 = E_2 \sin\theta_2$

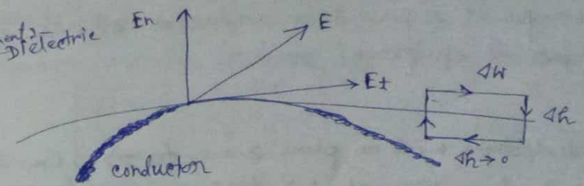
$\Rightarrow E_1 \cos\theta_1 = E_2 \cos\theta_2 \frac{\epsilon_{r2}}{\epsilon_{r1}}$

$\Rightarrow \tan\theta_1 = \tan\theta_2 \cdot \frac{\epsilon_{r1}}{\epsilon_{r2}}$

$$\boxed{\Rightarrow \frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}}$$

(ii) conductor – Dielectric :–

tangential component: Dielectric

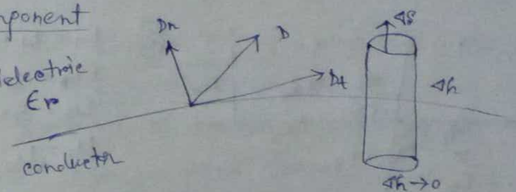


$\oint E \cdot dl = 0$

$E_t \Delta W - E_n \frac{\Delta h}{2} + 0\frac{\Delta h}{2} + 0\frac{\Delta h}{2} + 0\Delta W + 0\frac{\Delta h}{2} + E_n \frac{\Delta h}{2} = 0$

$$\boxed{\Rightarrow E_t = 0}$$

Normal – component



$\oint D \cdot ds = Q$

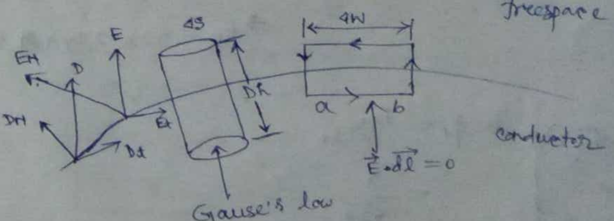
$D_n \Delta s = \rho_s \Delta s$

$\boxed{\Rightarrow D_n = \rho_s}$ surface charge density (ρ_s)

$E_t = 0$
$\Rightarrow D_t = \epsilon_0 \epsilon_r E_t = 0$
$D_n = \rho_s$
$D_n = \epsilon_0 \epsilon_r E_n = \rho_s$

(iii) conductor – free space :–

(a)



$\oint E \cdot \vec{dl} = 0$

$\int_a^b + \int_b^c + \int_c^d + \int_d^a = 0$

$E_t \Delta W + \left(E_{nb}\frac{\Delta h}{2} + E_{nc}\frac{\Delta h}{2}\right) + (E_t \Delta W) + \left(E_{nd} \cdot \frac{\Delta h}{2} + E \cdot N_a \frac{\Delta h}{2}\right) = 0$

$E_t \cdot \Delta W = 0$

$\boxed{E_t = 0} \longrightarrow$ tangential component of field $= 0$

(b) $\oint D \cdot \vec{ds} = Q_{in}$

$\int_{top} + \int_{bottom} + \int_{side} = Q_{in}$

$\Rightarrow D_N \Delta s + \int 0 + 0 = \rho_s \cdot \Delta s$

$\Rightarrow D_N \Delta s = \rho_s \Delta s$

$$\boxed{D_N = \rho_s}$$

$E_t = 0,\ D_t = 0$
$D_N = \rho_s$
$E_N = \frac{D_N}{\epsilon_0} = \frac{\rho_s}{\epsilon_0}$

$E_t = 0$.

1. E. field = 0
2. only normal component is present on conductor surface.
3. conductor surface is equipotent surface

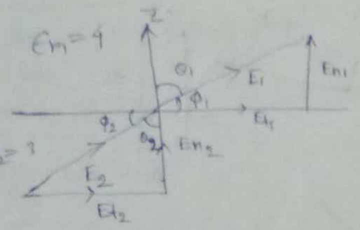
problems

two homogenous dielectrics meet on plane $z = 0$, for $z > 0$, $\epsilon_{r1} = 4$ & for $z < 0$, $\epsilon_{r2} = 3$. A uniform electric field $E_1 = 5a_x - 2a_y + 3a_z \frac{kV}{m}$ exists for $z \geq 0$. find (a) E_2 for $z \leq 0$

(b) the angles E_1 & E_2 with the interphase

(c) the energy density in both dielectrics.

⇒



(a) $E_1 = E_t + E_n = 5a_x - 2a_y + 3a_z$

$E_{n1} = 3a_z$

$E_{t1} = 5a_x - 2a_y$

$E_{t1} = E_{t2} = 5a_x - 2a_y$

$$\frac{E_{n2}}{E_{n1}} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$$

$$\Rightarrow \frac{E_{n2}}{E_{n1}} = 4/3$$

$\Rightarrow E_{n2} = 4/3 \times 3a_z$

$\Rightarrow E_{n2} = 4a_z$

$\Rightarrow E_2 = 5a_x - 2a_y + 4a_z$

(b)

$\theta_1 = 90° - \phi_1$

$\theta_2 = 90° - \phi_2$

$$\tan\theta_2 = \frac{E_{t2}}{E_{n2}} = \frac{\sqrt{29}}{4}$$

$\Rightarrow \theta_2 = \tan^{-1}\left(\frac{\sqrt{29}}{4}\right) = 53.39°$

$\Rightarrow \theta_1 = 90° - 53.39° = 36.61°$

$\tan\phi_1 = E_{n1}/E_{t1} = \frac{3}{\sqrt{29}}$

$\Rightarrow \phi_1 = \tan^{-1}\left(\frac{3}{\sqrt{29}}\right) = 29.12°$

(c) energy density

$W_E = \frac{1}{2}\epsilon|E|^2$

$W_{E1} = \left(\frac{1}{2} \times 8.854 \times 10^{-12} \times 4 \times (38) \times 10^6\right) J/m^3$

$W_{E2} = \left(\frac{1}{2} \times 8.854 \times 10^{-12} \times 3 \times (45) \times 10^6\right) J/m^3$

Poisson's equation :-

Poisson's Equation relates the electric potential to charge density.

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}}$$

where, V = electric potential

ρ_v = Volume charge density

ϵ = dielectric constant

→ the potential of a conductor is proportional to the charge it contains.

Proof :-

the relation b/w electric potential and field intensity is given as,

$$\boxed{E = -\nabla V} \quad \text{(i)}$$

'E' is the gradient of electric potential in opposite direction.

Gauss's law in point form:

$$\boxed{\nabla D = \rho_v} \quad \text{(ii)}$$

the divergence of electric flux density is equal to the volume charge density.

$\nabla . D = \rho_v$

$\nabla . (\epsilon E) = \rho_v$

$\Rightarrow \epsilon . (\nabla . E) = \rho_v$

$\Rightarrow \nabla . E = \frac{\rho_v}{\epsilon}$ —— (iii)

Sub equation (i) $E = -\nabla . V$ in 3

$$\nabla . (-\nabla . V) = \frac{\rho_v}{\epsilon}$$

$$\Rightarrow \nabla . \nabla . V = \frac{\rho_v}{\epsilon}$$

$$\boxed{\Rightarrow \nabla^2 V = -\frac{\rho_v}{\epsilon}}$$

this is called as poisson's equation.

for cartesian coordinate system, (x, y, z)

$$\nabla^2 V = \frac{\delta}{\delta x}\left(\frac{\delta v}{\delta x}\right) + \frac{\delta}{\delta y}\left(\frac{\delta v}{\delta y}\right) + \frac{\delta}{\delta z}\left(\frac{\delta v}{\delta z}\right)$$

$$\boxed{\frac{\delta^2 v}{\delta x^2} + \frac{\delta^2 V}{\delta y^2} + \frac{\delta^2 V}{\delta z^2} = -\frac{\rho_v}{\epsilon}}$$

for spherical coordinate system, (r, θ, ϕ)

$$\boxed{\nabla^2 V = \frac{1}{r^2}\frac{\delta}{\delta r}\left(r^2 \frac{\delta v}{\delta r}\right) + \frac{1}{r^2 \sin\theta}\frac{\delta}{\delta\theta}\left(\sin\theta \frac{\delta v}{\delta\theta}\right) + \frac{1}{r^2 \sin\theta} . \frac{\delta^2 v}{\delta\phi^2} = -\frac{\rho_v}{\epsilon}}$$

for cylindrical coordinate system (ρ, ϕ, z)

$$\boxed{\nabla^2 v = \frac{1}{\rho}\frac{\delta}{\delta\rho}\left(\rho \frac{\delta v}{\delta\rho}\right) + \frac{1}{\rho^2}\left(\frac{\delta^2 v}{\delta\phi^2}\right) + \frac{\delta^2 V}{\delta z^2} = -\frac{\rho_v}{\epsilon}}$$

Laplace Equation :-

If the enclosed surface does not have any volume charge ρ_v, then the poisson's equation becomes,

$$\boxed{\nabla^2 V = 0} \quad ; \rho_v = 0$$

this is called as laplace equation.

$[\nabla^2 \to$ laplacian operator$]$

→ the solution of laplace's equation are the harmonic function.

→ Harmonic functions are important is partial differential equation ~~Knownly~~ theory, physics and engineering fields.

Application :-

→ It is used in boundary value problems

→ In such cases, the charge distribution is involved on the surface of the conductor for which the volume charge density is zero.

ex: to obtain potential and capacitance is b/w two surface.

procedure for solving poisson's and Laplace's equation :-

Step-I :- Solve the laplace equation using the method of integration.

Step-II :- Determine the constants applying the boundary conditions given or known for

Step-III :- Then $\bar{E}$ can be obtained for the potential field v obtained using gradient operation $-\nabla V$.

Step-IV :- for homogeneous medium, $\bar{D}$ can be obtained as $\epsilon\bar{E}$.

Step-V :- At the surface, $\rho_s = D_N$

D_N = normal component to the surface is known.

Hence the charge induced on the conductor surface can be obtained as $Q = -\int \rho_s\, ds$.

Step-VI :- once the charge induced Q is known and potential V is known then the capacitance c of the system can be obtained.

$\rho_v \neq 0$ then similar procedure can be adopted to solve the poisson's equation.

problems :-

$V = 5x^3y^2z$ & $\epsilon = 2.25\epsilon_0$. find ρ_v at $(-3,1,2)$

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}$$

$$\Rightarrow \frac{\delta^2 V}{\delta x^2} + \frac{\delta^2 V}{\delta y^2} + \frac{\delta^2 V}{\delta z^2} = -\frac{\rho_v}{\epsilon}$$

$$\Rightarrow 5.6xy^2z + x^3z.5.2y + 0 = -\frac{\rho_v}{\epsilon}$$

$$\Rightarrow 30xy^2z + 10x^3z + 0 = -\frac{\rho_v}{\epsilon}$$

$$\Rightarrow 30xy^2z + 10x^3z + 0 = \frac{-\rho_v}{2.25 \times 8.854 \times 10^{-12}}$$

$$\Rightarrow \rho_v = -\left[30xy^2z + 10x^3z\right] \times (1.99215 \times 10^{-11})$$

$$= -\left[30.(-3).(1).(2) + 10.(-27).2\right] \times (1.99 \times 10^{-11})$$

$$= -\left[-180 - 540\right] \times (1.99 \times 10^{-11})$$

$$= 720 \times 1.99 \times 10^{-11}$$

$$= 1.434$$

$$= 14.34\ nc/m^3$$

(2) $V = \frac{z\cos\phi}{\rho}$

$$\Rightarrow \nabla^2 V = \nabla.(\nabla V)$$

$$\nabla V = \frac{\delta V}{\delta \rho} a_\rho + \frac{1}{\rho}\frac{\delta V}{\delta \phi} a_\phi + \frac{\delta V}{\delta z} a_z$$

$$= -\frac{z\cos\phi}{\rho^2} a_\rho - \frac{z\sin\phi}{\rho^2} a_\phi + \frac{\cos\phi}{\rho} a_z$$

$$\nabla^2 V = \frac{1}{\rho}\left[\frac{\delta}{\delta\rho}(\rho A_\rho) + \frac{\delta}{\delta\phi}(A_\phi) + \frac{\delta}{\delta z}(\rho\phi_z)\right]$$

$$= \frac{1}{\rho}\left[\frac{\delta}{\delta\rho}\left(\frac{-z\cos\phi}{\rho}\right) + \frac{\delta}{\delta\phi}\left(\frac{-z\sin\phi}{\rho^2}\right) + \frac{\delta}{\delta z}(\cos\phi)\right]$$

$$= \frac{1}{\rho}\left[\frac{z\cos\phi}{\rho^2} - \frac{z\cos\phi}{\rho^2}\right] + 0$$

$$\nabla^2 V = 0$$

(3) $V = \frac{30\cos\theta}{r^2}$

$$\nabla.V = \frac{\delta V}{\delta r} a_r + \frac{1}{r}\frac{\delta V}{\delta\theta} a_\theta + \frac{1}{r\sin\theta}\frac{\delta V}{\delta\phi} a_\phi$$

$$= -\frac{60\cos\theta}{r^3} a_r - \frac{30\sin\theta}{r^3} a_\theta + 0$$

$$\nabla^2 V = \frac{1}{r^2\sin\theta}\left[\frac{\delta}{\delta r}(r^2\sin\theta A_r) + \frac{\delta}{\delta\theta}(r\sin\theta A_\theta) + \frac{\delta}{\delta\phi}(r A_\phi)\right]$$

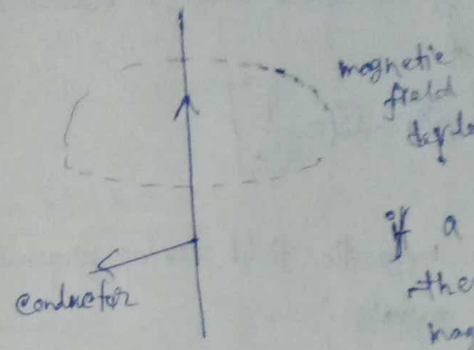
$$\nabla^2 V = \frac{1}{r^2\sin\theta}\left[\frac{\delta}{\delta r}\left(-\frac{60}{r}\cos\theta\sin\theta\right) + \frac{\delta}{\delta\theta}\left(r\sin\theta\left(-\frac{30\sin\theta}{r^3}\right)\right) + 0\right]$$

$$= \frac{1}{r^2\sin\theta}\left[\frac{60}{r^2}\cos\theta\sin\theta + \frac{\delta}{\delta\theta}\left(-\frac{30\sin^2\theta}{r^2}\right)\right]$$

$$= \frac{1}{r^2\sin\theta}\left[\frac{60}{r^2}\cos\theta\sin\theta - \frac{60}{r^2}\sin\theta\cos\theta\right]$$

4 Magneto-Static field

1820 – Mr. Austerd started this experiment.



developed (static nature)

If a static conductor surrounding current is apply. then each sides to conductor ~~~~ sol static magnetic field produced. this is called magneto-static field.

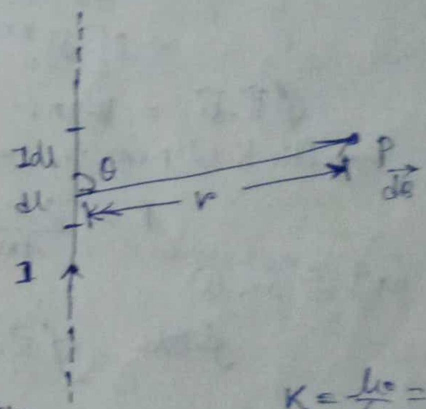
Bio-Savert law :-

According to this law, the magnitude of magnetic field at any point due to current element depends directly upon current element (Idl), sine of angle θ & Inversely proportional to square of distance b/w current element, & the point ~~where~~ where the magnetic field is to be calculated.

$dB \propto Idl$

$dB \propto \sin\theta$

$dB \propto \frac{1}{r^2}$



combine all factors

$dB \propto \frac{Idl \sin\theta}{r^2}$

$dB = K \frac{Idl \sin\theta}{r^2}$

$\boxed{dB = \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{r^2}}$ → simple form

$K = \frac{\mu_0}{4\pi} = 10^{-7}$

$\boxed{\Rightarrow \mu_0 = 10^{-7} \times 4\pi}$

unit & Dimension of μ_0 :

$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{r^2}$

$dB = \frac{\mu_0 Idl}{r^2}$

$\frac{dB r^2}{Idl} = \mu_0$

$\Rightarrow \frac{Tm^2}{Am} = \mu_0$

unit $\boxed{\Rightarrow \mu_0 = TmA^{-1}}$

$\mu_0 = \frac{dB r^2}{Idl}$

$= \frac{[M^1L^0T^{-2}A^{-1}][L^2]}{[A^1][L]}$

$\boxed{= M^1L^1T^{-2}A^{-1}}$

Vector form :–

where k is proportionality constant & μ_0 is permeability of free space

$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\vec{dl} \sin\theta}{r^2} \cdot \hat{r}$

$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$

$= \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{r^3} \cdot \vec{r}$

$\boxed{\vec{dB} = \frac{\mu_0}{4\pi} \cdot I \left(\frac{\vec{dl} \times \vec{r}}{r^3}\right)}$

→ Direction of $\vec{dl}$ is perpendicular to plane containing $\vec{dl}$ ~~and~~ & r.

⊛ Ampere's circuit law :-

It states that the line integral of magnetic field over a closed loop is equal to 'μ_0' times current enclosed

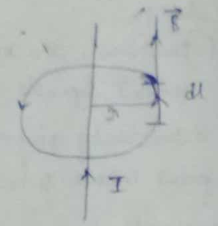
$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I \qquad \left[\oint \vec{E}\cdot\vec{dS} = \frac{q}{\epsilon_0}\right]$$

proof :-

Let a wire carrying current I then magnetic field develop around it & is in the form of concentric circles.

Let, r is radious of amperian ~~and~~ loop.

Net mag. field due to loop is given by,



$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl\cos\theta$$

$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl$$

$$= B\oint dl \qquad \theta = 0^\circ$$

$$= B\left[l\right]_0^{2\pi r} \qquad \cos 0^\circ = 1$$

$$\oint \vec{B}\cdot\vec{dl} = B\cdot 2\pi r \quad \text{——①}$$

magnetic field near the centre of wire

$$B = \frac{2\mu_0 I}{4\pi r} \quad \text{——②}$$

put ② in ①

$$\oint \vec{B}\cdot\vec{dl} = \frac{2\mu_0 I}{4\pi r}\times 2\pi r \qquad \begin{aligned}&\vec{B}\times\vec{dl}\\ &= \vec{B}\,\vec{dl}\sin\theta\end{aligned}$$

$$= \frac{2\mu_0 I}{2}$$

$$\boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 I} \quad \boxed{\text{proved}}$$

⊛ Magnetic flux :-

The total number of magnetic lines of force produced is called magnetic flux.(ϕ)

$\phi = \omega b$

$1\omega b = 10^8$ lines of force

Magnetic flux density :-

The magnetic flux density at a point is flux per unit area at angle to the flux at that point.

$$B = \frac{\phi}{A}\left(\frac{\omega b}{m^2}\right) \text{ or (tesla)}$$

permeability :-

The ability of a material to conduct magnetic lines of force through it.

$$\mu = \mu_0\mu_r \qquad \left[\begin{aligned}\mu_0 &= \text{permeability of air or vaccum}\\ &= 4\pi\times10^{-7}\ H/m\ \text{(constant)}\end{aligned}\right]$$

μ_r = relative ~~per~~ permeability of material (varing)

Relation b/w B & H :-

$B \propto H \longrightarrow$ (field intensity)

(magnetic flux density)

$B = \mu H$ or $H = \frac{B}{\mu}$

$B = \mu_0\mu_r H$ in medium

$B_0 = \mu_0 H$ in air ($\mu_r = 1$)

$\mu_0 = 4\pi\times10^{-7}$

Electromagnet :-

A magnetic material when obtains the magnetic property by providing a current it is called an electromagnet.

Magnetic Potential

- Scalar magnetic potential (V_m) ($J=0$)
- Vector magnetic potential ($\vec{A}$) ($J\neq0$)

Scalar ~~Static~~ Magnetic Potential (V_m)

Scalar magnetic potential is defined only in a region where $J=0$, i.e no current source is present.

→ It is the quantity whose negative gradient gives the magnetic intensity.

$$\boxed{H = -\nabla V_m} \quad \vec{J} = 0 \text{ (current density)}$$

$$V_m = -\int H\cdot dl \cdot \text{Ampere}$$

proof :- In magnetostatics,

$$\vec{H} = -\nabla V_m$$

taking curl on both sides, $\nabla\times\vec{H} = \nabla\times(-\nabla V_m)$

$$= -\nabla\times(\nabla V_m) \qquad \left[\begin{aligned}&\text{Using Intensity}\\ &\nabla\times(\nabla V)=0\\ &\nabla\cdot(\nabla\times\vec{A})=0\end{aligned}\right]$$

$$\boxed{\nabla\times\vec{H} = 0} \quad \text{——①}$$

According to Ampere's law,

$$\nabla\times\vec{H} = \vec{J} \quad \text{——②}$$

compare ① & ② $\quad \vec{J} = 0$ (It is proved)

$$\vec{H} = -\nabla V_m$$

$$\int V_m = -\int \vec{H}\cdot dl \quad \Rightarrow \boxed{V_m = -\oint \vec{H}\cdot dl} \text{ Ampere}$$

→ scalar potential statics laplace Equation —

$\boxed{\nabla^2 V_m = 0}$; $\vec{J} = 0$

~~take maxwells~~

∴ $\nabla \cdot (-\nabla V_m) = \nabla \times \vec{H}$

$\Rightarrow -\nabla^2 V_m = 0$ only for $\vec{J} = 0$

this is called Poison's equation for magnetic scalar potential (V_m)

S.I unit of SI = Ampere (V_m)

Poisson's equation for electric potential (V)

$\nabla^2 V = -\frac{\rho}{\epsilon_0}$

Vector Magnetic Potential (A):-

→ It is defined as a quantity whose curl gives the magnetic flux density. $\boxed{\vec{B} = \nabla \times \vec{A}}$ $\vec{J} \neq 0$ $\vec{J}$ (current density)

where, $\vec{A}$ is the vector magnetic potential

$\boxed{\vec{A} = \frac{\mu}{4\pi} \iiint \left(\frac{\vec{J}}{r}\right) dv}$ [weber/m]

→ It is used to find the magnetic field due to current $(J \neq 0)$

proof:-

According to magnetic Gauss law is point form

$\nabla \cdot \vec{B} = 0$ — ① [the magnetic flux through any closed surface = 0]

from the identity,

$\nabla \cdot (\nabla \times \vec{A}) = 0$ — ② $\boxed{\vec{B} = \nabla \times \vec{A}}$

compare ① & ②

$\boxed{\vec{B} = \nabla \times \vec{A}}$ — ③

take curl on both sides

$\nabla \times \vec{B} = \nabla \times (\nabla \times \vec{A})$ — ④

from the Identity,

$\nabla \times \nabla \times \vec{A} = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

for the steady current $\nabla \cdot A = 0$

$\nabla \times \nabla \times \vec{A} = -\nabla^2 \vec{A}$ — ⑤

substitute ⑤ & 4 equation

$\nabla \times \vec{B} = -\nabla^2 \vec{A}$ — ⑥

According to Ampere's law,

$\nabla \times \vec{H} = \vec{J}$

$\nabla \times \frac{\vec{B}}{\mu_0} = \vec{J}$

$\Rightarrow \nabla \times \vec{B} = \mu_0 \vec{J}$

$\Rightarrow \nabla^2 \vec{A} = \mu_0 \vec{J}$

[in free space, $\vec{B} = \mu_0 \vec{H}$ $\Rightarrow \vec{H} = \frac{\vec{B}}{\mu_0}$]

$\vec{A} = (A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z)$

$\vec{J} = (J_x \vec{a}_x + J_y \vec{a}_y + J_z \vec{a}_z)$

$\nabla^2 A_x \vec{a}_x + \nabla^2 A_y \vec{a}_y + \nabla^2 A_z \vec{a}_z = -\mu_0 (J_x \vec{a}_x + J_y \vec{a}_y + J_z \vec{a}_z)$

$\nabla^2 A_x = -\mu_0 J_x$

$\nabla^2 A_y = -\mu_0 J_y$

$\nabla^2 A_z = -\mu_0 J_z$

is the form of poisson's equation.

The magnetic potential can be written as,

$\vec{A_x} = \frac{\mu_0}{4\pi} \int_v \left(\frac{J_x}{r}\right) dv$

$\vec{A_y} = \frac{\mu_0}{4\pi} \int_v \left(\frac{J_y}{r}\right) dv$

$\vec{A_z} = \frac{\mu_0}{4\pi} \int_v \left(\frac{J_z}{r}\right) dv$

In electrostatics,

$V = \frac{q}{4\pi\epsilon_0 r}$;

$\nabla^2 V = -\frac{\rho_v}{\epsilon}$

$q = \int_v \rho_v dv$

In general, vector magnetic potential can be expressed as,

$\boxed{\vec{A} = \frac{\mu_0}{4\pi} \int_v (\vec{J}/r) dv \text{ , (Wb/m)}}$

Magnetic force due to magnetic field :-

force due to magnetic field $(\vec{B})$ is called magnetic force.

i) moving charge in $\vec{B}$ field

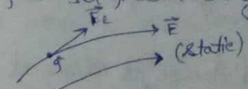
ii) on current element on external $\vec{B}$

iii) Between two current elements

(i) force on a charge particle :-

$\vec{F_e} = q\vec{E}$

+q = set free on moving

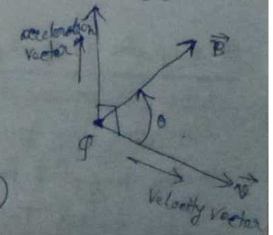


$\vec{F_e}$ does some work.

+q = remains at rest

↳ $\vec{F_m} = 0$ (does no work)

$\vec{F_m} = q(\vec{U} \times \vec{B})$



~~magnetic~~ $F_m = q\vec{u} \times \vec{B}$

$F = \vec{F_e} + \vec{F_m}$

$= q\vec{E} + q\vec{u} \times \vec{B}$

$\boxed{F = q(\vec{E} + \vec{U} \times \vec{B})}$ → Lorentz force equation — ①

from Newton second's law,

$$F = m\frac{du}{dv}$$

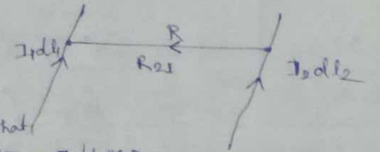
$$\therefore \boxed{m\frac{du}{dv} = q\left[E + u\times B\right]}$$

(ii) force on a current element :—

we have magnetic force on a magnetic charge $(F) = qu\times B$

$$dF = dq\,u\times B$$
$$= dq\frac{dl}{dt}\times B$$
$$= \frac{dq}{dt}\cdot dl\times B$$
$$\boxed{dF = I\,dl\times B}$$

(iii) force b/w two current element :-



we know that

$$dF = Idl\times B$$

force on I_1dl_1 due to I_2dl_2 is given by,

$$d(dF) = I_1dl_1\times B_2 \; \} \; dB_2 \text{ is field produced by } I_2dl_2$$

$$\therefore dF = I\,dl\times B$$
$$d(dF) = I_1d\vec{l}_1\times d\vec{B}_2$$

from bio savart's law, we have $dB_2 = \int \frac{\mu_0 I_2 dl_2\times \bar{R}_{21}}{4\pi R_{21}^2}$

substitute $d(dF) = I_1dl_1\times dB_2$

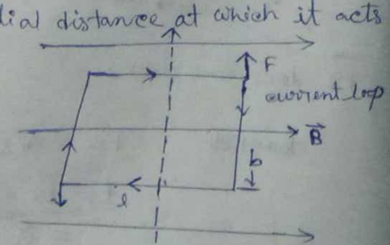
$$= I_1dl_1\times \frac{\mu_0 I_2 dl_2\times \bar{R}_{21}}{4\pi R_{21}^2}$$

$$F = \frac{\mu_0 I_1 I_2}{4\pi}\oint_{l_1}\oint_{l_2}\frac{d\vec{l}_1\times(d\vec{l}_2\times\vec{R}_{21})}{R_{21}^2}$$

Magnetic Torque ~~...~~ and moment :—

When a current loop is placed parallel to a magnetic field, forces act on the loop that tend to rotate it.

→ the tangential force multiplied by the radial distance at which it acts is called Torque. (T)



T = Force × Distance

total T = 2 × torque on each side

the force acting on the loop is

$$F = BIl\sin\theta$$

the total torque on the loop is,

$$T = 2\times \text{Torque on each side}$$
$$= 2\times \text{Distance}\times \text{Force}$$
$$= 2\times BIl\sin\theta\times b/2$$
$$T = BI(lb)\sin\theta$$
$$\boxed{T = BIA\sin\theta}$$

$$T = mB\sin\theta\,\hat{n}$$

where,

magnetic moment $= m = IA\hat{n}$

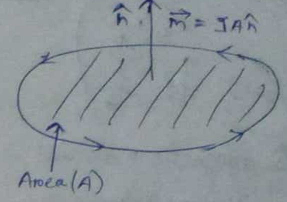
In Vector form, $\boxed{\vec{T} = \vec{m}\times\vec{B}}$

$$|T| = mB\sin\theta$$

from

$$\boxed{m = \frac{T}{B}}$$

Magnetic Dipole Moment / Magnetic Moment :—



→ unit is Am^2.

→ Direction is perpendicular to the surface.

Magnetization :—

magnetization is magnetic dipole moment per unit Volume.

→ magnetization is depending on material.

→ unit $= \left[\frac{\text{magnetic moment }(\vec{m})}{\text{Volume }(V)} = \frac{Am^2}{m^3} = A/m\right]$

$\vec{m}$ (magnetic moment) $= \chi_m$ (magnetic susceptability) $\times$ $\vec{H}$ (magnetic field intensity)

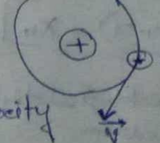
$\boxed{\mu_r = 1 + \chi_m}$ μ_r = relative permeability

Magnetization in Materials :—

→ All magnetic materials are affected by external magnetic field $\vec{B}$.

→ as magnetic materials acquires some magnetic moment or magnetic material dipole moment $(\vec{m})$

→ As we know that ~~magnetic~~ material consist of atoms. atom consist of neuclious and electron that revolves in a circular orbit

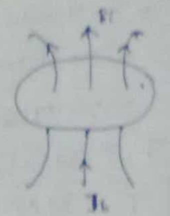


$\vec{v}$ – Velocity

→ electron revolves of own axis that is called electron spin.

→ When some magnetic field produces ~~the~~ due to the electron spin that is called internal magnetic field (B_i)

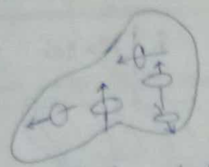
→ ~~the direction of magnetic field~~ current is following anticlockwise direction so the direction of the magnetic field is given by right hand thumb rule.



I_b = Bound current (bound to the atom)

this case magnetic moment $(\vec{m}) = I_b\, S\hat{a}_n$

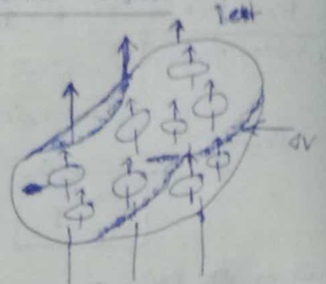
Influence of external $\vec{B}$ field



↑ magnetic material having Volume ΔV

$\vec{B}_{ex}$ (~~external~~ external magnetic field) $= 0$

$\Sigma \vec{m} = 0$ (magnetic moment) — Due to random motion



$\vec{m}$ = more or align themselves with $\vec{B}$

So, $\Sigma \vec{m} \neq 0$

Magnetization of Magnetic material $\boxed{(\vec{M}) = \lim_{\Delta v \to 0} \frac{\sum_{K=1}^{N} \vec{m}_K}{\Delta V}}$

Note :- When $\vec{B}_{ext} = 0, \Sigma \vec{m} = 0, \ \& \ \vec{M} = 0$

$\vec{B}_{ext} \neq 0, \Sigma \vec{m} \neq 0 \ \& \ \vec{M} \neq 0$ - - - magnetized medium or material

magnetic flux density in magnetic material Increases due to $\vec{M}$

$\therefore \vec{B} = \underbrace{\mu_0 \vec{H}}_{\text{Due to free space}} + \underbrace{\mu_0 \vec{M}}_{\text{Due to magnetic material}}$ . . . . A/m

$\Rightarrow \vec{B} = \mu_0(\vec{H} + \vec{M})$ ——— (i)

$\because \vec{B} = \mu \vec{H}$
$= \mu_0 \mu_r \vec{H}$ ——— (ii)

$\therefore$ (i) and (ii) solved this equation

$\mu_0(\vec{H} + \vec{M}) = \mu_0 \mu_r \vec{H}$

$\Rightarrow \vec{M} = (\mu_r - 1)\vec{H}$

$\boxed{\Rightarrow \vec{M} = X_m \vec{H}}$

$[(\mu_r - 1) = X_m]$

[X_m = magnetic susceptibility of the medium]

μ_r = relative permeability (dimensionless)

Also, $\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$

$= \mu_0 \vec{H} + \mu_0 X_m \vec{H}$ $\quad \because \vec{M} = X_m \vec{H}$

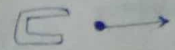
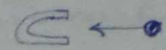
$\vec{B} = \mu_0 (1 + X_m) \vec{H}$

$\boxed{\vec{B} = \mu_0 (1 + X_m)}$ $\quad$ Where, $\vec{B}_0 = \mu_0 \vec{H}$ . . . free space

⊛ magnetic Material :-

Materials that can be magnetized, in the presence of magnetic field.

i) Diamagnetic materials
ii) paramagnetic
iii) Ferromagnetic

Diamagnetic	paramagnetic	Ferromagnetic
i) weakly repelled by a magnet	i) weakly attracted by a magnet	i) strongly attracted by a magnet.
		
ii) weakly magnetized in opposite direction of magnetic field. Value = (-ve), small	ii) weakly magnetized in the same direction Value = +ve, small	ii) strongly magnetized in the same direction of magnetic field. Value +ve, large
iii) they loss their magnetism on removal of external magnetic field. → No unpair electron	iii) same but unpaired electron present.	iii) behaves as permanent magnets. unpaired electron present
iv) Solid, liquid, Gases ex - water, copper, Mercury, Gold, H_2	iv) Solid, liquid, gases Al, platinum, Na, crown glass.	iv) only solid Fe, Ni, Cu etc.

⊛ Boundary condition for Magnetic field inductance $(\vec{B})$:-

The condition that the magnetic field induction $\vec{B}$ must satisfy at the interface seperating 2 difference media is called boundary condition for $\vec{B}$.

To obtain boundary condition for $\vec{B}$, let us consider a pill box like surface at the interface b/w 2 difference media as shown in fig

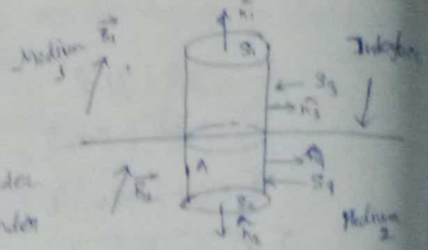
The pill box is composed of 4 surfaces S_1, S_2, S_3, S_4

S_1 = to upper surface of pill box

S_2 = lower most surface of pill box

S_3 = curved surface of upper cylinder

S_4 = curved surface of lower cylinder



Let $\vec{B_1}$ & $\vec{B_2}$ be the magnetic field inductance in medium 1 & 2 respectively. from Maxwell's equation for magnetic field inductance ($\vec{B}$)

$$\nabla\cdot\vec{B} = 0 \quad [\text{integrating } \text{the volume enclosed by pill box}]$$

$$\int_V \nabla\cdot\vec{B}\,dv = 0 \quad \text{——— (1)}$$

According to Gauss's Divergence theorem we have,

$$\int_V \nabla\cdot\vec{B}\,dv = \oint_S \vec{B}\cdot\hat{n}\,ds \quad \text{——— (2)}$$

comparing (1) & (2) equation

$$\oint_S \vec{B}\cdot\hat{n}\,ds = 0$$

$$\int_{S_1} \vec{B_1}\hat{n_1}\,ds + \int_{S_2} \vec{B_2}\cdot\hat{n_2}\,ds + \int_{S_3} \vec{B_3}\hat{n_3}\,ds + \int_{S_4} \vec{B_4}\hat{n_4}\,ds = 0$$

Now $\Delta h \to 0$, then there will be no current contribution from surfaces S_3 & S_4.

$$\Rightarrow \int_{S_1} \vec{B_1}\hat{n_1}\,ds + \int_{S_2} \vec{B_2}\cdot\hat{n_2}\,ds = 0$$

$$\Rightarrow \int_A \left(\vec{B_1}\hat{n_1} + \vec{B_2}\hat{n_2}\right)ds = 0 \quad [\because S_1 = S_2 = A]$$

$$\Rightarrow \left(\vec{B_1}\hat{n_1} + \vec{B_2}\hat{n_2}\right)A = 0 \quad \left[\begin{array}{l}\vec{B_1} = \vec{B_{1t}} + \vec{B_{1n}} \\ \vec{B_2} = \vec{B_{2t}} + \vec{B_{2n}}\end{array}\right]$$

$$\Rightarrow \vec{B_1}\hat{n_1} + \vec{B_2}\hat{n_2} = 0 \quad [\because A \neq 0]$$

$$\Rightarrow \left(\vec{B_{1t}} + \vec{B_{1n}}\right)\hat{n_1} + \left(\vec{B_{2t}} + \vec{B_{2n}}\right)\cdot\hat{n_2} = 0$$

$$\Rightarrow \vec{B_{1t}}\cdot\hat{n_1} + \vec{B_{1n}}\hat{n_1} + \vec{B_{2t}}\hat{n_2} + \vec{B_{2n}}\cdot\hat{n_2} = 0$$

$$\Rightarrow \vec{B_{1n}}\hat{n_1} + \vec{B_{2n}}\cdot\hat{n_2} = 0$$

$$\Rightarrow \vec{B_{1n}}\hat{n_1} - \vec{B_{2n}}\cdot\hat{n_1} = 0 \quad (\hat{n_2} = -\hat{n_1})$$

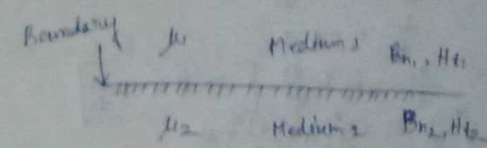
$$\boxed{\Rightarrow \vec{B_{1n}} = \vec{B_{2n}}}$$

Hence normal component of magnetic field induction $\vec{B}$ is continuous across the interface b/w 2 different medium.

Boundary condition for Magnetic field :-

→ Magnetic field is continuous in a single medium.

→ At the interface b/w two magnetic materials the magnetic field changes both in the magnitude & direction.



Conditions :-

i) the normal component of magnetic flux density is continuous across the boundary. $\boxed{B_{n_1} = B_{n_2}}$

2) the tangential component of magnetic field intensity is discontinuous by the amount equal to surface current density (J_s)

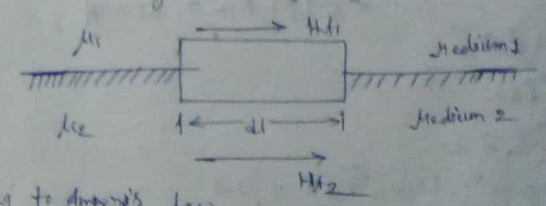
$$\boxed{H_{t_1} - H_{t_2} = J_s}$$

If there is no 'J_s' the magnetic field intensity is continuous across the boundary.

$$\boxed{H_{t_1} = H_{t_2}}$$

$H_{t_1} - H_{t_2} = J_s$ (Magnetic field intensity)

Consider a small rectangular box of length 'dl' at the boundary of two medium



According to Ampere's law,

$$\oint_c H\cdot dl = I$$

$$H_{t_1}\Delta l - H_{t_2}\Delta l = J_s\Delta l$$

$$\boxed{\Rightarrow H_{t_1} - H_{t_2} = J_s}$$

∴ the tangential component of magnetic field intensity is discontinuous at the boundary by the amount of 'J_s' - surface current density.

If $J_s = 0$ $\boxed{H_{t_1} = H_{t_2}}$

If there is no surface current density at the interface, then the tangential component of $\vec{H}$ is continuous across the boundary.

Energy stored in Magnetic field :-

$W = q\,\Delta V$

work done by the magnetic field is against the EMF.

So, $W = -qE$ $\quad \therefore \frac{dq}{dt} = I$

$$\frac{dw}{dt} = -E\frac{dq}{dt}$$

$$\frac{dw}{dt} = -EI$$

$$\frac{dw}{dt} = LI\frac{dI}{dt} \quad \left[\because E = -L\frac{dI}{dt}\right]$$

$$\int\left(\frac{dw}{dt}\right)dt = \int\left(LI\frac{dI}{dt}\right)dt$$

$$\int dw = L\int I\,dI$$

$$W = L\frac{I^2}{2}$$

$$\Rightarrow W = \frac{1}{2}LI^2 \quad \text{—— (1)}$$

$\phi_B = MI$ (mutual inductance)

$\phi_B = LI$ (self inductance) —— (2)

$$\phi = \int \vec{B}\cdot d\vec{a}$$

$$\vec{B} = (\vec{\nabla}\times\vec{A}) \quad \therefore \vec{A} \text{ vector potential of } \vec{B}$$

$$\phi = \int(\vec{\nabla}\times\vec{A})\,d\vec{a}$$

By stokes law.

$$\phi = \oint \vec{A}\cdot d\vec{l} \quad \text{—— (3)}$$

Now, from equation no (1) $W = \frac{1}{2}LI^2$

$$= \frac{1}{2}I(LI)$$

$$= \frac{1}{2}I\phi \quad [\text{from 2, } LI = \phi]$$

$$\therefore W = \frac{1}{2}I\oint \vec{A}\cdot d\vec{l}$$

$$I = \int \vec{J}\,da$$

$$W = \frac{1}{2}\int_v \vec{J}\cdot\vec{A}\,dz$$

By using Ampere's law, $\vec{\nabla}\times\vec{B} = \mu_0 J$

$$W = \frac{1}{2}\mu_0\int \vec{A}(\vec{\nabla}\times\vec{B})\,dz \quad \text{—— (4)}$$

$$\therefore \vec{A}\cdot(\vec{\nabla}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{\nabla}\cdot(\vec{A}\times\vec{B})$$

putting in equation no 4.

$$W = \frac{1}{2}\mu_0\left[\int_v \vec{B}\cdot(\vec{\nabla}\times\vec{A})\,dz - \int_v \nabla\cdot(\vec{A}\times\vec{B})\,dz\right]$$

$$W = \frac{1}{2}\mu_0\left[\int_v B^2\,dz - \int_s (\vec{A}\times\vec{B})\,dz\right]$$

$$W = \frac{1}{2}\mu_0\int_v B^2\,dz$$

$$\boxed{W = \frac{1}{2}\mu_0\int_v B^2\,dz}$$

force on Magnetic Material :-

$$\psi = BS$$

$$\Rightarrow B = \frac{\psi}{S}$$

energy density $(W_E) = \frac{1}{2}\mu H^2$

$B = \mu_0 H \quad W_E = \frac{1}{2}\mu\left(\frac{B^2}{\mu_0^2}\right)$

$$\Rightarrow H = \frac{B}{\mu_0}$$

$$\frac{-F\,dl}{\cancel{q}} = dw = \frac{1}{2}\frac{B^2}{\mu_0}S\,dl + \frac{1}{2}\frac{B^2}{\mu_0}S\,dl$$

$$\Delta W = \frac{1}{2}\mu H^2 \underbrace{\Delta x\Delta y\Delta z}_{\Delta V}$$

$$\boxed{F = \frac{-B^2 S}{\mu_0}}$$

~~Prest~~ $P = \frac{F}{S} = \frac{B^2}{2\mu_0} = \boxed{\frac{1}{2}BH}$

Unit-5 / Electromagnetic fields /

~~Faraday's law~~

electrostatic = charges at rest

~~electro~~ magnetostatic = charges are in motion

$$a = \frac{dv}{dt}$$

faraday's law :-

EMF :-

$$V = \frac{d\phi}{dt}$$

Vemf in any closed ckt is equal to the time rate of change of magnetic flux linked by the ckt.

$$V_{emf} = -N\frac{d\phi}{dt} \quad [N = \text{no of the turn}$$

$\downarrow$

- sign, lenz's law ($d\phi$ opposing the V)

$$V_{emf} = \text{potential difference} = \frac{\text{work by the external force}}{\text{charge } (q)}$$

$$V_{emf} = -\int \vec{E}.\vec{dl} = IR$$

transformer emf | motional emf

Faradays law of electromagnetic induction :-

First law :- Faraday's first law of electromagnetic induction states that whenever magnetic flux linked with a circuit changes, induced emf is produced.

2nd law :- that law states that the magnetitude of induced e.m.f is directly proportional to the time rate of change in magnetic flux linked with the circuit.

$$e \propto \frac{d\phi_B}{dt}$$

$$\Rightarrow e = -k.\frac{d\phi_B}{dt}$$

$$\Rightarrow \boxed{e = -N\frac{d\phi_B}{dt}}$$

N = no of turns of the coil

ϕ_B = magnetic flux

(-) = which states that the induced emf opposes the flux producing it.

Lenz's law :-

It states that the induced current produced in a ckt due to change magnetic flux always flows in such a direction that it opposes the cause that produced it.

1st :- If flux is change then current produced. when emf produced.

2nd :- ~~emf~~ . ~~emf~~ ~~electromagnetic induction~~ ~~created~~ which amount magnetitude is produced of created electromagnetic induction

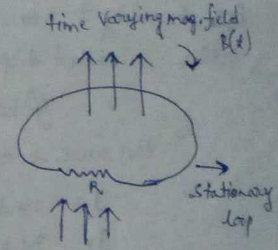
Lenzs :- this law state that the direction of induced emf.

~~Transfor~~ induced emf are two types :-

i) transformer emf

ii) motional emf

i) transformer emf :-



the e.m.f induced in a stationary loop in a time varying magnetic field is called 'transformer' emf.

from faraday's law

$$V_{emf} = -N\frac{d\phi}{dt}$$

if, N = 1, $V_{emf} = -\frac{d\phi}{dt}$

$$\boxed{\text{transformer emf} = \int_S \frac{\delta B}{\delta t}.ds}$$

$$V_{emf} = \oint \vec{E}.dl = -\frac{d}{dt}\int_S B.ds$$

(V/m × m)

$$V_{emf} = \oint_l E.dl = -\frac{d}{dt}\int_S B.ds$$

Apply stoks theorem to line integral ($\int_l - \iint_s$)

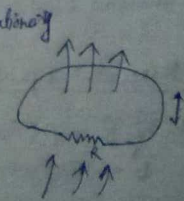
$$\boxed{V_{emf} = \oint_A E\,dl = \int_S (\nabla \times E).ds = \int -\frac{dB}{\delta t}.ds}$$

this is called Maxwell's 3rd equation in integral form.

By comparing surface integral

$$\boxed{\nabla \times E = -\frac{\delta B}{\delta t}}$$

ii) Motional Emf / Flux cutting emf :-



the emf induced in a moving loop in a stationary magnetic field is called "motional emf".

charge 'q' is moving with velocity 'u' in magnetic field 'B'

$$\boxed{F_m = qu \times B}$$

the motional electric field as $E_m = \frac{F_m}{q}$

$$\Rightarrow \frac{F_m}{q} = \cancel{F_m} \times u \times B = E_m$$

$$\boxed{\text{the motional emf} = \oint_l F_m \cdot dl - \oint_l (u \times B) \cdot dl}$$

(*) Displacement current :-

According to Ampere circuit law $\oint B\,dl = \mu_0 I$

But according to maxwell, there is another current in the circuit which is called displacement current.

$$\oint B\,dl = \mu_0 (I + I_D)$$

this rule is called Ampere's maxwell law

I = conduction current

I_D = Displacement current

→ Displacement current is that current which comes into play in the region in which the electric field and electric flux is changing with time.

$$\phi = \frac{q}{\epsilon_0}$$

$$\Rightarrow \frac{d\phi}{dt} = \frac{1}{\epsilon_0}\frac{dq}{dt} \quad \text{[differentiate both sides w.r.t -- t]}$$

$$\Rightarrow \frac{d\phi}{dt} = \frac{1}{\epsilon_0} I_D \quad \left[\frac{dq}{dt} = I_D\right]$$

$$\Rightarrow \epsilon_0 \frac{d\phi}{dt} = I_D$$

$$\boxed{\Rightarrow I_D = \epsilon_0 \frac{d\phi}{dt}}$$

Now, Ampere Maxwell rule can be written as

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 \left[I + \epsilon_0 \frac{d\phi}{dt}\right]}$$

→ changing magnetic field → electric field generate.

→ changing electric field → magnetic field generate.

(*) Maxwell's Equation :-

① $\vec{\nabla} \cdot \vec{D} = \rho$ (Gauss's law of electrostatics)

$\vec{D}$ = Displacement density

ρ = Volume charge density

$\nabla \cdot$ = divergence
$\nabla \times$ = curl
∇ = gradient

② $\vec{\nabla} \cdot \vec{B} = 0$ (Gauss's law for magneto statics)

where, $\vec{B}$ = magnetic field/magnetic flux density

③ $\vec{\nabla} \times \vec{E} = -\frac{\delta B}{\delta t}$ (Faraday's law of electro magnetic mutual Induction)

where, $\vec{E}$ = electric field

④ $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\delta \vec{D}}{\delta t}$ (Modified Amper's circuital law)

$\vec{J}$ = current density (I/A)

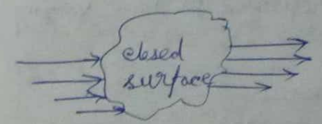
$\frac{\delta \vec{D}}{\delta t}$ → Displacement current

1) $\int \vec{E} \cdot \vec{ds} = \int_V \rho\,dv \left(\frac{1}{\epsilon_0}\right)$

$\vec{ds}$ = surface area

$\vec{E}$ = electric field

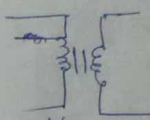
②



Net field $(\vec{B}) = 0$

Div. $\vec{B} = 0$

③



$$\int_l \vec{E} \cdot \vec{dl} = -\frac{d}{dt}\oint_s \vec{B} \cdot \vec{ds}$$

$$emf = -\frac{d\phi_B}{dt}$$

ϕ_B = magnetic flux

dt = change in magnetic flux

$-$ = negative change in m.f (lenz's law)

(*) Integral :-

① $\oint_S \vec{D} \cdot \vec{ds} = \int_V \rho\,dv$

② $\oint_S \vec{B} \cdot \vec{ds} = 0$

③ $\oint_l \vec{E} \cdot \vec{dl} = -\frac{d}{dt}\oint_s \vec{B} \cdot \vec{ds}$

④ $\int_l \vec{H} \cdot \vec{dl} = \oint_J \left(\vec{J} + \frac{\vec{dD}}{\delta t}\right) \cdot \vec{ds}$

Application :-

i) Audio ~~B~~ comm

ii) communication system (a radio communication, satelite communication, antena communication)

Time Varying Potential :

Examine the potential V and $\vec{A}$ for time varying fields.

for time varying field

$$\nabla \times \vec{E} \neq 0$$

to the $\vec{E} = -\nabla V$

$$\nabla \times \vec{E} = \nabla \times (-\nabla V)$$
$$= -\nabla \times \nabla V = 0$$

$\vec{E} = -\nabla V$ is not valid when a time varying magnetic field in present therefore new approach to potential is required for time varying fields

According to Maxwell's equation $\nabla \cdot \vec{B} = 0$

$$\therefore \vec{B} = \nabla \times \vec{A}$$

and $\nabla \times \vec{E} = -\frac{\delta B}{\delta t} = -\frac{\delta}{\delta t}(\nabla \times \vec{A})$

$$\therefore \nabla \times \left(E + \frac{\delta A}{\delta t}\right) = 0$$

curl of gradient of a scalar field is zero. the solution is

$$E + \frac{\delta A}{\delta t} = -\nabla V$$

$$\boxed{\Rightarrow E = -\nabla V - \frac{\delta A}{\delta t}} \quad \text{——— (1)}$$

Also, $\nabla \cdot \vec{B} = \rho_v$

$$\Rightarrow \nabla \cdot \vec{E} = \frac{\rho_v}{\epsilon}$$

$$\Rightarrow \nabla \cdot \left(-\nabla V - \frac{\delta A}{\delta t}\right) = \frac{\rho_v}{\epsilon}$$

$$\Rightarrow -\nabla^2 V - \nabla \frac{dA}{\delta t} = \frac{\rho_v}{\epsilon}$$

$$\boxed{\Rightarrow \nabla^2 V + \nabla \frac{\delta A}{\delta t} = -\frac{\rho_v}{\epsilon}} \quad \text{——— (2)}$$

Also using maxwell equation $\nabla \times \vec{H} = \vec{J} \times \frac{\delta \vec{D}}{\delta t}$

but $\vec{B} = \mu \vec{H}$

$$\therefore \nabla \times \frac{\vec{B}}{\mu} = \vec{J} + \frac{\delta \vec{D}}{\delta t}$$

$$\Rightarrow \nabla \times \vec{B} = \mu\left(\vec{J} + \frac{\delta \vec{D}}{dt}\right)$$

$$\Rightarrow \nabla \times \nabla \times \vec{A} = \mu\left(\vec{J} + \frac{\delta \vec{D}}{\delta t}\right) \quad [\vec{B} = \nabla \times \vec{A}]$$

$$\Rightarrow \nabla \times \nabla \times \vec{A} = \mu \vec{J} + \mu\epsilon \frac{\delta \vec{E}}{\delta t} \quad [\vec{D} = \epsilon \vec{E}]$$

replacing from equation (1) above equation gives.

$$\Rightarrow \nabla \times \nabla \times \vec{A} = \mu \vec{J} + \mu\epsilon \frac{\delta}{\delta t}\left(-\nabla V - \frac{\delta \vec{A}}{\delta t}\right)$$

$$\nabla(\nabla \times \vec{A}) - (\nabla \cdot \nabla)\vec{A} = \mu \vec{J} - \mu\epsilon \nabla\left(\frac{\delta V}{\delta t}\right) - \mu\epsilon \frac{\delta^2}{\delta t^2}\vec{A}$$

$$\Rightarrow \nabla^2 \vec{A} - \nabla(\nabla \cdot \vec{A}) = -\mu \vec{J} + \mu\epsilon \nabla\left(\frac{\delta V}{\delta t}\right) + \mu\epsilon \frac{\delta^2 \vec{A}}{\delta t^2} \quad \text{——— (3)}$$

According to Lorenz condition for potential

$$\nabla \cdot \vec{A} = -\mu\epsilon \frac{\delta V}{\delta t} \quad \text{——— (4)}$$

equation (3) put (1)

$$\nabla^2 A - \nabla \cdot \left(-\mu\epsilon \frac{\delta V}{\delta t}\right) = -\mu \vec{J} + \mu\epsilon \nabla\left(\frac{\delta V}{\delta t}\right) + \mu\epsilon \frac{\delta^2 \vec{A}}{\delta t^2}$$

$$\Rightarrow \nabla^2 A + \nabla \mu\epsilon \frac{\delta V}{\delta t} = -\mu \vec{J} + \mu\epsilon \nabla\left(\frac{\delta V}{\delta t}\right) + \mu\epsilon \frac{d^2 \vec{A}}{dt^2}$$

$$\Rightarrow \nabla^2 A = -\mu \vec{J} + \mu\epsilon \frac{d^2 \vec{A}}{dt^2}$$

$$\boxed{\Rightarrow \nabla^2 A - \mu\epsilon \frac{\delta^2 \vec{A}}{\delta t^2} = -\mu \vec{J}} \quad \text{——— (5)}$$

Also replacing $\nabla \cdot \vec{A}$ from equation (4) in equation (2) fields

$$\nabla^2 V + \nabla \frac{\delta \vec{A}}{\delta t} = \frac{-\rho_v}{\epsilon}$$

$$\Rightarrow \nabla^2 V + \frac{\delta}{\delta t}\left(-\mu\epsilon \frac{\delta V}{\delta t}\right) = -\frac{\rho_v}{\epsilon}$$

$$\boxed{\Rightarrow \nabla^2 V - \mu\epsilon \frac{\delta^2 V}{\delta t^2} = -\frac{\rho_v}{\epsilon}} \quad \text{——— (6)}$$

$\boxed{\nabla^2 A - \mu\epsilon \frac{\delta^2 \vec{A}}{\delta t^2} = -\mu \vec{J}}$ this equation are called the wave equation. This solution of above equation will give the potential V and $\vec{A}$ of time varying field. therefore

$$V = \int_v \frac{[\rho_v]\, dv}{4\pi\epsilon R} \quad \text{——— (8)}$$

&

$$\vec{A} = \int_v \frac{\mu [\vec{J}]\, dv}{4\pi R} \quad \text{——— (9)}$$

Time Harmonic Potential (Sinusoidal Variations of fields) :–

→ time harmonics fields is one that varies periodically or sinusoidally with time or

phasor form time harmonic (x, y, z, t)

→ A phasor Z is a complex number that can be written as

$$Z = x + jy \rightarrow \text{Rectangular form}$$
$$= r\angle\phi \rightarrow \text{polar form}$$

→ Electric field / magnetic field depends on $E[x, y, z]$

→ time-varying field — $E\,\boxed{x, y, z, t}$

$Z = r\angle\phi$ $\qquad \phi = \omega t + \theta$ (time include)

$Z = r\angle \omega t + \theta$ or $r.e^{j\omega t}.e^{j\theta}$

$$\boxed{Z = r.e^{j\theta} . e^{j\omega t}} \qquad \left\{\begin{array}{l} Z_1 = Re\{Z\} = r\cos(\omega t + \theta) \\ Z_2 = Im(Z) = r\sin(\omega t + \theta) \end{array}\right\}$$

Instantaneous Value

$$I(t) = I_0 \cos(\omega t + \theta)$$

$$I(t) = Re\{I_0 . e^{j(\omega t + \theta)}\}$$

$$= Re\{\boxed{I_0 . e^{j\theta}} \boxed{e^{j\omega t}}\} \rightarrow e^{j\omega t}$$

Real part (I_s)

$$= Re\{I_s . e^{j\omega t}\} \qquad (I_s = \text{phasor form})$$

$$\Rightarrow \boxed{I(t) = Re\{I_s . e^{j\omega t}\}}$$

$I(t) \rightarrow$ Instantaneous form

$I_s \rightarrow$ phasor form

$$\frac{d}{dt}\{I(t)\} = \frac{d}{dt} Re\{I_s . e^{j\omega t}\}$$

$$= Re\{I_s . \frac{d}{dt}(e^{j\omega t}) + e^{j\omega t} . 0\}$$

$$= Re\{I_s . e^{j\omega t} . j\omega\}$$

$$= Re\{I_s j\omega . e^{j\omega t}\}$$

$$\boxed{\frac{d}{dt} I(t) \longrightarrow j\omega I_s}$$

Similarly, $\boxed{\int I(t)\, dt \longrightarrow \frac{I_s}{j\omega}}$

question :-

Given, $A = 10\cos(10^8 t - 10x + 60^\circ)\, a_z$ and

$B_s = \frac{20}{j} a_x + 10 e^{\frac{j2\pi x}{3}} a_y$, express A in phasor form and B_s in instantaneous form.

$\Rightarrow$ (I) $\qquad$ I - P [Instantaneous to phasor]

$A = 10\cos(10^8 t - 10x + 60^\circ)\, a_z \qquad (\omega) = 10^8$

$Re\{10 e^{j(10^8 t - 10x + 60^\circ)} a_z\}$

$Re\{10 . e^{j(-10x + 60^\circ)} a_z . e^{j10^8 t}\}$

$$\boxed{A_s = 10 e^{j(-10x + 60^\circ)} a_z}$$

(II) $\quad B_s = \frac{20}{j} a_x + 10 . e^{\frac{j2\pi x}{3}} a_y$

$\boxed{B = Re\{B_s . e^{j\omega t}\}} \longrightarrow$ Instantaneous Value

$B_s = 20 . e^{-j\pi/2} a_x + 10 e^{\frac{j2\pi x}{3}} a_y$

$$B = Re\{20 e^{-j\pi/2} e^{j\omega t} + 10 e^{j\frac{2\pi x}{3}} . e^{j\omega t} a_y\}$$

$$B = Re\{20 . e^{j(\omega t - \pi/2)} a_x + 10 . e^{j(\omega t + \frac{2\pi x}{3})} a_y\}$$

$$B = 20\cos(\omega t - \pi/2) a_x + 10\cos(\omega t + \frac{2\pi x}{3}) a_y$$

$$\boxed{B = 20 \sin\omega t\, a_x + 10\cos(\omega t + \frac{2\pi x}{3})\, a_y}$$

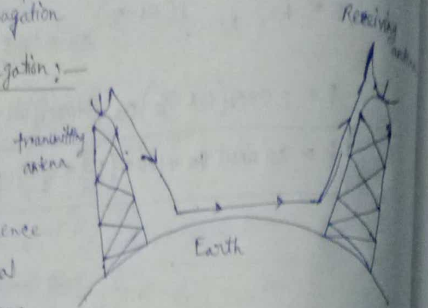
unit - 6

Electromagnetic wave Propagation

there are four mode of propagation of waves.

i) Ground wave Propagation
ii) sky wave propagation
iii) space wave propagation
iv) Satelite propagation

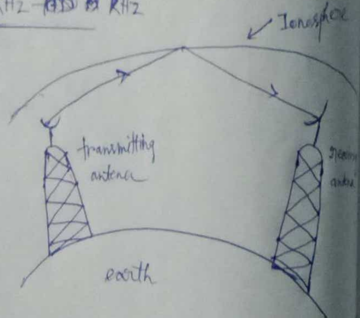
① Ground wave Propagation :—



It is the mode of propagation in which ground has strong influence on the propagation of signal from the transmitting antena to receiving antena.

→ In this propagation, the signal wave glides over the surface of earth. therefore, the loss in this type of propagation are large and hence its range is short. The general frequency range for this propagation is 530 KHz - 1710 KHz

② Sky wave propagation :—



In this type of propagation, the rays are first transmitted from transmitting antena. through Ionosphere. After reflection from Ionosphere, they reach at receiving antena.

→ In this type propagation loss is very less and range is high.
→ the general frequency range for sky wave propagation is 1710 KHz to 40 MHz.

→ why we can not use the wave propagation for propagation the signals of frequency more than 40MHz ?
It is due to the reason that signals of frequency more than 40 MHz can not be reflected by ionosphere. Therefore, they will pass out from the ionosphere and hence transmission is not possible.

Wave equation of electromagnetic wave propagation :—

As per faraday's law

$$\Rightarrow \nabla \times E = -\frac{dB}{dt}$$

take curl on both sides

$$\Rightarrow \nabla \times \nabla \times E = -\frac{d(\nabla \times B)}{dt}$$

$$\Rightarrow \nabla(\nabla \cdot E) - \nabla^2 E = -\frac{d(\nabla \times B)}{dt} \quad \text{—— ①}$$

As per gauss's law of electric field

$$\nabla \cdot D = \rho_v$$
$$\rightarrow \nabla \cdot (\epsilon_0 E) = \rho_v$$
$$\Rightarrow \nabla \cdot E = \frac{\rho_v}{\epsilon_0}$$

– for free space $\rho_v = 0$

$$\Rightarrow \boxed{\nabla \cdot E = 0}$$

from no ① equation

$$-\nabla^2 E = -\frac{d(\nabla \times B)}{dt}$$

$$\Rightarrow \nabla^2 E = \frac{d(\nabla \times B)}{dt} \quad \left[B = \mu_0 H = \mu_0 \frac{d}{dt}(\nabla \times H) \right]$$

As per Ampere circuit law for time varying field

$$\Rightarrow \nabla \times H = J + \frac{\delta D}{\delta t}$$

[for free space $J = 0$
(J = conduction current density

$$\Rightarrow \nabla \times H = \frac{\delta D}{\delta t} = \epsilon_0 \frac{\delta E}{\delta t}$$

from above equation

$$\nabla^2 E = \mu_0 \epsilon_0 \frac{\delta^2 E}{\delta t^2}$$

$$\Rightarrow \nabla^2 E = \frac{1}{c^2} \frac{\delta^2 E}{\delta t^2}$$

[for free space, $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$
$\Rightarrow \sqrt{\mu_0 \epsilon_0} = 1/c$]

$$\boxed{\Rightarrow c^2 \nabla^2 E = \frac{\delta^2 E}{\delta t^2}}$$

this is wave equation (a)

that we can say, $\boxed{\nabla^2 E = \gamma^2 E}$ —— (b) this is wave equation

⊛ Electromagnetic wave :—

"Maxwell equation related to EM Wave"
"waves are mean of transporting energy or Information".
ex:- radio waves, TV signals, radder beams, light rays etc.

Three fundamental characteristics :-

i) They all travel at high velocity.
ii) In travelling, they assume the properties of wave.
iii) They radiate outward from a source.

Medium of EM wave :-

i) free space : ($\sigma = 0$; $\epsilon = \epsilon_0$; $\mu = \mu_0$)
ii) lossless dielectric : ($\sigma = 0$; $\epsilon = \epsilon_0\epsilon_r$; $\mu = \mu_0\mu_r$) / ($\sigma << \omega\epsilon$)
iii) lossy dielectric : ($\sigma \neq 0$; $\epsilon = \epsilon_0\epsilon_r$; $\mu = \mu_0\mu_r$)
iv) Good conductor : ($\sigma = \infty$, $\epsilon = \epsilon_0$, $\mu = \mu_0\mu_r$) / ($\sigma >> \omega\epsilon$)

σ = conductivity
ω = angular velocity of wave
ϵ = relative permittivity
μ = relative permibility

Wave propagation in lossy dielectrics :-

→ Imperfect dielectric
→ As wave propagate loses power

lossy dielectric is partially conducting medium (or imperfect conductor) ($\sigma \neq 0$)

Note :- In lossless dielectric (perfect or good dielectric ($\sigma = 0$)

$\sigma \neq 0$

consider the charge free region $\rho_v = 0$

$$\nabla \cdot D = \rho_v$$
$$\Rightarrow \nabla \cdot D = 0$$
$$\Rightarrow \nabla \cdot \epsilon E = 0$$
$$\Rightarrow \epsilon \nabla \cdot E = 0 \quad \text{—— (1)}$$

$$\nabla \cdot B = 0 \Rightarrow \nabla \cdot \mu H = 0$$
$$\Rightarrow \mu \nabla H = 0 \quad \text{—— (2)}$$

$$\nabla \times E = -\frac{\delta B}{\delta t} = -\mu \frac{\delta H}{\delta t} \quad \text{—— (3)}$$

$$\nabla \times H = \sigma E + \frac{\delta D}{\delta t} = \sigma E + \epsilon \frac{\delta E}{\delta t} \quad \text{—— (4)} \qquad [\because \sigma E = J]$$

we know that,

$$E = E_0 e^{j\omega t} \quad \text{—— (5)} \quad \text{and} \quad H = H_0 \cdot e^{j\omega t} \quad \text{—— (6)}$$

(6) in equation (3)

$$\nabla \times E = -\mu \frac{\delta (H_0 e^{j\omega t})}{\delta t}$$

$$\Rightarrow \nabla \times E = -\mu \frac{\delta}{\delta t}\left[H_0 e^{j\omega t}\right]$$

$$\boxed{\Rightarrow \nabla \times E = -j\omega\mu H} \quad \text{—— (7)}$$

(5) in equation (4)

$$\nabla \times H = \sigma E_0 e^{j\omega t} + \epsilon \frac{\delta}{\delta t}(E_0 \cdot e^{j\omega t})$$
$$= \sigma E_0 e^{j\omega t} + \epsilon E_0 \cdot (j\omega) \cdot e^{j\omega t}$$
$$= (\sigma + j\epsilon\omega) E_0 e^{j\omega t}$$

$$\boxed{\Rightarrow \nabla \times H = (\sigma + j\omega\epsilon)E} \quad \text{—— (8)}$$

Decoupling curl on both sides equation no (7) & (8)

$$\nabla \times E = -j\omega\mu H$$
$$\nabla \times (\nabla \times E) = -j\omega\mu(\nabla \times H)$$
$$\nabla(\nabla \cdot E) - \nabla^2 E = -j\omega\mu(\nabla \times H)$$
$$\Rightarrow -\nabla^2 E = -j\omega\mu(\nabla \times H)$$

(from equation 8)

$$\Rightarrow -\nabla^2 E = -j\omega\mu(\sigma + j\omega\epsilon)E$$
$$\Rightarrow \nabla^2 E = j\omega\mu(\sigma + j\omega\epsilon)E \quad \text{—— (9)}$$

$\nabla \times H = (j\omega\epsilon + \sigma)E$

$\Rightarrow \nabla \times \nabla \times H = (j\omega\epsilon + \sigma)(\nabla \times E)$

$\Rightarrow \underset{(0)}{\nabla(\nabla \cdot H)} - \nabla^2 H = (j\omega\epsilon + \sigma)(-j\omega\mu H)$

$\Rightarrow + \boxed{\nabla^2 H = + j\omega\mu H (j\omega\epsilon + \sigma)}$ —— (10)

(9) and (10) are the wave equation for lossy dielectric medium.

→ and this two equation are called "Helmholtz equations".

plane wave in medium :—

carries information from one place to another over a medium.

ex- radiowaves, TV signals, etc :

Characteristics:—

i) propagation constant (γ)
ii) Attenuation constant (α)
iii) Phase shift (β) (Phase constant)
iv) velocity of EM waves (v)
v) wavelength (λ)

types of medium :—

i) lossless medium ($\sigma = 0$)
ii) lossy medium :— → low loss dielectric ($\sigma \neq 0$)
→ Good conductor ($\sigma = \infty$)

plane wave in a lossless dielectric :—

$(\sigma = 0, \epsilon = \epsilon_0\epsilon_r, \mu = \mu_0\mu_r)$ $(\sigma \ll \omega\epsilon)$

→ lossless dielectric medium here conductivity is zero ($\sigma = 0$)

Air, and Vaccum space have no conductivity.

→ here no power loss.

General formula

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)}$$

As $\sigma = 0$,

$$\Rightarrow \alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+0} - 1\right)}$$

$$\Rightarrow \alpha = \omega\sqrt{\frac{\mu\epsilon}{2}(1-1)}$$

$$\Rightarrow \alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\times 0}$$

$\therefore \boxed{\Rightarrow \alpha = 0}$

α = Attenation constant

β :- (Phase shift constant) :—

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1}$$ since $\sigma = 0$

$$\Rightarrow \beta = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1+0} + 1}$$

$$\Rightarrow \beta = \omega\sqrt{\frac{\mu\epsilon}{2}\sqrt{1} + 1}$$

$$\Rightarrow \beta = \omega\sqrt{\frac{\mu\epsilon}{\cancel{2}}\times\cancel{2}}$$

$$\boxed{\Rightarrow \beta = \omega\sqrt{\mu\epsilon}}$$

λ (wave length) :—

$\lambda = \frac{2\pi}{\beta}$ (m)

v (wave velocity) :—

$$v = \frac{\omega}{\beta}$$

$$= \frac{\omega}{\omega\sqrt{\mu\epsilon}}$$

$$\boxed{v = \frac{1}{\sqrt{\mu\epsilon}}} \text{ m/s}$$

Intrinsic Impedance (η) :—

$$\eta = \sqrt{\frac{\mu/\epsilon}{1+\frac{\sigma}{j\omega\epsilon}}}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \text{ ; } (\Omega) \angle 0^\circ$$

phase value (θ) :—

$$\tan\theta = \frac{\sigma}{\omega\epsilon} = 0$$

$\boxed{\theta = 0}$

Both $\vec{E}$ & $\vec{H}$ are same phase in lossless medium.

Plane wave in free space :—

$\sigma = 0$, $\epsilon_r = 1$, $\mu_r = 1$

characteristics of plane waves in free space.

i) $\alpha = 0$

ii) $\beta = \omega\sqrt{\mu\epsilon}$

$= \omega\sqrt{\mu_0\mu_r \times \epsilon_0\epsilon_r}$

$\mu = \mu_0\mu_r$
$\epsilon = \epsilon_0\epsilon_r$

$\because \mu_0 = 4\pi\times10^{-7}$ H/m
$\epsilon_0 = 8.85\times10^{-12}$ F/m

$$\boxed{\beta = \omega\sqrt{\mu_0\epsilon_0}}$$

iii) $v = \frac{\omega}{\beta}$

$$= \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$\boxed{\Rightarrow v = \frac{1}{\sqrt{\mu_0\epsilon_0}}} = 3\times10^8 \text{ m/s} = c$$

speed of light

→ electromagnetic waves travel in free space at the speed of the light.

(iv) Intrinsic Impedance (η) :-

$$\eta = \eta_0 = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$= \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}}$$

$$\boxed{\eta_0 = 120\pi \simeq 377\,\Omega}$$

(v) wave length (λ) = $\frac{2\pi}{\beta}$

(vi) $\tan\theta_n = \frac{\sigma}{\omega\epsilon}$ ($\sigma = 0$)

$\boxed{\Rightarrow \theta_n = 0}$ ... Both $\vec{E}$ & $\vec{H}$ are in same phase.

✱ Plane Wave in Good Conductors :-

$\frac{\sigma}{\omega\epsilon} \gg 1$ (or) $\frac{\omega\epsilon}{\sigma} \ll 1$

$\Rightarrow \sigma \gg \omega\epsilon$

$\sigma = \infty$, $\epsilon = \epsilon_0$, $\mu = \mu_0\mu_r$

the propagation constant, $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

$$= \sqrt{j\omega\mu\sigma\left(1 + \frac{j\omega\epsilon}{\sigma}\right)}$$

$$\gamma = \sqrt{j\omega\mu\sigma}\sqrt{\left(1 + \frac{j\omega\epsilon}{\sigma}\right)}$$

Now as $\frac{\omega\epsilon}{\sigma} \ll 1$ for good conductor

$\gamma \simeq \sqrt{j\omega\mu\sigma}$ Also, $\gamma = |\gamma| \angle \theta_\gamma$

$\boxed{\text{propagation constant } (\gamma) = \sqrt{\omega\mu\sigma} \angle 45^\circ}$

Since, $\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\frac{\sigma}{\omega\epsilon}\right)}$ [$\sigma = 0$]

$\alpha = \sqrt{\frac{\omega^2\mu\sigma}{2\omega}}$

$= \sqrt{\frac{\mu\omega\sigma}{2}} = \sqrt{\pi f \mu\sigma}$

$\boxed{\alpha = \sqrt{\pi f \mu\sigma}}$

$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{\left\{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right\}} - 1\right]}$

$= \omega\sqrt{\frac{\mu\epsilon}{2}}\left[1 - 1\right]\frac{\sigma}{\omega\epsilon}$

$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$

$= \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}} + 1\right]}$

$= \omega\sqrt{\frac{\mu\epsilon}{2}}\sqrt{1 +}$

~~propagation constant :-~~

~~$\gamma = \alpha + j\beta$~~

α (Attenuation constant) :-

$\sigma = 0$, $\epsilon = \epsilon_0$, $\mu = \mu_0\mu_r$

$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$

$= \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\infty}{\omega\epsilon}\right)^2} - 1\right]}$

$= \omega\sqrt{\frac{\mu\epsilon}{2}} \cdot \infty$

$\boxed{\alpha = \infty}$ Nepers/m.

β (phase shift constant) :-

$\beta = \omega\sqrt{\frac{\epsilon\mu}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$ rad/m

$\Rightarrow \beta = \omega\sqrt{\frac{\epsilon\mu}{2}} \cdot \infty$

$\boxed{\Rightarrow \beta = \infty}$

γ (propagation constant) :-

$\boxed{\gamma = \alpha + j\beta}$

$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$ $\left(\frac{\omega\epsilon}{\sigma} \lll 1\right)$

$\gamma^2 = j\omega\mu\sigma\left(1 + \frac{j\omega\epsilon}{\sigma}\right)$

$\gamma = \sqrt{j\omega\mu\sigma\left(1 + \frac{j\omega\epsilon}{\sigma}\right)}$

$\gamma = \sqrt{j\omega\mu\sigma(1 + 0)}$

$\Rightarrow \gamma = \sqrt{j\omega\mu\sigma}$

$\Rightarrow \gamma = \sqrt{\omega\mu\sigma}\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$ $\left[\sqrt{j} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right]$

η (Intrinsic Impedence) :-

$\eta = \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{1}{\sqrt{1 - j\frac{\sigma}{\epsilon\omega}}}$ (Ω) [put $\sigma = \infty$]

$\Rightarrow \eta = \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{1}{\sqrt{1 - j\frac{\infty}{\epsilon\omega}}}$

$\boxed{\eta = 0\,\Omega}$

v (Velocity of propagation) :-

$v = \frac{1}{\sqrt{\frac{\mu\epsilon}{2}\left\{\sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}} + 1\right\}}}$ [$\sigma = \infty$]

$\boxed{v = 0 \text{ m/s}}$

λ (wavelength) :—

$$\lambda = \frac{2\pi}{\omega\sqrt{\frac{\mu\epsilon}{2}\left\{\sqrt{1+\frac{\sigma^2}{\epsilon^2\omega^2}}+1\right.}} \quad [\sigma = \alpha]$$

$$\boxed{\lambda = c\ m}$$

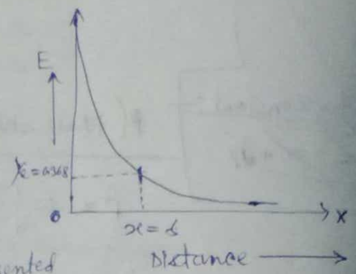
Skin depth (Depth of Penetration) :-

It may be defined as the depth in which the strength of electric field associated with the electromagnetic wave reduces to 1/e times to its ~~ea~~ Initial Value.

the amplitude of strength of electric field of an electromagnetic wave decreases by a factor

$e^{-\alpha x}$

$[\alpha = \text{attenuation constant}]$



→ If depth of penetration is represented by δ, then

$x = \delta$

will satisfy the equation

$\alpha x = 1$

or, $\alpha\delta = 1$

$\Rightarrow \delta = \frac{1}{\alpha}$ —— (i)

thus skin ~~effect~~ depth or depth of penetration $\boxed{\delta = \frac{1}{\alpha}}$

Hence, the reciprocal of attenuation constant is called skin depth or depth of penetration.

the attention constant is given by,

$$\alpha = \omega\left[\frac{\mu\epsilon}{2}\left(\sqrt{\left(1+\frac{\sigma^2}{\omega^2\epsilon^2}\right)}-1\right)\right]^{1/2}$$

$$\therefore \text{skin depth } (\delta) = \frac{1}{\omega\left[\frac{\mu\epsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\epsilon^2\omega^2}}-1\right)\right]^{1/2}}$$

(i) for good conductor :—

$$\text{skin depth } (\delta) = \frac{1}{\alpha}$$

$$= \sqrt{\frac{2}{\omega\mu\sigma}} \qquad \boxed{\delta = \sqrt{\frac{1}{\pi f\mu\sigma}}}$$

~~for poor conductors or good dielectric or ins~~

(ii) for poor conductors or Good dielectric or insulator :—

$$\text{Strain depth } (\delta) = \frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$$

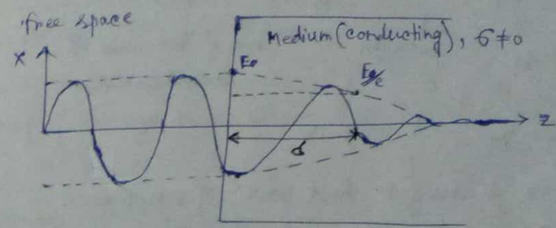
Skin effect :-

When signal passing through any conducting medium, it has loss as it passes through medium.

$$E = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\,\hat{a}_x$$

E_0 → Amplitude

$[z = \text{direction of propagation}]$



→ When amplitude at signal decreases to $(1/e)^{th}$ of it's initial amplitude at ~~medium~~ certain depth δ. that depth is reflected as skin depth.

$$\frac{E_0}{e} = E_0 \cdot e^{-\alpha\delta}$$

$$\Rightarrow e = e^{\alpha\delta}$$

$$\boxed{\Rightarrow \delta = 1/\alpha}$$

→ α for loss medium $(\alpha) = \sqrt{\frac{\mu\sigma\omega}{2}}$

$$\boxed{\therefore \delta = \sqrt{\frac{2}{\mu\sigma\omega}}}$$

Poyenting Vector :—

The cross product $\vec{E}\times\vec{H}$ is known as poyenting vector. $(\vec{P})$

$$\boxed{\vec{P} = \vec{H}\times\vec{E}}$$

→ unit = watt/m²

→ the rate of energy flow through a unit area perpendicular to the direction of electromagnetic wave propagation is known as poyenting vector.

If du energy passes through dA in time interval dt then

$$S = \frac{du}{dA\,dt} \qquad \text{S.I unit} \rightarrow \frac{J}{s\,m^2} = \frac{W}{m^2}$$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$\boxed{\text{poyenting vector } (\vec{P}) = \frac{\text{Energy of power transformed}}{\text{Area}}}$$

App → antena, communication system uses.

mathematically expression :-

$$\oint_S (\vec{E} \times \vec{H}) \cdot \vec{ds} = -\int_V (\vec{E} \cdot \vec{J})\, dv - \frac{d}{\delta t} \int_V \left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2} \right) dV$$

↓ rate of energy flow ↓ power loss ↓ rate of decrease of stored energy

Poyenting Theorem :-

→ poynting theorem is used to find power of EM wave.

→ poynting theorem explains power density of EM wave.

→ power density $\vec{P} = \vec{E} \times \vec{H}$

$$\Rightarrow \vec{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z + \theta_1)\hat{a}_x$$

$$\Rightarrow \vec{H} = H_0 e^{-\alpha z} \cos(\omega t - \beta z + \theta_2)\hat{a}_y$$

$$\therefore \vec{P} = E_0 . e^{-\alpha z} \cos(\omega t - \beta z + \theta_1)\hat{a}_x \times H_0 e^{-\alpha z} \cos(\omega t - \beta z + \theta_2)\hat{a}_y$$

$$\Rightarrow \vec{P} = E_0 H_0 e^{-2\alpha z} \cos(\omega t - \beta z + \theta_1) \cos(\omega t - \beta z + \theta_2)\hat{a}_z$$

$$\Rightarrow \vec{P} = \frac{E_0 H_0 e^{-2\alpha z}}{2} \left[\cos(2\omega t - 2\beta z + \theta_1 + \theta_2) + \cos(\theta_1 - \theta_2) \right] \hat{a}_z$$

$[2\cos A \cos B = \cos(A+B) + \cos(A-B)]$

→ Average Power density $\{$ $P_{avg} = \frac{1}{T} \int_0^T P\,dt$

$$= \frac{1}{T} \int_0^T \frac{E_0 H_0 e^{-2\alpha z}}{2} \left[\cos(\omega t - 2\beta z + \theta_1 + \theta_2) + \cos(\theta_1 - \theta_2) \right] dt$$

$$= \frac{1}{T} \left[\frac{E_0 H_0 e^{-2\alpha z}}{2} \right] \left[0 + \cos(\theta_1 - \theta_2) \right] \times [T]$$

$$= \frac{E_0 H_0 e^{-2\alpha z}}{2} \cos(\theta_1 - \theta_2)$$

→ Intrinsic Impedance $(\eta) = \frac{E \angle \theta_1}{H \angle \theta_2} = \frac{E_0}{H_0} \angle \theta_1 - \theta_2$

$$= \frac{E_0}{H_0} \angle \theta_n$$

$$P_{avg} = \frac{E_0 H_0}{2} . e^{-2\alpha z} . \cos\theta_n$$

$$\rightarrow \eta = \frac{E_0}{H_0}$$

$$\Rightarrow H_0 = \frac{E_0}{\eta}$$

$$P_{avg} = \frac{E_0^2}{2\eta} . e^{-2\alpha z} \cos\theta_n$$

→ for lossless medium $\alpha = 0$, $\theta_n = 0$

$$\boxed{P_{avg} \text{ (average power density)} = \frac{E_0^2}{2\eta}}$$

→ unit of average power density $(P_{avg}) = \left[\frac{V}{m}\right]\left[\frac{A}{m}\right]$

$$\left[= \frac{W}{m^2} \right]$$

→ Average power (P_{avg})

$$\boxed{P = P_{avg} \times S}$$

[P = avg = average power density
S = Surface area]

⊛ power :-

$$\text{power} = \frac{\text{energy}}{\text{time}} = \frac{\text{Joule}}{\text{sec}} = \text{watt}$$

⊛ power density :-

$$P = E \times H$$

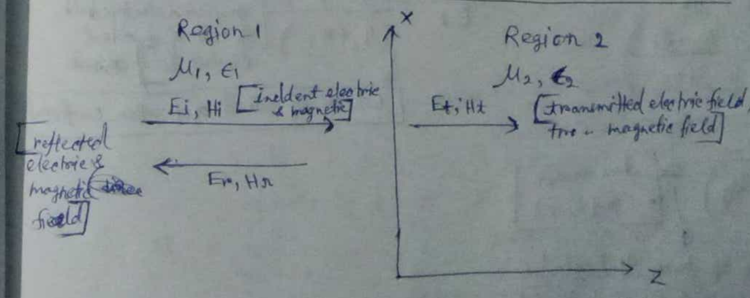
$$= \frac{\text{Volt}}{m} \times \frac{\text{Amp}}{m}$$

$$= \frac{\text{watt}}{m^2}$$

E = electric field intensity

H = magnetic field intensity

⊛ Reflection of plane wave at Normal Incidence :-



E_i = incident electric field

H_i = incident magnetic field

$E_i = E_{i0} e^{-jk_1 z}$ [direction $\hat{a} = +\hat{a}_z$ so equation is $+ve$]

$H_i = H_{i0} e^{-jk_1 z}$

$$\eta = \frac{E}{H}$$

$$\Rightarrow H = \frac{E}{\eta}$$

μ_1 = permeability

ϵ_1 = permitivity

direction (Negative) = sign is (+)

$E_t = E_{to} e^{-jk_2 z}$

$H_t = H_{to} e^{-jk_2 z}$

$H_t = \frac{E_{to}}{\eta_2} e^{-jk_2 z}$

$H_i = \frac{E_{io}}{\eta_1} e^{-jk_1 z}$ [$(-ve)$ that means there direction is positive]

from boundary condition

$H_{tan1} = H_{tan2}$

$H_i = H_t$

$\frac{E_{io}}{\eta_1} e^{-jk_1 z} = \frac{E_{to}}{\eta_2} e^{-jk_2 z}$

$E_r = E_{ro} e^{jk_1 z}$

$H_r = -H_{ro} e^{jk_1 z}$

$H_r = \frac{-E_{ro}}{\eta_1} \cdot e^{jk_1 z}$

At $z = 0$.

$E_i + E_r = E_t$

$H_i + H_r = H_t$

$\frac{E_{io}}{\eta_1} + \left(-\frac{E_{ro}}{\eta_1}\right) = \frac{E_{to}}{\eta_2}$

$\Rightarrow \frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}$

and $\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2} = \frac{E_i + E_r}{\eta_2}$

$\Rightarrow \frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_i}{\eta_2} + \frac{E_r}{\eta_2}$

$\Rightarrow \frac{E_i}{\eta_1} - \frac{E_i}{\eta_2} = \frac{E_r}{\eta_2} + \frac{E_r}{\eta_1}$

$\Rightarrow E_i\left(\frac{1}{\eta_1} - \frac{1}{\eta_2}\right) = E_r\left(\frac{1}{\eta_2} + \frac{1}{\eta_1}\right)$

$\Rightarrow E_i\left(\frac{\eta_2 - \eta_1}{\eta_1 \eta_2}\right) = E_r\left(\frac{\eta_1 + \eta_2}{\eta_2 \eta_1}\right)$

$\Rightarrow E_i(\eta_2 - \eta_1) = E_r(\eta_1 + \eta_2)$

$\Rightarrow \frac{E_r}{E_i} = \frac{(\eta_2 - \eta_1)}{(\eta_1 + \eta_2)}$ [It is the ratio of reflected electric field to that of incident electric field]

[which is called reflection coefficient]

$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2}$

$\Rightarrow \frac{E_i}{\eta_1} - \left(\frac{E_t - E_i}{\eta_1}\right) = \frac{E_t}{\eta_2}$ [$E_t = E_i + E_r$]

$\Rightarrow \frac{E_i}{\eta_1} - \frac{E_t}{\eta_1} + \frac{E_i}{\eta_1} = \frac{E_t}{\eta_2}$

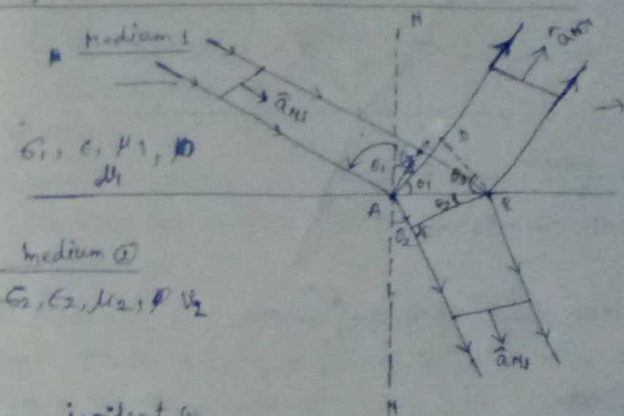
$\Rightarrow \frac{E_i}{\eta_1} + \frac{E_i}{\eta_1} = \frac{E_t}{\eta_1} + \frac{E_t}{\eta_2}$

$\Rightarrow \frac{2E_i}{\eta_1} = E_t\left(\frac{\eta_2 + \eta_1}{\eta_1 \eta_2}\right)$

$\boxed{\Rightarrow \frac{E_t}{E_i} = \frac{2\eta_2}{(\eta_2 + \eta_1)}}$ [transmission coefficient]

[It is the ratio of transmitted electric field to that of incident electric field]

Reflection of a plane wave at oblique Incidence :-



NN = Normal plane

AC = incident ω

BD = reflected

BE = transmitted

$t = \frac{CB}{v_1} = \frac{AD}{v_1} = \frac{AE}{v_2}$ —— ①

$\frac{CB}{v_1} = \frac{AD}{v_1}$

$\boxed{\Rightarrow CB = AD}$

$\triangle ACD$

$\angle CAB = \theta_1$

$CB = AB \sin\theta_1$

$AD = AB \sin\theta_3$

$\therefore CB = AD$

$\Rightarrow AB\sin\theta_1 = AB\sin\theta_3$

$\boxed{\Rightarrow \therefore \theta_1 = \theta_3}$ → angle of reflection

angle of incidence

$\Rightarrow$ [angle of Incidence = angle of reflection]

$1 \cdot \frac{CB}{v_1} = \frac{AE}{v_2}$

$AE = AB\sin\theta_2$

$CB = AB\sin\theta_1$

$\therefore \frac{AB\sin\theta_1}{v_1} = \frac{AB\sin\theta_2}{v_2}$

$\boxed{\Rightarrow \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}}$

law of sines / snell's law

v_1 = Velocity of propagation medium 1

v_2 = Velocity of propagation medium 2

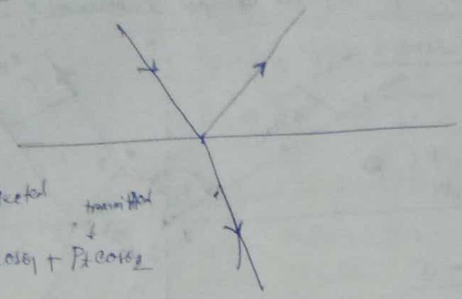
$\therefore v_1 = \frac{1}{\sqrt{\mu_0 \epsilon_1}}$

$v_2 = \frac{1}{\sqrt{\mu_0 \epsilon_2}}$

$\therefore \frac{\sin\theta_1}{\sin\theta_2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$ [$\epsilon = \epsilon_0 \epsilon_r$]

$\boxed{\Rightarrow \frac{\sin\theta_1}{\sin\theta_2} = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}}}$

power density :—



Incidence ↓ reflected ↓ transmitted ↓

$P_i \cos\theta_1 = P_r \cos\theta_1 + P_t \cos\theta_2$

$$\frac{E_i^2}{\eta_1}\cos\theta_1 = \frac{E_r^2}{\eta_1}\cdot\cos\theta_1 + \frac{E_t^2}{\eta_2}\cos\theta_2$$

$$\left(\frac{E_r}{E_i}\right)^2 = 1-\left(\frac{E_t}{E_i}\right)^2 \frac{\eta_1\cos\theta_2}{\eta_2\cos\theta_1}$$

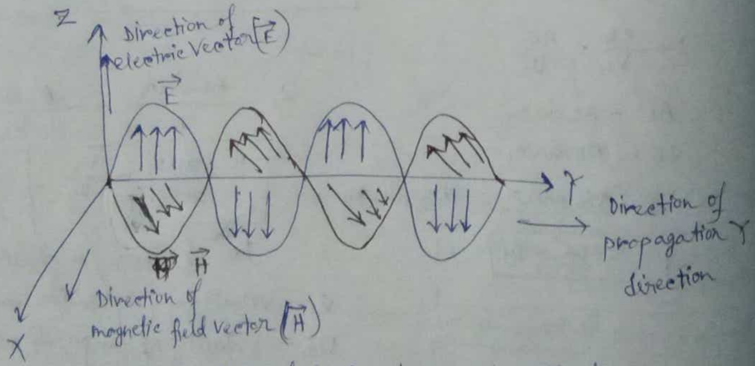
$$\boxed{\gamma_R^2 = 1-\{\gamma_T\}^2 \frac{\eta_1\cos\theta_2}{\eta_2\cos\theta_1}}$$ relation b/w reflection coefficient and transient coefficient.

⊛ polarisation :—

polarization in form of electric vector E.

→ polarization of plane wave : It show direction electric Vector E align during the pass at least one full cycle.

→ Electric vector E and magnetic Vector H mutually perpendicular to each other.



→ Wave is ~~said~~ Vertical polarized field vector E is vertical.

→ Wave is horizontal polarized electric field Vector E is horizontal.

uniform plane wave :—

$$\left.\begin{aligned}\vec{E}\times\vec{H} &= \vec{P}\\ \vec{a}_E \times \vec{a}_H &= \vec{a}_K\\ \vec{a}_x\times\vec{a}_y &= \vec{a}_z\end{aligned}\right\}$$ orthogonal relationship (always true)

polarization of wave :—

Above relation may change as a function of time & position depend on how the wave was generated or on what type of medium it is propagating through.

complete description of EM wave not only include parameters (wavelength, phase velocity & Power), but also the instantaneous orientation of its field vectors.

Wave Polarization :—

the electric field vector orientation as a function of time, at a fixed point in space.

[only the electric field direction is sufficient, since magnetic field is readily found from $\vec{E}$ using Maxwell's equation]

Definition :—

Polarization of a wave is defined as the direction of the electric field at a given point as a function of time.

polarization
- Linear polarization
  - → Horizontal Polarization
  - → Vertical Polarization
  - → thetha Polarization
- circular polarization
- Elliptical polarization

• Linear Polarization :—

$\vec{E}$ remain along a straight line as a function of time at some point in the medium.

→ When wave travels in Z- direction both $\vec{E}$s & $\vec{H}$s lying in x-y plane

i) $E_{ys} = 0$ & E_{xs} = present [x- polarized or horizontal Polarised]

ii) $E_{xs} = 0$ & E_{ys} = present [y-polarised or Vertical polarised]

iii) E_{xs} & E_{ys} present & in phase [θ-~~plan~~ polarized]

$$\boxed{\theta = \tan^{-1}\frac{E_y}{E_x}}$$

Circular Polarization :-

$\vec{E}$ traces a circle

→ E_{xs} & E_{ys} have equal magnitude & a 90° phase difference, the locus of the resultant E_s is a circle & the wave is a circle & wave is circularly polarised.

$$\boxed{E_{xs}^2 + E_{ys}^2 = E_K^2}$$

Elliptical Polarization :-

→ E_{xs} & E_{ys} are not equal in magnitude & they differ by 90° phase, then the tip of the resultant electric vector traces an ellipse. The wave is said to be elliptically polarised.

$$\boxed{\frac{E_{xs}^2}{a^2} + \frac{E_{ys}^2}{b^2} = 1}$$

a & b are constant

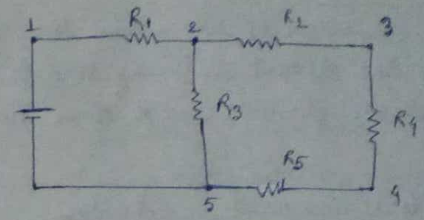
⟶ equation of ellipse.

unit-7

Transmission Line

Lumped parameter :-

→ Here no of definite number of element [R, L, C]



→ the physical size of the parameter is negligibly small when compared to the wavelength of the electromagnetic wave propagation.

example :- Resistor, Inductor, capacitor etc.

→ KVL and KCL apply this type circuit.

→ Here all elements parameters value determined is possible.

Distributed Parameter :-

→ Here very large no of elements (R, L, C) are present.

→ transmission line is an example of Distributed parameter

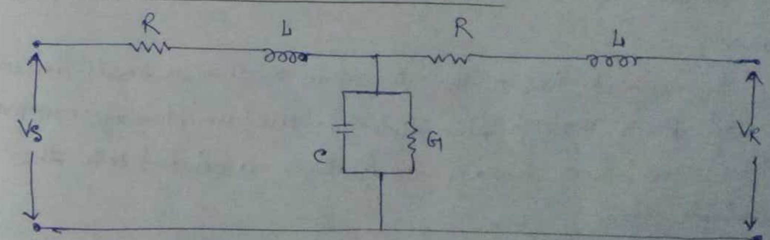
→ KVL and KCL are not applicable this parameter.

transmission line Parameter :-

4 parameters effect performance of a transmission line -

1. L (Inductance) - H/unit length
2. R (Resistance) - Ω/unit length
3. C (capacitance) - F/unit length
4. G (conductance) ℧/unit length

equivalent circuit of transmission line :-



the parameters R, L, C and G are uniformly distributed along the transmission line.

Distributed Parameter :-

Series Impedance : $\boxed{Z = R + j\omega L}$

Shunt Admittance :-

$$\boxed{Y = G + j\omega c}$$

Resistance :-

Each conductor of transmission line has a certain length & diameter
→ When the current is flowing through the conductor, it must have resistance per unit length of the line.

$$R = \frac{\rho l}{a}$$

→ It is measured in ohms/Km

Inductance :-

When the current flows through the transmission line, it induces the magnetic flux.

→ When the current changes, the magnetic flux also varies due to which emf induced in the line

$$L = \frac{N\phi}{I} = \frac{\psi}{I} \qquad \phi = \text{flux line}$$

→ emf produced in the transmission line resists the flow of current.

→ It is measured as the inductance of the line (Henry/Km)

Capacitance :-

In the transmission line, air acts as a dielectric medium b/w the conductors.

→ It produces the capacitive effect.

→ Due to this there exists a capacitance associated with the transmission line.

$$c = \frac{\epsilon A}{d} = \frac{q}{V}$$

→ It is denoted as 'c' and measured in farad/Km.

Conductance :-

→ Due to this imperfections of the dielectric medium, a small amount of current flows through the dielectric medium. [leakage current]

→ this gives rise to a leakage conductance associated with the transmission line.

→ It is denoted by 'G'

→ measured in mho/km.

Primary constant :-

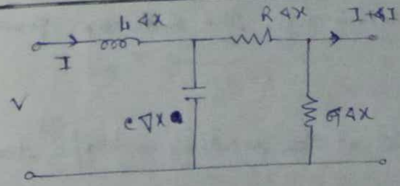
Resistance (R), Inductance (L), capacitance (c), conductance (G) are the primary constants of the transmission line.

→ All those constants are assumed to be independent of frequency.

Secondary constant :-

→ characteristics Impedance (Z_0)
→ propagation constant (γ)
→ ~~Attention~~ Attenuation constant - α
→ phase shift constant - β
→ All this constant are fixed at one particular frequency.
→ these parameters change their values as the frequency changes.

Transmission line equation :-



$$\Delta V \quad (R\Delta x + j\omega L\Delta x) I$$

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = -(R + j\omega L) I$$

$$\Rightarrow \frac{dV}{dx} = -(R + j\omega L) I \quad \text{—— (1)}$$

After differentiation

$$\frac{d^2V}{dx^2} = -(R + j\omega L)\frac{dI}{dx}$$

$$\Delta I = -(G\Delta x + j\omega c\Delta x) V$$

$$\lim_{\Delta x \to 0} \frac{\Delta I}{\Delta x} = -(G + j\omega c) V$$

$$\frac{dI}{dx} = -(G + j\omega c) V \quad \text{—— (2)}$$

$$\Rightarrow \frac{d^2V}{dx^2} = -(R + j\omega L)[-(G + j\omega c)V]$$

$$= (R + j\omega L)(G + j\omega c) V$$

$$\boxed{\Rightarrow \frac{d^2V}{dx^2} = \gamma^2 V}$$ ← transmission line equation.

Solution :-

$$\frac{d^2V}{dx^2} = \gamma^2 V$$

$$\Rightarrow V(x,t) = (V^+ e^{-\gamma x} + V^- e^{+\gamma x}) e^{j\omega t} \quad [\gamma = \text{propagation constant}]$$

$$\gamma = \alpha + j\beta$$

α = attenuation constant

β = phase shift constant

for lossless medium ($\alpha = 0$)

$$V(x,t) = V^+ e^{-(\alpha + j\beta)x} e^{j\omega t} + V^- e^{-(\alpha + j\beta)x} e^{j\omega t}$$

$$= V^+ e^{-j\beta x} e^{j\omega t} + V^- e^{+j\beta x} e^{j\omega t}$$

$$= V^+ e^{j(\beta x - \omega t)} + V^- e^{j(\beta x + \omega t)}$$

$$V(x,t) = \underbrace{V^+ \cos(\omega t - \beta x)}_{\text{this waves travelling +ve direction}} + \underbrace{V^- \cos(\omega t + \beta x)}_{\text{this waves travelling -ve direction}}$$

$$\vec{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\hat{x}$$

$$\vec{P} = E_0 \cos(\omega t - \beta z)\hat{x} \; [\text{lossless medium}]$$

→ one voltage wave travelling in forward and another wave travelling in backward.

→ current and voltage travel in the form of wave along the length of the transmission line.

Physical significance of transmission line equation :—

transmission line equation :—

$$V = V_R \cosh\sqrt{zy}\,x + I_R Z_0 \sinh\sqrt{zy}\,x$$

$$I = I_R \cosh\sqrt{zy}\,x + \frac{V_R}{Z_0}\sinh\sqrt{zy}\,x$$

these are the equations for voltage and current of a ~~transmission~~ transmission line at any distance 'x' from the receiving end of the transmission line.

- transmission line equations are useful to find :—
  i) V_s & I_s (ii) Input Impedance (Z_s)
  iii) the length of the line which gives the maximum power transfer.

V_s & I_s :—

V_s the equation for voltage & current at the sending end of a transmission line of length 'l' are given by,

$$V = V_R \cosh\sqrt{zy}\,x + I_R Z_0 \sinh\sqrt{zy}\,x$$

If, $x = l$, then $V = V_s$, $\gamma = \sqrt{zy}$

$$V_s = V_R \cosh\gamma l + I_R Z_0 \sinh\gamma l$$

$$= V_R \cosh\gamma l + \frac{V_R}{Z_R} Z_0 \sinh\gamma l$$

$$\boxed{V_s = V_R\left[\cosh\gamma l + \frac{Z_0}{Z_R}\sinh\gamma l\right]} \quad \text{—— (i)}$$

I_s :— current at the sending end.

$$I = I_R \cosh\sqrt{zy}\,x + \frac{V_R}{Z_0}\sinh\sqrt{zy}\,x$$

If $x = l$, $I = I_s$, $\sqrt{zy} = \gamma$

$$I_s = I_R \cosh\gamma l + \frac{V_R}{Z_0}\sinh\gamma x$$

$$= I_R \cosh\gamma l + \frac{I_R Z_R}{Z_0}\sinh\gamma x$$

$$\boxed{I_s = I_R\left[\cosh\gamma l + \frac{Z_R}{Z_0}\sinh\gamma l\right]} \quad \text{—— (ii)}$$

If the line is terminated with its characteristics impedance ($Z_R = Z_0$), then

$$I_s = I_R\left[\cosh\gamma l + \sinh\gamma l\right]$$

$$\Rightarrow \boxed{I_s = I_R \cdot e^{\gamma l}} \qquad \boxed{\because \cosh x + \sinh x = e^{x}}$$

I/P Impedance :—

the input impedance of the transmission line is defined as,

$$\boxed{Z_s = \frac{V_s}{I_s}}$$

(i) sub equation (i) & (ii)

$$Z_s = \frac{V_R\left[\cosh\gamma l + \frac{Z_0}{Z_R}\sinh\gamma l\right]}{I_R\left[\cosh\gamma l + \frac{Z_R}{Z_0}\sinh\gamma l\right]}$$

$$= \frac{I_R Z_R\left[\cosh\gamma l + \frac{Z_0}{Z_R}\sinh\gamma l\right]}{I_R\left[\cosh\gamma l + \frac{Z_R}{Z_0}\sinh\gamma l\right]}$$

$$Z_s = \frac{Z_R \cosh\gamma l + Z_0 \sinh\gamma l}{\cosh\gamma l + \frac{Z_R}{Z_0}\sinh\gamma l}$$

$$\Rightarrow Z_s = Z_0\left[\frac{Z_R \cosh\gamma l + Z_0 \sinh\gamma l}{Z_0 \cosh\gamma l + Z_R \sinh\gamma l}\right]$$

$$= Z_0 \cdot \frac{\cosh\gamma l}{\cosh\gamma l}\left[\frac{Z_R + Z_0 \frac{\sinh\gamma l}{\cosh\gamma l}}{Z_0 + Z_R \frac{\sinh\gamma l}{\cosh\gamma l}}\right]$$

$$\boxed{\Rightarrow Z_s = Z_0\left[\frac{Z_R + Z_0 \tanh\gamma l}{Z_0 + Z_R \tanh\gamma l}\right]}$$

If the transmission line is terminated with its characteristic impedance $\boxed{Z_R = Z_0}$, then the input impedance is $\boxed{Z_s = Z_0}$

(ii) Another form of i/p impedance :—

V_s :
$$V_s = \frac{V_R}{2}\left[\left(1 + \frac{Z_0}{Z_R}\right)e^{\gamma l} + \left(1 - \frac{Z_0}{Z_R}\right)e^{-\gamma l}\right]$$

$$= \frac{V_R}{2}\left[\frac{Z_R + Z_0}{Z_R}\cdot e^{\gamma l} + \frac{Z_R - Z_0}{Z_R}\cdot e^{-\gamma l}\right]$$

$$V_s = \frac{V_R}{2}\left(\frac{Z_R + Z_0}{Z_R}\right)\left[e^{\gamma l} + \frac{Z_R - Z_0}{\cancel{Z_R}} \times \frac{\cancel{Z_R}}{Z_R + Z_0}\cdot e^{-\gamma l}\right]$$

$$\Rightarrow V_s = \frac{V_R}{2}\left(\frac{Z_R + Z_0}{Z_R}\right)\left[e^{\gamma l} + \frac{Z_R - Z_0}{Z_R + Z_0}\cdot e^{-\gamma l}\right] \quad \text{—— (3)}$$

Is :-

$$I_S = \frac{I_R}{2}\left[\left(1+\frac{Z_R}{Z_0}\right)e^{\gamma l} + \left(1-\frac{Z_R}{Z_0}\right)e^{-\gamma l}\right]$$

$$= \frac{I_R}{2}\left[\left(\frac{Z_0+Z_R}{Z_0}\right)e^{\gamma l} + \left(\frac{Z_0-Z_R}{Z_0}\right)\cdot e^{-\gamma l}\right]$$

$$= \frac{I_R}{2}\left(\frac{Z_0+Z_R}{Z_0}\right)\left[e^{\gamma l} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\gamma l}\right]$$

the i/p impedance of the transmission line is,

$$Z_S = \frac{V_S}{I_S} = \frac{\frac{V_R}{2}\left(\frac{Z_R+Z_0}{Z_R}\right)\left[e^{\gamma l} + \frac{Z_R-Z_0}{Z_R+Z_0}e^{-\gamma l}\right]}{\frac{I_R}{2}\left(\frac{Z_R+Z_0}{Z_0}\right)\left[e^{\gamma l} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)\cdot e^{-\gamma l}\right]}$$

$$= \frac{I_R Z_R \times Z_0}{I_R \times Z_R}\left[\frac{e^{\gamma l} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\gamma l}}{e^{\gamma l} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\gamma l}}\right]$$

$$\boxed{\Rightarrow Z_S = Z_0\left[\frac{e^{\gamma l} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)\cdot e^{-\gamma l}}{e^{\gamma l} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\gamma l}}\right]}$$

If, $Z_R = Z_0$, then.

$$\boxed{Z_S = Z_0}$$

Transfer Impedance :-

Transfer impedance of a transmission line is defined as the ratio of voltage at the sending end to the current at the receiving end.

$$Z_T = \frac{\text{transmitted Voltage}}{\text{Received current}}$$

$$\boxed{Z_T = \frac{V_S}{I_R}}$$

→ Z_T is used to determine the current at the receiving end if the transmitted Voltage is known.

Derivation :- From the transmission line equation,

$$V_S = \frac{V_R}{2}\left[\frac{Z_R+Z_0}{Z_R}\right]\left[e^{\gamma l} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\gamma l}\right]$$

$$= \frac{I_R Z_R}{2}\left(\frac{Z_R+Z_0}{Z_R}\right)\left(e^{\gamma l} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)\cdot e^{-\gamma l}\right)$$

$$V_S = \frac{I_R}{2}\left[Z_R e^{\gamma l} + Z_0 e^{\gamma l} + Z_R e^{-\gamma l} - Z_0 e^{-\gamma l}\right]$$

$$\Rightarrow V_S = \frac{I_R}{2}\left[Z_R\left(e^{\gamma l} + e^{-\gamma l}\right) + Z_0\left(e^{\gamma l} - e^{-\gamma l}\right)\right]$$

$$\therefore Z_T = \frac{V_S}{I_R} = Z_R\left(\frac{e^{\gamma l} + e^{-\gamma l}}{2}\right) + Z_0\left(\frac{e^{\gamma l} - e^{-\gamma l}}{2}\right)$$

$$\boxed{\Rightarrow Z_T = Z_R \cosh \gamma l + Z_0 \sinh \gamma l}$$

⊛ Propagation constant (P) :-

→ It show the nature of Voltage and current vary with distance x.

$V = V_S \cosh Px - I_S Z_0 \sinh Px$ $\qquad P = \sqrt{ZY}$

$I = I_S \cosh Px - \frac{V_S}{Z_0} \sinh Px$ $\qquad P = \alpha + j\beta$

$P^2 = (R + j\omega L)(G + j\omega C)$

$\Rightarrow P = \sqrt{(R + j\omega L)(G + j\omega C)}$ per unit area.

→ propagation constant define as natural logarithm of vector ratio steady state Voltage and current entering and leaving in structure.

$(8)\ P = \log_e \frac{I_S}{I_R}$

$(8)\ P = \log_e \frac{V_S}{V_R}$

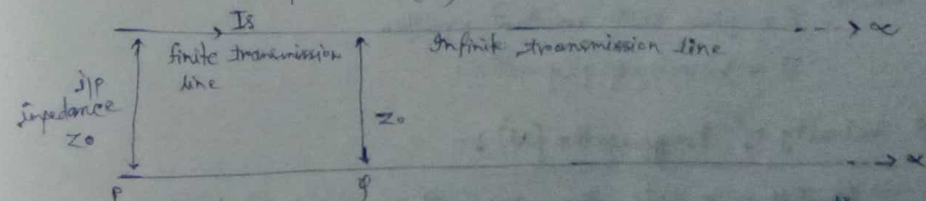
→ propagation constant multiple by the length of transmission line. It is termed as electrical length of line.

→ unit less / Dimensionless.

⊛ characteristics Impedance (Z_0) :-

It is steady state vector ratio of Voltage to current at input of infinite line.

Characteristics Impedance (Z_0) = Surge Impedance



$$-\frac{dv}{dx} = I(R + j\omega L)$$

$$\boxed{V = V_S \cdot e^{-Px}}$$

$$I = \frac{V_S \cdot e^{-Px}}{Z_0}$$

$$I = I_S \cdot e^{-Px}$$

$$-\frac{d}{dx}\left[V_s e^{-Px}\right] = I_s \cdot e^{-Px}(R + j\omega L)$$

$$\Rightarrow -V_s \cdot e^{-Px}(-P) = I_s \cdot e^{-Px}(R + j\omega L)$$

$$\Rightarrow +V_s e^{-Px} \cdot P = I_s \cdot e^{-Px}(R + j\omega L)$$

$$\Rightarrow \frac{V_s}{I_s} = \frac{R + j\omega L}{P}$$

$$= \frac{R + j\omega L}{\sqrt{(R + j\omega L)(G + j\omega C)}}$$

$$Z_0 \text{ (characteristics Impedance)} = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$$

$$\Rightarrow Z_0 = \frac{V_s}{I_s} = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$$

$$\Rightarrow Z_0 = \frac{V_s}{I_s} = \sqrt{\frac{Z}{Y}}$$

$R + j\omega L = Z$ series impedance
$G + j\omega C = Y$ shunt impedance

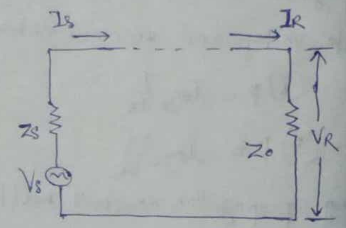
→ characteristics impedance is not depend on length.

⇒ depend = ~~to~~ terminated load.

Wavelength (λ) :-

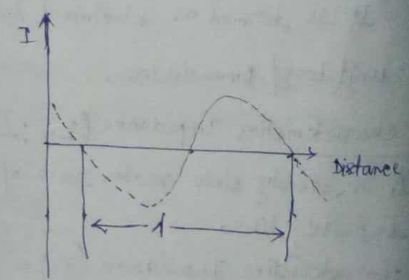
~~the distance~~

In a transmission line the wavelength is defined as the distance the wave travel along the line while the phase angle is changing to 2π radian.



$$\boxed{\lambda = \frac{2\pi}{\beta}}$$

$$\boxed{\lambda = \frac{v}{f}}$$



where, β = phase shift constant ; rad/Km

v = Velocity of propagation

⊛ Velocity of Propagation (v) :-

→ It represents the speed of light at which an electrical signal can propagate through the transmission line with respect to speed of light.

$$v = \lambda f$$

$$= \frac{2\pi}{\beta} f$$

$$= \frac{2\pi}{2\pi} \times \lambda f$$

$$\boxed{v = \lambda f}$$

$$\left[\beta = \frac{2\pi}{\lambda} ; \frac{1}{\beta} = \frac{\lambda}{2\pi} \quad \omega = 2\pi f\right]$$

for a perfect transmission line, $\beta = \omega\sqrt{LC}$

$$\Rightarrow v = \frac{\omega}{\omega\sqrt{LC}}$$

$$\boxed{\Rightarrow v = \frac{1}{\sqrt{LC}}}$$ → velocity of propagation for an ideal line.

→ It is measured m/sec.